

Chapter 10 Solutions

1. The wavelength of the peak is

$$\begin{aligned}\lambda_{pic} &= \frac{2.898 \times 10^{-3} mK}{T} \\ &= \frac{2.898 \times 10^{-3} mK}{3273 K} \\ &= 885 nm\end{aligned}$$

This corresponds to infrared radiation.

2. The temperature is found with

$$\begin{aligned}\lambda_{pic} &= \frac{2.898 \times 10^{-3} mK}{T} \\ 502 \times 10^{-9} m &= \frac{2.898 \times 10^{-3} mK}{T} \\ T &= 5773 K\end{aligned}$$

3. The power is

$$\begin{aligned}P &= \sigma A (T^4 - T_0^4) \\ &= 5.67 \times 10^{-8} \frac{W}{m^2 K^4} 4\pi (3.2 \times 10^{10} m)^2 ((6015 K)^4 - (0 K)^4) \\ &= 9.55 \times 10^{29} W\end{aligned}$$

This is about 2500 times brighter than the Sun.

4. a) The power is

$$\begin{aligned}P &= \sigma A (T^4 - T_0^4) \\ &= 5.67 \times 10^{-8} \frac{W}{m^2 K^4} \cdot 1.8 m^2 \cdot ((310 K)^4 - (293 K)^4) \\ &= 190.4 W\end{aligned}$$

(This is the same power as the power required to go biking with a little effort).

b) The power is

$$\begin{aligned} P &= \sigma A (T^4 - T_0^4) \\ &= 5.67 \times 10^{-8} \frac{W}{m^2 K^4} \cdot 1.8 m^2 \cdot \left((310 K)^4 - (243 K)^4 \right) \\ &= 586.7 W \end{aligned}$$

(This is equivalent to a really strenuous exercising. Go on a stationary bike that shows the power and try to achieve this power...)

5. The temperature will be found from the power with

$$P = \sigma A (T^4 - T_0^4)$$

To find it, we need the area of the filament.

The filament is a cylinder whose area is

$$\begin{aligned} A &= 2\pi r l \\ &= 2\pi \cdot 0.0005 m \cdot 0.1 m \\ &= 3.1416 \times 10^{-4} m^2 \end{aligned}$$

Therefore, the temperature is

$$\begin{aligned} P &= \sigma A (T^4 - T_0^4) \\ 60 W &= 5.67 \times 10^{-8} \frac{W}{m^2 K^4} \cdot 3.1416 \times 10^{-4} m^2 \cdot \left(T^4 - (293 K)^4 \right) \\ T &= 1355 K = 1082^\circ C \end{aligned}$$

6. The energy is

$$\begin{aligned} E_\gamma &= \frac{1240 e V nm}{\lambda} \\ &= \frac{1240 e V nm}{550 nm} \\ &= 2.25 eV \end{aligned}$$

7. The number of photons is given by

$$N = \frac{\text{Total energy emitted}}{\text{Energy of one photon}}$$

The energy of a photon is

$$\begin{aligned} E_{\gamma} &= \frac{1240 \text{ eVnm}}{\lambda} \\ &= \frac{1240 \text{ eVnm}}{632 \text{ nm}} \\ &= 1.962 \text{ eV} \\ &= 3.143 \times 10^{-19} \text{ J} \end{aligned}$$

The energy emitted per second is

$$\begin{aligned} E &= Pt \\ &= 0.001 \text{ W} \cdot 1 \text{ s} \\ &= 0.001 \text{ J} \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned} N &= \frac{\text{Total energy emitted}}{\text{Energy of one photon}} \\ &= \frac{0.001 \text{ J}}{3.143 \times 10^{-19} \frac{\text{J}}{\text{photons}}} \\ &= 3.182 \times 10^{15} \text{ photons} \end{aligned}$$

