Chapter 9 Solutions

1. a) The heat transfer rate is

$$\begin{aligned} P &= k\frac{A}{\ell}\Delta T \\ &= 34.7\tfrac{W}{m°C}\cdot\frac{\pi\cdot(0.02m)^2}{2m}\cdot 100°C \\ &= 2.180W \end{aligned}$$

In 1 hour, the total energy is

$$\begin{aligned} E &= P\cdot\Delta t \\ &= 2{,}180W\cdot 3600s \\ &= 7849J \end{aligned}$$

b) Between the end on the left of the rod and the place 10 cm from this end, the length is 10 cm. As P is the same everywhere in the rod, we must have, for the small 10 cm part of the rod,

$$\begin{aligned} P &= k\frac{A}{\ell}\Delta T \\ 2.180W &= 34.7\tfrac{W}{m°C}\cdot\frac{\pi\cdot(0.02m)^2}{0.1m}\cdot\Delta T \\ \Delta T &= 5°C \end{aligned}$$

Since one end is at 100°C, a difference of 5°C means that the temperature at 10 cm from the end is 95°C.

2. We have

$$\begin{aligned} P &= k\frac{A}{\ell}\Delta T \\ 20W &= 427\tfrac{W}{m°C}\cdot\frac{(0.03m\cdot 0.03m)}{4m}\Delta T \\ \Delta T &= 208.2°C \end{aligned}$$

3. a)

Since the heat that passes through rod 1 must then pass through rod 2, the values of P must be identical for both rods.

For the 1st rod, we have

$$\begin{aligned} P &= k_1 \frac{A}{\ell} \Delta T \\ &= k_1 \frac{A}{\ell} \left(100°C - T_j\right) \end{aligned}$$

where T_j is the temperature at the junction of the rods. For rod 2, we have

$$\begin{aligned} P &= k_2 \frac{A}{\ell} \Delta T \\ &= k_2 \frac{A}{\ell} \left(T_j - 0°C\right) \end{aligned}$$

Since the P's are equal, we have

$$\begin{gathered} k_1 \frac{A}{\ell} \left(100°C - T_j\right) = k_2 \frac{A}{\ell} \left(T_j - 0°C\right) \\ k_1 \left(100°C - T_j\right) = k_2 \left(T_j - 0°C\right) \\ 34.7 \tfrac{W}{m°C} \cdot \left(100°C - T_j\right) = 79.5 \tfrac{W}{m°C} \cdot \left(T_j - 0°C\right) \\ 3470°C - 34.7 \cdot T_j = 79.5 \cdot T_j \\ 3470°C = 114.2 \cdot T_j \\ T_j = 30.39°C \end{gathered}$$

b)

The transfer rate can be found by looking at any of the 2 rods. Let's take rod 1.

$$\begin{aligned} P &= k_1 \frac{A}{\ell} \Delta T \\ &= 34.7 \tfrac{W}{m°C} \cdot \frac{\pi \cdot (0.02m)^2}{1m} \cdot \left(100°C - 30.39°C\right) \\ &= 3.036W \end{aligned}$$

4. Since the heat that passes through rod 1 must then pass through rod 2, the values of *P* must be identical for both rods.

For the 1st rod, we have

$$\begin{aligned} P &= k_1 \frac{A}{\ell} \Delta T \\ &= k_1 \frac{A}{\ell}\left(100°C - 40°C\right) \\ &= k_1 \frac{A}{\ell} \cdot 60°C \end{aligned}$$

where T_j is the temperature at the junction of the rods. For rod 2, we have

$$\begin{aligned} P &= k_2 \frac{A}{\ell} \Delta T \\ &= k_2 \frac{A}{\ell}\left(40°C - 0°C\right) \\ &= k_2 \frac{A}{\ell} \cdot 40°C \end{aligned}$$

Since the *P*'s are equal, we have

$$\begin{aligned} k_1 \frac{A}{\ell} \cdot 60°C &= k_2 \frac{A}{\ell} \cdot 40°C \\ k_1 \cdot 60°C &= k_2 \cdot 40°C \\ 100 \tfrac{W}{m°C} \cdot 60°C &= k_2 \cdot 40°C \\ k_2 &= 150 \tfrac{W}{m°C} \end{aligned}$$

5. a) We have

$$\begin{aligned} P &= k \frac{A}{\ell} \Delta T \\ &= 0.837 \tfrac{W}{m°C} \cdot \frac{\left(1.25m \cdot 0{,}5m\right)}{0.012m} \cdot 35°C \\ &= 1526W \end{aligned}$$

b) Since the power that passes through the 1st pane must then pass through the air space and then into the 2nd pane, the values of *P* must be the same for the windows and the air space.

