7 DIFFRACTION

How far is a car when it is possible to distinguish, with the naked eye, the two front lights of the car? The lights are 1.5 m apart.

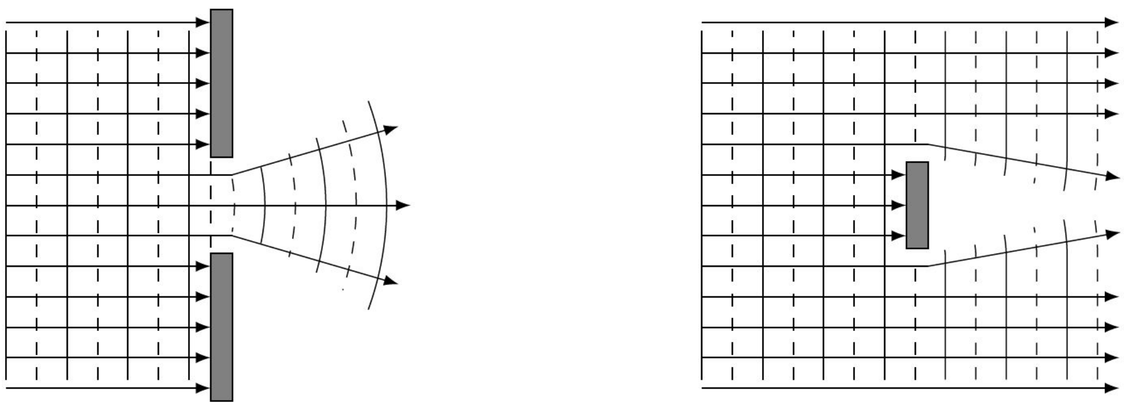
scienceblogs.com/startswithabang/2011/02/18/open-wide-what-do-you-mean-my/

Discover how to solve this problem in this chapter.

7.1 WHAT IS DIFFRACTION?

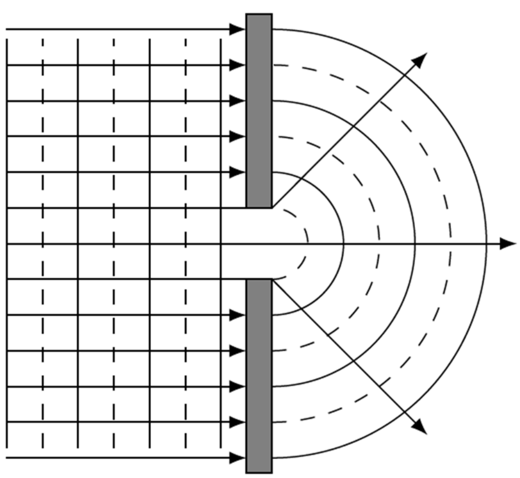
Wave Spreading

Diffraction is a phenomenon that occurs when a wave encounters an object or passes through a hole whose dimensions are not much larger than the wavelength. In this case, the wave spreads a little passing around the object or through the hole.



$a > \lambda$

(a represents the size of the object or the hole.)