8. The number of photons is given by

$$N = \frac{\text{Total energy received}}{\text{Energy of one photon}}$$

The energy of a photon is

$$\begin{aligned}
 E_{\gamma} &= \frac{1240 \text{ eVnm}}{\lambda} \\
 &= \frac{1240 \text{ eVnm}}{585 \text{ nm}} \\
 &= 2.12 \text{ eV} \\
 &= 3.396 \times 10^{-19} \text{ J}
 \end{aligned}$$

The energy received in 20 seconds is

$$\begin{aligned}
 E &= IA_{\text{receiver}}t \\
 &= 50 \frac{\text{W}}{\text{m}^2} \cdot 3 \text{ m}^2 \cdot 20 \text{ s} \\
 &= 3000 \text{ J}
 \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned}
 N &= \frac{\text{Total energy received}}{\text{Energy of one photon}} \\
 &= \frac{3000 \text{ J}}{3.396 \times 10^{-19} \frac{\text{J}}{\text{photons}}} \\
 &= 8.835 \times 10^{21} \text{ photons}
 \end{aligned}$$

9. The number of photons is given by

$$N = \frac{\text{Total energy received}}{\text{Energy of one photon}}$$

The energy of a photon is

$$\begin{aligned}
 E_{\gamma} &= \frac{1240 \text{ eVnm}}{\lambda} \\
 &= \frac{1240 \text{ eVnm}}{470 \text{ nm}} \\
 &= 2.638 \text{ eV} \\
 &= 4.227 \times 10^{-19} \text{ J}
 \end{aligned}$$

The energy received per second is

$$\begin{aligned}
 E &= IA_{\text{receiver}} t \\
 &= 200 \frac{\text{W}}{\text{m}^2} \cdot \pi (0.0025\text{m})^2 \cdot 1\text{s} \\
 &= 0.003927\text{J}
 \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned}
 N &= \frac{\text{Total energy received}}{\text{Energy of one photon}} \\
 &= \frac{0.003927\text{J}}{4.227 \times 10^{-19} \frac{\text{J}}{\text{photons}}} \\
 &= 9.291 \times 10^{15} \text{photons}
 \end{aligned}$$

10. The maximal energy of the electrons is found with

$$E_{k\text{max}} = hf - \phi$$

The photon energy is

$$\begin{aligned}
 E_{\gamma} &= \frac{1240\text{eVnm}}{\lambda} \\
 &= \frac{1240\text{eVnm}}{150\text{nm}} \\
 &= 8.267\text{eV}
 \end{aligned}$$

The maximum energy of the ejected electrons is, therefore,

$$\begin{aligned}
 E_{k\text{max}} &= hf - \phi \\
 &= 8.267\text{eV} - 4.5\text{eV} \\
 &= 3.767\text{eV}
 \end{aligned}$$

11. The maximal energy of the electrons is found with

$$E_{k\text{max}} = hf - \phi$$

The work function of cesium is

$$\begin{aligned}
 \phi &= \frac{1240\text{eVnm}}{\lambda_0} \\
 &= \frac{1240\text{eVnm}}{686\text{nm}} \\
 &= 1.808\text{eV}
 \end{aligned}$$

a) With a wavelength of 690 nm, the energy of the photons is

$$\begin{aligned}
 E_\gamma &= \frac{1240\text{eVnm}}{\lambda} \\
 &= \frac{1240\text{eVnm}}{690\text{nm}} \\
 &= 1.797\text{eV}
 \end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned}
 E_{k\max} &= hf - \phi \\
 &= 1.797\text{eV} - 1.808\text{eV} \\
 &= -0.011\text{eV}
 \end{aligned}$$

This means that there are no electrons ejected since a negative kinetic energy is impossible. Photons don't have enough energy to eject electrons.

b) With a wavelength of 450 nm, the energy of the photons is

$$\begin{aligned}
 E_\gamma &= \frac{1240\text{eVnm}}{\lambda} \\
 &= \frac{1240\text{eVnm}}{450\text{nm}} \\
 &= 2.756\text{eV}
 \end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned}
 E_{k\max} &= hf - \phi \\
 &= 2.756\text{eV} - 1.808\text{eV} \\
 &= 0.948\text{eV}
 \end{aligned}$$