For the 1st pane (the inside one), we have

$$\begin{aligned} P &= k_v \frac{A}{\ell} \Delta T \\ &= k_v \frac{A}{\ell} \left(15°C - T_{j1}\right) \end{aligned}$$

where T_{j1} is the temperature at the junction between the 1st pane and the air between the panes. For the air space, we have

$$\begin{aligned} P &= k_a \frac{A}{\ell} \Delta T \\ &= k_a \frac{A}{\ell} \left(T_{j1} - T_{j2}\right) \end{aligned}$$

where T_{j2} is the temperature at the junction between the air between the panes and the 2nd pane. For the 2nd pane, we have

$$\begin{aligned} P &= k_v \frac{A}{\ell} \Delta T \\ &= k_v \frac{A}{\ell} \left(T_{j2} - -20°C\right) \end{aligned}$$

Since the P's are equal, we have, for the 2 panes of glass,

$$\begin{aligned} k_v \frac{A}{\ell} \left(15°C - T_{j1}\right) &= k_v \frac{A}{\ell} \left(T_{j2} - -20°C\right) \\ 15°C - T_{j1} &= T_{j2} + 20°C \end{aligned}$$

and for the air and the 2nd pane

$$\begin{aligned} k_a \frac{A}{\ell} \left(T_{j1} - T_{j2}\right) &= k_v \frac{A}{\ell} \left(T_{j2} - -20°C\right) \\ k_a \left(T_{j1} - T_{j2}\right) &= k_v \left(T_{j2} + 20°C\right) \end{aligned}$$

Let's solve for T_{j1} in the first equation

$$\begin{aligned} 15°C - T_{j1} &= T_{j2} + 20°C \\ T_{j2} &= -T_{j1} - 5°C \end{aligned}$$

and use this value in the 2nd equation.

$$k_a\left(T_{j1}-T_{j2}\right)=k_v\left(T_{j2}+20°C\right)$$
$$k_a\left(T_{j1}-\left(-T_{j1}-5°C\right)\right)=k_v\left(-T_{j1}-5°C+20°C\right)$$
$$k_a\left(2T_{j1}+5°C\right)=k_v\left(-T_{j1}+15°C\right)$$
$$2k_aT_{j1}+k_a\cdot 5°C=-k_vT_{j1}+k_v\cdot 15°C$$
$$2k_aT_{j1}+k_vT_{j1}=k_v\cdot 15°C-k_a\cdot 5°C$$
$$\left(2k_a+k_v\right)T_{j1}=k_v\cdot 15°C-k_a\cdot 5°C$$
$$T_{j1}=\frac{k_v\cdot 15°C-k_a\cdot 5°C}{2k_a+k_v}$$

With the values, we obtain

$$\begin{aligned}T_{j1}&=\frac{k_v\cdot 15°C-k_a\cdot 5°C}{2k_a+k_v}\\&=\frac{0.837\,\frac{W}{m°C}\cdot 15°C-0.0234\,\frac{W}{m°C}\cdot 5°C}{2\cdot 0.0234\,\frac{W}{m°C}+0.837\,\frac{W}{m°C}}\\&=14.0733°C\end{aligned}$$

The power passing through the 1st pane is therefore

$$\begin{aligned}P&=k_v\frac{A}{\ell}\Delta T\\&=0.837\,\frac{W}{m°C}\cdot\frac{1.25m\cdot 0.5m}{0.006m}\left(15°C-14.0733°C\right)\\&=80.79W\end{aligned}$$

It’s a pretty good reduction compared to the 1526 W without the air layer!