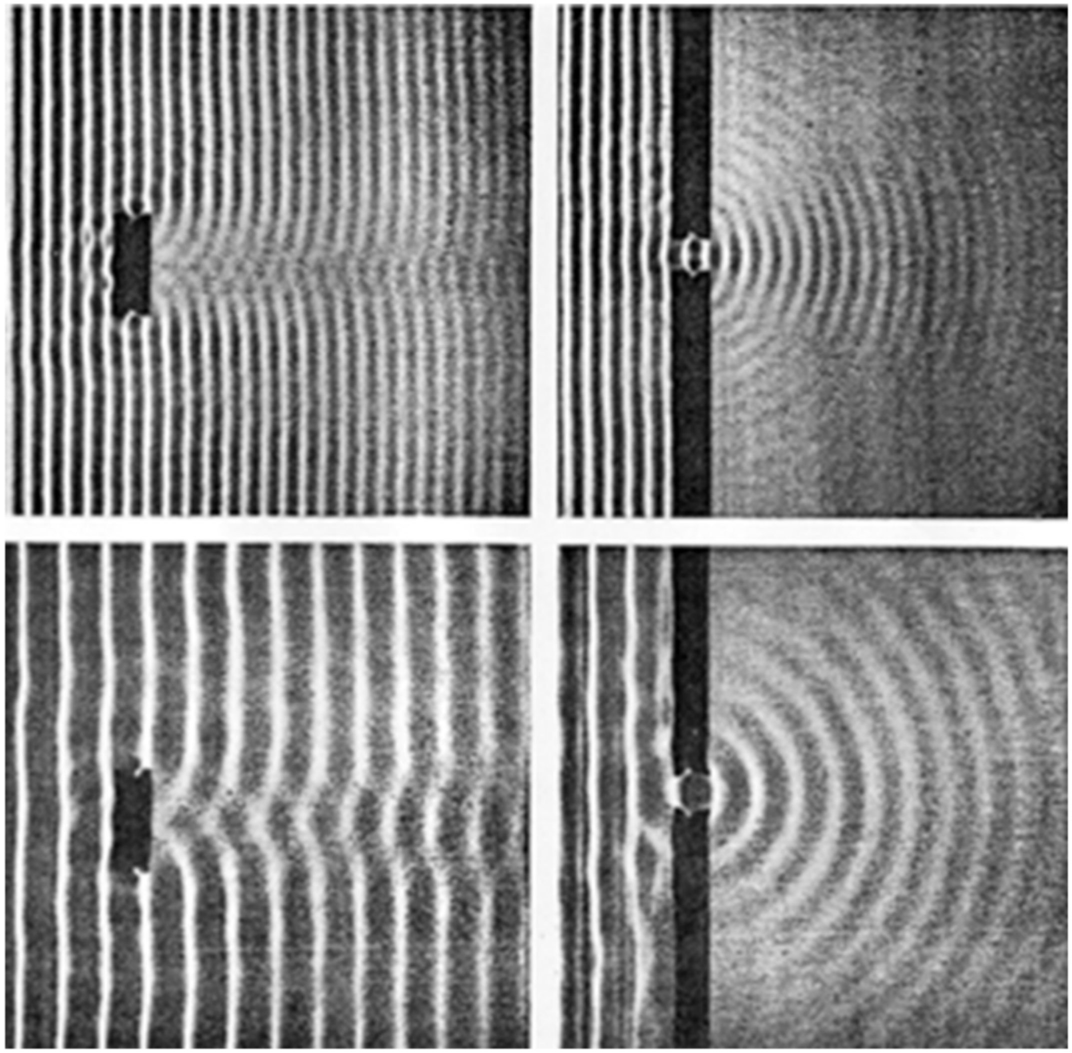


$a < \lambda$

If light encounters an even smaller object or if it passes through an even smaller hole, the spreading of the wave is much more important.

This deflection of rays is called diffraction.

The following images were obtained by sending waves into small openings or by placing an obstacle in the path of these waves. In each of these pictures, the diffraction is obvious. In the bottom images, the wavelength is greater than on the top images. It is easy to see on the bottom images that the waves spread further than on the top images. This is normal behaviour since there is more diffraction when the obstacles have dimensions closer to the wavelength.



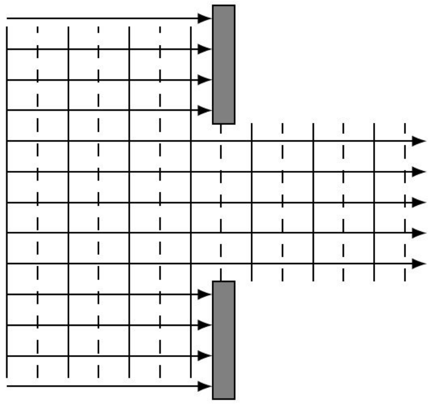
www.joeruff.com/artruff/physics/Student_Pages/Wave_Behavior/moooo.wps.htm