12. a) The threshold wavelength is

$$\phi = \frac{1240 \text{ eVnm}}{\lambda_0}$$

$$3.2 \text{ eV} = \frac{1240 \text{ eVnm}}{\lambda_0}$$

$$\lambda_0 = 387.5 \text{ nm}$$

- b) The maximal speed is found with the maximum energy of the electrons, which is found with

$$E_{k \max} = hf - \phi$$

With a wavelength of 250 nm, the energy of the photons is

$$E_\gamma = \frac{1240 \text{ eVnm}}{\lambda}$$

$$= \frac{1240 \text{ eVnm}}{250 \text{ nm}}$$

$$= 4.96 \text{ eV}$$

The energy of the ejected electrons is then

$$E_{k \max} = hf - \phi$$

$$= 4.96 \text{ eV} - 3.2 \text{ eV}$$

$$= 1.76 \text{ eV}$$

$$= 2.82 \times 10^{-19} \text{ J}$$

Therefore, the speed of the electrons is

$$E_{k \max} = \frac{1}{2} m v_{\max}^2$$

$$2.82 \times 10^{-19} \text{ J} = \frac{1}{2} \cdot 9.1094 \times 10^{-31} \text{ kg} \cdot v_{\max}^2$$

$$v_{\max} = 7.868 \times 10^5 \frac{\text{m}}{\text{s}}$$

- 13.** The threshold wavelength is found with the work function, and this work function is found with

$$E_{k \max} = hf - \phi$$

The maximum kinetic energy of the electrons is

$$\begin{aligned}
 E_{k\max} &= \frac{1}{2}mv_{\max}^2 \\
 &= \frac{1}{2} \cdot 9.1094 \times 10^{-31} \text{ kg} \cdot \left(5 \times 10^5 \frac{\text{m}}{\text{s}}\right)^2 \\
 &= 1.139 \times 10^{-19} \text{ J} \\
 &= 0.711 \text{ eV}
 \end{aligned}$$

The energy of the photons is

$$\begin{aligned}
 E_{\gamma} &= \frac{1240 \text{ eVnm}}{\lambda} \\
 &= \frac{1240 \text{ eVnm}}{400 \text{ nm}} \\
 &= 3.1 \text{ eV}
 \end{aligned}$$

The work function is then found with

$$\begin{aligned}
 E_{k\max} &= hf - \phi \\
 0.711 \text{ eV} &= 3.1 \text{ eV} - \phi \\
 \phi &= 2.389 \text{ eV}
 \end{aligned}$$

Therefore, the threshold wavelength is

$$\begin{aligned}
 \phi &= \frac{1240 \text{ eVnm}}{\lambda_0} \\
 2,389 \text{ eV} &= \frac{1240 \text{ eVnm}}{\lambda_0} \\
 \lambda_0 &= 519 \text{ nm}
 \end{aligned}$$

14. Since 3 % of the photons eject electrons, the number of ejected electrons is

$$N_{\text{electrons}} = 0.03 \cdot N_{\text{photons}}$$

The energy of a photon received is given by

$$N_{\text{photons}} = \frac{\text{Total energy}}{\text{Energy of one photon}}$$

$$\begin{aligned}
 E_{\gamma} &= \frac{1240 \text{ eVnm}}{\lambda} \\
 &= \frac{1240 \text{ eVnm}}{450 \text{ nm}} \\
 &= 2.756 \text{ eV} \\
 &= 4.414 \times 10^{-19} \text{ J}
 \end{aligned}$$

The energy received per second per square centimetre is

$$\begin{aligned}
 E &= IA_{\text{receiver}}t \\
 &= 40 \frac{\text{W}}{\text{m}^2} \cdot 0.0001 \text{ m}^2 \cdot 1 \text{ s} \\
 &= 0.004 \text{ J}
 \end{aligned}$$