6. The wavelength of the peak is

$$\begin{aligned}\lambda_{peak}&=\frac{2.898\times 10^{-3}mK}{T}\\&=\frac{2.898\times 10^{-3}mK}{3273K}\\&=885nm\end{aligned}$$

It’s in the infrared part of the spectrum.

7. We have

$$\lambda_{peak} = \frac{2.898\times 10^{-3} mK}{T}$$
$$502\times 10^{-9} m = \frac{2.898\times 10^{-3} mK}{T}$$
$$T = 5773K$$

8. The power is

$$\begin{aligned} P &= \varepsilon\sigma A T^4 \\ &= 1\cdot 5.67037\times 10^{-8} \tfrac{W}{m^2K^4}\cdot 4\pi\left(3.2\times 10^{10} m\right)^2\cdot\left(6015K\right)^4 \\ &= 9.551\times 10^{29} W \end{aligned}$$

This is about 2500 times brighter than the Sun.

9. a) The power is

$$\begin{aligned} P &= \varepsilon\sigma A\left(T^4 - T_0^4\right) \\ &= 0.98\cdot 5.67037\times 10^{-8} \tfrac{W}{m^2K^4}\cdot 1.8m^2\cdot\left(\left(310K\right)^4 - \left(293K\right)^4\right) \\ &= 186.6W \end{aligned}$$

(That's the same power as riding a bike with a small sustained effort.)

b) The power is

$$\begin{aligned} P &= \varepsilon\sigma A\left(T^4 - T_0^4\right) \\ &= 0.98\cdot 5.67037\times 10^{-8} \tfrac{W}{m^2K^4}\cdot 1.8m^2\cdot\left(\left(310K\right)^4 - \left(243K\right)^4\right) \\ &= 575.0W \end{aligned}$$

(This is equivalent to a very sustained effort. Go on a stationary bike that displays the power and try to reach that power...)

10. The temperature is found from the power with

$$P = \varepsilon\sigma A\left(T^4 - T_0^4\right)$$

To find it, we need to find the area of the filament.

The filament is a cylinder whose area is (neglecting the ends)

$$\begin{aligned} A &= 2\pi rl \\ &= 2\pi \cdot 0.0005m \cdot 0,1m \\ &= 3.1416 \times 10^{-4} m^2 \end{aligned}$$

Therefore

$$\begin{aligned} P &= \varepsilon\sigma A\left(T^4 - T_0^4\right) \\ 60W &= 0.20 \cdot 5.67037 \times 10^{-8} \tfrac{W}{m^2K^4} \cdot 3.1416 \times 10^{-4} m^2 \cdot \left(T^4 - \left(293K\right)^4\right) \\ T &= 2026K = 1752°C \end{aligned}$$

11. The equilibrium temperature of the Earth is given by

$$T_e = \sqrt[4]{\frac{Q\left(1-A\right)}{\sigma}}$$

where Q is given by

$$Q = \frac{I}{4}$$

and where I is the intensity of the radiation arriving from the Sun. This intensity is

$$I = \frac{P_{star}}{4\pi D^2}$$

When the Earth is at perihelion, the intensity of the radiation is

$$\begin{aligned} I &= \frac{P_{star}}{4\pi D^2} \\ &= \frac{3.828 \times 10^{26} W}{4\pi \cdot \left(1.471 \times 10^{11} m\right)^2} \\ &= 1407.79 \tfrac{W}{m^2} \end{aligned}$$

The value of Q is then

$$
\begin{aligned}
Q &= \frac{I}{4} \\
&= \frac{1407.79 \frac{W}{m^2}}{4} \\
&= 351.95 \frac{W}{m^2}
\end{aligned}
$$

and the equilibrium temperature is

$$
\begin{aligned}
T_e &= \sqrt[4]{\frac{Q(1-A)}{\sigma}} \\
&= \sqrt[4]{\frac{351.95 \frac{W}{m^2} \cdot (1-0.30)}{5.67037 \times 10^{-8} \frac{W}{m^2 K^4}}} \\
&= 256.74 K
\end{aligned}
$$

When the Earth is at aphelion, the intensity of the radiation is

$$
\begin{aligned}
I &= \frac{P_{star}}{4\pi D^2} \\
&= \frac{3.828 \times 10^{26} W}{4\pi \cdot \left(1.521 \times 10^{11} m\right)^2} \\
&= 1316.75 \frac{W}{m^2}
\end{aligned}
$$

The value of Q is then

$$
\begin{aligned}
Q &= \frac{I}{4} \\
&= \frac{1316.75 \frac{W}{m^2}}{4} \\
&= 329.19 \frac{W}{m^2}
\end{aligned}
$$

and the equilibrium temperature is

$$
\begin{aligned}
T_e &= \sqrt[4]{\frac{Q(1-A)}{\sigma}} \\
&= \sqrt[4]{\frac{329.19 \frac{W}{m^2} \cdot (1-0.30)}{5.67037 \times 10^{-8} \frac{W}{m^2 K^4}}} \\
&= 252.48 K
\end{aligned}
$$

The temperature difference is therefore

$$256.74\ \text{K} - 252.48\ \text{K} = 4.26\ \text{K}$$

12. The equilibrium temperature of the Earth is given by

$$T_e = \sqrt[4]{\frac{Q(1-A)}{\sigma}}$$

where Q is given by

$$Q = \frac{I}{4}$$

and where I is the intensity of the radiation arriving from the Sun. This intensity will be

$$\begin{aligned} I &= \frac{P_{star}}{4\pi D^2} \\ &= \frac{3.828\times10^{26}W\cdot1.1}{4\pi\cdot\left(1.496\times10^{11}m\right)^2} \\ &= 1497.24\tfrac{W}{m^2} \end{aligned}$$

The value of Q will then be

$$\begin{aligned} Q &= \frac{I}{4} \\ &= \frac{1497.24\tfrac{W}{m^2}}{4} \\ &= 374.31\tfrac{W}{m^2} \end{aligned}$$

and the equilibrium temperature will be

$$\begin{aligned} T_e &= \sqrt[4]{\frac{Q(1-A)}{\sigma}} \\ &= \sqrt[4]{\frac{374.31\tfrac{W}{m^2}\cdot(1-0.3)}{5.67037\times10^{-8}\tfrac{W}{m^2K^4}}} \\ &= 260.72K \end{aligned}$$

Since the equilibrium temperature is 254.58 K now, this corresponds to an increase of 6.14 °C.

13. The equilibrium temperature of the Earth is given by

$$T_e = \sqrt[4]{\frac{Q(1-A)}{\sigma}}$$

If we want the temperature to be 273.15 K, we must have

$$273.15K = \sqrt[4]{\frac{340.275\frac{W}{m^2}\cdot(1-A)}{5.67037\times10^{-8}\frac{W}{m^2K^4}}}$$
$$A = 0.0723$$

14. The equilibrium temperature of the Earth is given by

$$T_e = \sqrt[4]{\frac{Q(1-A)}{\sigma}}$$

If we want the temperature to be 353.15 K, we must have

$$353{,}15K = \sqrt[4]{\frac{Q\cdot(1-0.3)}{5.67037\times10^{-8}\frac{W}{m^2K^4}}}$$
$$Q = 1259.94\frac{W}{m^2}$$

This means that the intensity received should be

$$Q = \frac{I}{4}$$
$$1259.94\frac{W}{m^2} = \frac{I}{4}$$
$$I = 5039.76\frac{W}{m^2}$$

To receive this intensity, we must have

$$I = \frac{P_{star}}{4\pi D^2}$$

$$5039.76 \tfrac{W}{m^2} = \frac{3.828\times10^{26}W}{4\pi D^2}$$

$$D = 7.775\times10^{10}m$$

$$D = 77\,750\,000km$$

That's about half the current distance.