Therefore, the number of photons received is

$$\begin{aligned}
 N &= \frac{\text{Total energy}}{\text{Energy of one photon}} \\
 &= \frac{0.004 \text{ J}}{4.414 \times 10^{-19} \frac{\text{J}}{\text{photons}}} \\
 &= 9.091 \times 10^{15} \text{ photons}
 \end{aligned}$$

If only 3% of the photons eject an electron, then the number of ejected electrons is

$$\begin{aligned}
 N_{\text{electrons}} &= 0.03 \cdot N_{\text{photons}} \\
 &= 0.03 \cdot 9.091 \times 10^{15} \\
 &= 2.718 \times 10^{15}
 \end{aligned}$$

15. a) The wavelength shift is

$$\begin{aligned}
 \Delta\lambda &= 2.4263 \times 10^{-3} \text{ nm} \cdot (1 - \cos \theta) \\
 &= 2.4263 \times 10^{-3} \text{ nm} \cdot (1 - \cos 45^\circ) \\
 &= 0.0007106 \text{ nm}
 \end{aligned}$$

b) The wavelength of the incident photon is

$$E_{\gamma} = \frac{1240 \text{ eVnm}}{\lambda}$$

$$62\,000 \text{ eV} = \frac{1240 \text{ eVnm}}{\lambda}$$

$$\lambda = 0.02 \text{ nm}$$

The new wavelength is thus

$$\lambda' = \lambda + \Delta\lambda$$

$$= 0.02 \text{ nm} + 0.0007106 \text{ nm}$$

$$= 0.0207106 \text{ nm}$$

c) The new energy of the photon is

$$E'_{\gamma} = \frac{1240 \text{ eVnm}}{\lambda'}$$

$$= \frac{1240 \text{ eVnm}}{0.0207106 \text{ nm}}$$

$$= 59\,873 \text{ eV}$$

d) The kinetic energy of the electron is

$$E_{\gamma} = E'_{\gamma} + E_{ke}$$

$$62\,000 \text{ eV} = 59\,873 \text{ eV} + E_{ke}$$

$$E_{ke} = 2127 \text{ eV}$$

e) The angle with the conservation of y-component of the momentum.

$$0 = p'_{\gamma} \sin \theta - p'_e \sin \phi$$

The momentum of the photon is found with

$$E'_{\gamma} = p'_{\gamma} c$$

$$59\,873 \cdot 1.602 \times 10^{-19} \text{ J} = p'_{\gamma} \cdot 3 \times 10^8 \frac{\text{m}}{\text{s}}$$

$$p'_{\gamma} = 3.197 \times 10^{-23} \frac{\text{kgm}}{\text{s}}$$

The momentum of the electron is found with

$$E_e = \frac{p^2}{2m}$$

$$2127 \cdot 1.602 \times 10^{-19} \text{ J} = \frac{p_e'^2}{2 \cdot 9.1094 \times 10^{-31} \text{ kg}}$$

$$p_e' = 2.491 \times 10^{-23} \frac{\text{kgm}}{\text{s}}$$

The conservation equation then becomes

$$0 = p_\gamma' \sin \theta - p_e' \sin \phi$$

$$0 = 3.197 \times 10^{-23} \frac{\text{kgm}}{\text{s}} \cdot \sin 45^\circ - 2.491 \times 10^{-23} \frac{\text{kgm}}{\text{s}} \cdot \sin \phi$$

$$0 = 3.197 \cdot \sin 45^\circ - 2.491 \cdot \sin \phi$$

$$\phi = 65.1^\circ$$

16. The diffusion angle is found with

$$\Delta\lambda = 2.4263 \times 10^{-3} \text{ nm} \cdot (1 - \cos \theta)$$

To find the angle, we need the wavelength shift. This shift is found with the wavelengths before and after the collision.