15. The average temperature would be

$$\begin{aligned} T_s &= T_e\sqrt[4]{1-\ln\sqrt{1-\varepsilon}} \\ &= 254.58K\cdot\sqrt[4]{1-\ln\sqrt{1-0.80}} \\ &= 254.58K\cdot1.1591 \\ &= 295.07K \\ &= 21.92°C \end{aligned}$$

16. For the Earth's temperature to be 323.15 K, we must have

$$T_s = T_e\sqrt[4]{1-\ln\sqrt{1-\varepsilon}}$$

$$323.15K = 254.58K\cdot\sqrt[4]{1-\ln\sqrt{1-\varepsilon}}$$

$$1.2682 = \sqrt[4]{1-\ln\sqrt{1-\varepsilon}}$$

$$2.5871 = 1-\ln\sqrt{1-\varepsilon}$$

$$1.5871 = -\ln\sqrt{1-\varepsilon}$$

$$-1.5871 = \ln\sqrt{1-\varepsilon}$$

$$e^{-1.5871} = \sqrt{1-\varepsilon}$$

$$0.2045 = \sqrt{1-\varepsilon}$$

$$0.04182 = 1-\varepsilon$$

$$\varepsilon = 0.9582$$

17. a) The equilibrium temperature of Mars is given by

$$T_e = \sqrt[4]{\frac{Q(1-A)}{\sigma}}$$

where Q is given by

$$Q = \frac{I}{4}$$

and where I is the intensity of the radiation arriving from the Sun. This intensity is

$$\begin{aligned} I &= \frac{P_{star}}{4\pi D^2} \\ &= \frac{3.828\times 10^{26} W}{4\pi\cdot\left(2.2734\times 10^{11} m\right)^2} \\ &= 589.40 \tfrac{W}{m^2} \end{aligned}$$

The value of Q on Mars is therefore

$$\begin{aligned} Q &= \frac{I}{4} \\ &= \frac{589.40 \tfrac{W}{m^2}}{4} \\ &= 147.35 \tfrac{W}{m^2} \end{aligned}$$

The equilibrium temperature is therefore

$$\begin{aligned} T_e &= \sqrt[4]{\frac{Q(1-A)}{\sigma}} \\ &= \sqrt[4]{\frac{147.35 \tfrac{W}{m^2}\cdot(1-0.25)}{5.67037\times 10^{-8} \tfrac{W}{m^2K^4}}} \\ &= 210.11K \\ &= -63,04°C \end{aligned}$$

Since there is no greenhouse effect, this temperature is the true average temperature on Mars.

b) With the new atmosphere, we have

$$\begin{aligned} T_s &= T_e \sqrt[4]{1 - \ln \sqrt{1-\varepsilon}} \\ &= 210.11K \cdot \sqrt[4]{1 - \ln \sqrt{1-0.70}} \\ &= 210.11K \cdot 1.125 \\ &= 236.38K \\ &= -36.77°C \end{aligned}$$

18. If we want the average temperature to be 278.15 K, we must have

$$\begin{aligned} T_s &= T_e \sqrt[4]{1 - \ln \sqrt{1-\varepsilon}} \\ 278.15K &= T_e \cdot \sqrt[4]{1 - \ln \sqrt{1-0.71}} \\ 278.15K &= T_e \cdot 1.1280 \\ T_e &= 246.59K \end{aligned}$$

We must therefore have

$$\begin{aligned} T_e &= \sqrt[4]{\frac{Q(1-A)}{\sigma}} \\ 246.59K &= \sqrt[4]{\frac{340.275 \frac{W}{m^2} \cdot (1-A)}{5.67037 \times 10^{-8} \frac{W}{m^2 K^4}}} \\ A &= 0.3839 \end{aligned}$$

19. If the absorption coefficient is 0.71, then the transmission coefficient is 0.29. With 50 layers, we must have

$$\begin{aligned} t_c^{50} &= 0.29 \\ t_c &= \sqrt[50]{0.29} \\ t_c &= 0.9755 \end{aligned}$$

If the transmittance coefficient of each layer is 0.9755, then the absorption coefficient of each layer is

$$\begin{aligned} \varepsilon_c &= 1 - t_c \\ &= 0.0244 \end{aligned}$$

20. In 1850, the temperature was 286.75 K. The value of ε must therefore have been

$$T_s = T_e \sqrt[4]{1 - \ln\sqrt{1-\varepsilon}}$$
$$286.75K = 254.58K \cdot \sqrt[4]{1 - \ln\sqrt{1-\varepsilon}}$$
$$1.1264 = \sqrt[4]{1 - \ln\sqrt{1-\varepsilon}}$$
$$1.6096 = 1 - \ln\sqrt{1-\varepsilon}$$
$$0.6096 = -\ln\sqrt{1-\varepsilon}$$
$$-0.6096 = \ln\sqrt{1-\varepsilon}$$
$$e^{-0.6096} = \sqrt{1-\varepsilon}$$
$$0.5436 = \sqrt{1-\varepsilon}$$
$$0.2954 = 1-\varepsilon$$
$$\varepsilon = 0.7045$$

In 2023, the temperature was 287.95 K. The value of ε must therefore have been

$$T_s = T_e \sqrt[4]{1 - \ln\sqrt{1-\varepsilon}}$$
$$287.95K = 254.58K \cdot \sqrt[4]{1 - \ln\sqrt{1-\varepsilon}}$$
$$1.1311 = \sqrt[4]{1 - \ln\sqrt{1-\varepsilon}}$$
$$1.6367 = 1 - \ln\sqrt{1-\varepsilon}$$
$$0.6367 = -\ln\sqrt{1-\varepsilon}$$
$$-0.6367 = \ln\sqrt{1-\varepsilon}$$
$$e^{-0.6367} = \sqrt{1-\varepsilon}$$
$$0.5290 = \sqrt{1-\varepsilon}$$
$$0.2799 = 1-\varepsilon$$
$$\varepsilon = 0.7201$$

The value of ε has therefore increased from 0.7045 to 0.7201.

21. At 1000 ppm, the radiative forcing would be

$$\begin{aligned}\Delta F_{CO2} &\approx 5.35\tfrac{W}{m^2}\cdot\ln\left(\frac{C}{278\,ppm}\right)\\ &\approx 5.35\tfrac{W}{m^2}\cdot\ln\left(\frac{1000\,ppm}{278\,ppm}\right)\\ &\approx 6.85\tfrac{W}{m^2}\end{aligned}$$

The temperature increase would then be

$$\begin{aligned}\Delta T &\approx 0.55\tfrac{°Cm^2}{W}\cdot\Delta F_{CO2}\\ &\approx 0.55\tfrac{°Cm^2}{W}\cdot 6.85\tfrac{W}{m^2}\\ &\approx 3.8°C\end{aligned}$$

22. As 41% of emissions remain in the atmosphere, the amount of carbon added will be

$$0.41\cdot 6000Gt = 2460Gt$$

Such a quantity of carbon corresponds to an increase in the concentration of

$$\frac{2460Gt}{2.214\tfrac{Gt}{ppm}} = 1111\,ppm$$

The concentration would the be

$$278\,ppm + 1111\,ppm = 1389\,ppm$$

The radiative forcing would be

$$\begin{aligned}\Delta F_{CO2} &\approx 5.35\tfrac{W}{m^2}\cdot\ln\left(\frac{C}{278\,ppm}\right)\\ &\approx 5.35\tfrac{W}{m^2}\cdot\ln\left(\frac{1389\,ppm}{278\,ppm}\right)\\ &\approx 8.61\tfrac{W}{m^2}\end{aligned}$$

The temperature increase would be

$$\begin{aligned}\Delta T &\approx 0.55\tfrac{°Cm^2}{W}\cdot\Delta F_{CO2}\\ &\approx 0.55\tfrac{°Cm^2}{W}\cdot 8.61\tfrac{W}{m^2}\\ &\approx 4.7°C\end{aligned}$$

23. With a temperature of 20°C, the warming is 6.4°C compared to pre-industrial temperatures. This corresponds to a radiative forcing of

$$\Delta T \approx 0{,}55 \tfrac{°Cm^2}{W} \cdot \Delta F_{CO2}$$
$$6.4°C \approx 0.55 \tfrac{°Cm^2}{W} \cdot \Delta F_{CO2}$$
$$\Delta F_{CO2} \approx 11.64 \tfrac{W}{m^2}$$