The initial wavelength is

$$E_\gamma = \frac{1240 \text{ eVnm}}{\lambda}$$

$$50\,000 \text{ eV} = \frac{1240 \text{ eVnm}}{\lambda}$$

$$\lambda = 0.0248 \text{ nm}$$

The wavelength after the scattering is

$$E_\gamma' = \frac{1240 \text{ eVnm}}{\lambda'}$$

$$49\,500 \text{ eV} = \frac{1240 \text{ eVnm}}{\lambda'}$$

$$\lambda' = 0.02505 \text{ nm}$$

So, the wavelength shift is

$$\begin{aligned}
 \Delta\lambda &= \lambda' - \lambda \\
 &= 0.02505\text{nm} - 0.0248\text{nm} \\
 &= 0.00025\text{nm}
 \end{aligned}$$

Therefore, the angle is

$$\begin{aligned}
 \Delta\lambda &= 2.4263 \times 10^{-3} \text{nm} \cdot (1 - \cos \theta) \\
 0.00025\text{nm} &= 2.4263 \times 10^{-3} \text{nm} \cdot (1 - \cos \theta) \\
 \theta &= 26.3^\circ
 \end{aligned}$$

17. The wavelength is

$$\begin{aligned}
 \lambda &= \frac{h}{p} \\
 &= \frac{h}{mv} \\
 &= \frac{6.626 \times 10^{-34} \text{Js}}{1.6726 \times 10^{-27} \text{kg} \cdot 10^4 \frac{\text{m}}{\text{s}}} \\
 &= 3.96 \times 10^{-11} \text{m} = 0.0396\text{nm}
 \end{aligned}$$

18. As the speed is close to the speed of light, the relativistic momentum formula must be used. The wavelength is, therefore,

$$\begin{aligned}
 \lambda &= \frac{h}{p} \\
 &= \frac{h}{\gamma mv} \\
 &= \frac{6.626 \times 10^{-34} \text{Js}}{\frac{1}{\sqrt{1 - \left(\frac{2 \times 10^8 \frac{\text{m}}{\text{s}}}{3 \times 10^8 \frac{\text{m}}{\text{s}}}\right)^2}} \cdot 1.6726 \times 10^{-27} \text{kg} \cdot 2 \times 10^8 \frac{\text{m}}{\text{s}}} \\
 &= 1.476 \times 10^{-15} \text{m}
 \end{aligned}$$

19. The wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

The speed will be found from the kinetic energy.

With a 10 eV kinetic energy, the speed of the electron is

$$\begin{aligned}
 E_k &= \frac{1}{2}mv^2 \\
 10 \cdot 1.602 \times 10^{-19} \text{ J} &= \frac{1}{2} \cdot 9.1094 \times 10^{-31} \text{ kg} \cdot v^2 \\
 v &= 1.875 \times 10^6 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

Thus, the wavelength is

$$\begin{aligned}
 \lambda &= \frac{h}{mv} \\
 &= \frac{6.626 \times 10^{-34} \text{ Js}}{9.1094 \times 10^{-31} \text{ kg} \cdot 1.875 \times 10^6 \frac{\text{m}}{\text{s}}} \\
 &= 3.879 \times 10^{-10} \text{ m} = 0.3879 \text{ nm}
 \end{aligned}$$

20. With a 6 eV kinetic energy, the speed of the electron is

$$\begin{aligned}
 E_k &= \frac{1}{2}mv^2 \\
 6 \cdot 1.602 \times 10^{-19} \text{ J} &= \frac{1}{2} \cdot 9.1094 \times 10^{-31} \text{ kg} \cdot v^2 \\
 v &= 1.453 \times 10^6 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

Thus, the wavelength is

$$\begin{aligned}
 \lambda &= \frac{h}{p} \\
 &= \frac{h}{mv} \\
 &= \frac{6.626 \times 10^{-34} \text{ Js}}{9.1094 \times 10^{-31} \text{ kg} \cdot 1.453 \times 10^6 \frac{\text{m}}{\text{s}}} \\
 &= 5.007 \times 10^{-10} \text{ m} = 0.5007 \text{ nm}
 \end{aligned}$$

When U increases to 2 eV, the kinetic energy decreases to 4 eV. The speed of the electron is then

$$E_k = \frac{1}{2}mv^2$$

$$4 \cdot 1.602 \times 10^{-19} J = \frac{1}{2} \cdot 9.1094 \times 10^{-31} kg \cdot v^2$$

$$v = 1.186 \times 10^6 \frac{m}{s}$$

And the wavelength is

$$\lambda = \frac{h}{p}$$

$$= \frac{h}{mv}$$

$$= \frac{6.626 \times 10^{-34} Js}{9.1094 \times 10^{-31} kg \cdot 1.186 \times 10^6 \frac{m}{s}}$$

$$= 6.132 \times 10^{-10} m = 0.6132 nm$$

The change in wavelength is, therefore,

$$\Delta\lambda = \lambda' - \lambda$$

$$= 0.6132 nm - 0.5007 nm$$

$$= 0.1125 nm$$

- 21.** The distance x is the distance between the order-2- maxima. The position of these maxima will be found with

$$d \sin \theta = m\lambda$$

We have d but not λ . We will find this wavelength with h/p .

With a 2 eV kinetic energy, the speed of the electron is

$$E_k = \frac{1}{2}mv^2$$

$$2 \cdot 1.602 \times 10^{-19} J = \frac{1}{2} \cdot 9.1094 \times 10^{-31} kg \cdot v^2$$

$$v = 8.3877 \times 10^5 \frac{m}{s}$$

The wavelength is

$$\begin{aligned}
 \lambda &= \frac{h}{p} \\
 &= \frac{h}{mv} \\
 &= \frac{6.626 \times 10^{-34} \text{ Js}}{9.1094 \times 10^{-31} \text{ kg} \cdot 8.3877 \times 10^5 \frac{\text{m}}{\text{s}}} \\
 &= 8.672 \times 10^{-10} \text{ m} = 0.8672 \text{ nm}
 \end{aligned}$$

Therefore, the angle of the order-2 maximum is

$$\begin{aligned}
 d \sin \theta &= m\lambda \\
 0.1 \times 10^{-6} \text{ m} \cdot \sin \theta &= 2 \cdot 8.672 \times 10^{-10} \text{ m} \\
 \theta &= 0.9938^\circ
 \end{aligned}$$

The distance from the central maximum to the order 2 maximum is, therefore,

$$\begin{aligned}
 \tan \theta &= \frac{y}{L} \\
 \tan(0.9938^\circ) &= \frac{y}{300 \text{ cm}} \\
 y &= 5.204 \text{ cm}
 \end{aligned}$$

The distance between the two order 2 maxima is twice as big. Therefore, it is 10.408 cm.

22. The uncertainty of the momentum is

$$\begin{aligned}
 \Delta p &= p_{\max} - p_{\min} \\
 &= 2.05 \times 10^{-23} \frac{\text{kgm}}{\text{s}} - 2 \times 10^{-23} \frac{\text{kgm}}{\text{s}} \\
 &= 5 \times 10^{-25} \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

Therefore, the uncertainty of the position is

$$\begin{aligned}
 \Delta x \Delta p &= h \\
 \Delta x \cdot 5 \times 10^{-25} \frac{\text{kgm}}{\text{s}} &= 6.626 \times 10^{-34} \text{ Js} \\
 \Delta x &= 1.325 \times 10^{-9} \text{ m} = 1.325 \text{ nm}
 \end{aligned}$$

23. The uncertainty of the energy is

$$\Delta E \Delta t = h$$

$$\Delta E \cdot 10^{-8} s = 6.626 \times 10^{-34} Js$$

$$\Delta E = 6.626 \times 10^{-26} J = 4.136 \times 10^{-7} eV$$