To have such a forcing, the concentration of CO2 should be

$$\Delta F_{CO2} \approx 5.35 \tfrac{W}{m^2} \cdot \ln\left(\frac{C}{278\,ppm}\right)$$
$$11.64 \tfrac{W}{m^2} \approx 5.35 \tfrac{W}{m^2} \cdot \ln\left(\frac{C}{278\,ppm}\right)$$
$$2.175 \approx \ln\left(\frac{C}{278\,ppm}\right)$$
$$e^{2.175} = \frac{C}{278\,ppm}$$
$$C = 2447\,ppm$$

24. The temperature of the Earth is given by

$$T_S = T_e \sqrt[4]{1-\ln\sqrt{1-\varepsilon}}$$
$$T_S = \sqrt[4]{\frac{Q(1-A)}{\sigma}} \sqrt[4]{1-\ln\sqrt{1-\varepsilon}}$$
$$T_S^4 = \frac{Q(1-A)}{\sigma}\left(1-\ln\sqrt{1-\varepsilon}\right)$$

If we want the temperature to be the same in 1850 and 2023, we have

$$\frac{Q(1-A)}{\sigma}\left(1-\ln\sqrt{1-\varepsilon}\right) = \frac{Q(1-A')}{\sigma}\left(1-\ln\sqrt{1-\varepsilon'}\right)$$

We then have

$$\frac{Q(1-A)}{\sigma}\left(1-\ln\sqrt{1-\varepsilon}\right)=\frac{Q(1-A')}{\sigma}\left(1-\ln\sqrt{1-\varepsilon'}\right)$$
$$(1-A)\left(1-\ln\sqrt{1-\varepsilon}\right)=(1-A')\left(1-\ln\sqrt{1-\varepsilon'}\right)$$
$$(1-0,3)\left(1-\ln\sqrt{1-0.7045}\right)=(1-A')\left(1-\ln\sqrt{1-0.7201}\right)$$
$$(1-0.3)(1.6095)=(1-A')(1.6367)$$
$$0.6884=1-A'$$
$$A'=0.3116$$

25. Since each kWh generates 0.68 kg of CO_2, the amount of CO_2 produced is

$$\begin{aligned} m_{CO_2} &= 1.1\times10^{11}kWh\cdot 0.68\tfrac{kg}{kWh} \\ &= 7.5\times10^{10}kg \\ &= 75Mt \end{aligned}$$

These 75 million tonnes of CO_2 correspond to

$$\frac{12}{44}\cdot 75Mt=20Mt$$

of carbon. That's nearly 0.2% of all the carbon emitted.

26. a) The force made on 1 m² is

$$\begin{aligned} F &= PA \\ &= 101\,300Pa\cdot 1m^2 \\ &= 101\,300N \end{aligned}$$

b) The weight of the atmosphere above 1 m² is therefore 101 300 N. The mass of this weight is

$$F=mg$$
$$101\,300N=m\cdot 9.8\tfrac{N}{kg}$$
$$m=10\,337kg$$

c) If the radius of the Earth is 6371 km, then its surface is

$$\begin{aligned} A &= 4\pi r^2 \\ &= 4\pi \cdot \left(6.371\times 10^6 m\right)^2 \\ &= 5.1\times 10^{14} m^2 \end{aligned}$$

Since there are 10 337 kg for each m², the total mass is

$$\begin{aligned} m_{tot} &= 10\,337 \tfrac{kg}{m^2} \cdot 5.1\times 10^{14} m^2 \\ &= 5.3\times 10^{18} kg \end{aligned}$$

d) If the atmosphere was 90 times more massive, then its mass was

$$90 \cdot 5.3\times 10^{18} kg = 4.8\times 10^{20} kg$$

e) If 95% of this mass was CO_2, then the mass of CO_2 was

$$0.95 \cdot 4.8\times 10^{20} kg = 4.6\times 10^{20} kg$$

This corresponds to a mass of carbon of

$$\begin{aligned} \frac{12}{44} \cdot 4.6\times 10^{20} kg &= 1.2\times 10^{20} kg \\ &= 120\,000\,000 Gt \end{aligned}$$