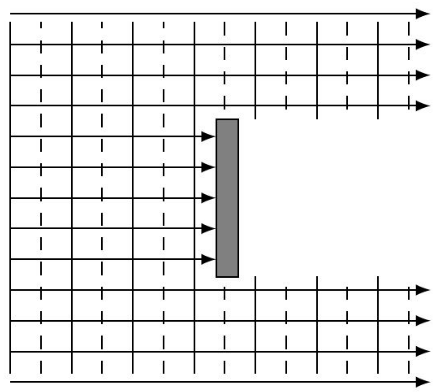
Wave diffraction can also be seen in this video.
http://www.youtube.com/watch?v=lIn-BLJNXpY

First Observation With Light

Diffraction is a phenomenon specific to waves. So, it's easy to think that this phenomenon could have easily helped determine the nature of the light. By passing the light through a hole, it should have been easy to see if the wave is spread out or not. If it spreads, light is a wave. If it does not spread, light is a particle.

However, it is not as simple. The spreading of the wave depends on the width of the hole compared to the wavelength. The hole is smaller, the wave spreads more. However, when the hole is much larger than the wavelength, there is practically no spreading of the wave.





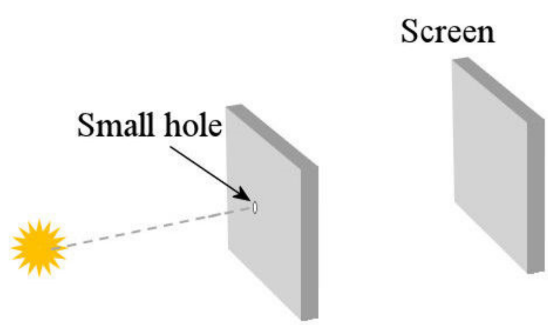
$a \gg \lambda$

As the wavelength of light is only a few hundred nanometres, the dimensions of the holes and objects are often much larger than the wavelength of light. A very small hole must be used to observe diffraction with visible light.

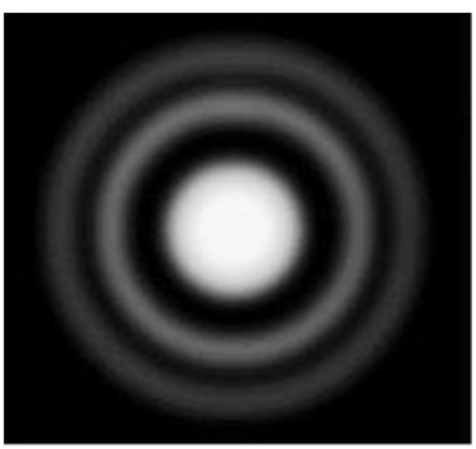
Despite these difficulties, light diffraction had been observed a little before 1665 by Francesco Maria Grimaldi. (The book was published in 1665, 2 years after Grimaldi's death.) Although it was a proof of the wave nature of light, diffraction was not a very convincing evidence then. Supporters of the corpuscular theory of light explained it by invoking an attraction between particles of light and the walls of the hole or the object.

More Than a Simple Spreading of Light

Diffraction is actually much more complex than a simple deviation of rays. To show this complexity, let's see what can be seen on a screen after the passage of the light through a small circular hole.

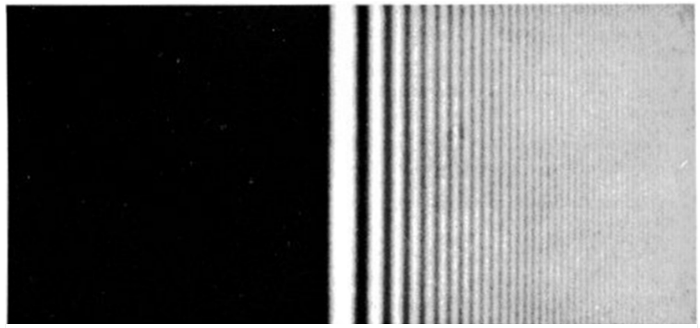


www.vision-doctor.com/en/optic-quality/limiting-resolution-and-mtf.html



On the screen, something like the image on the left can be seen. Of course, the wave spreads out when it passes through the hole and a bright spot larger than the hole is obtained (this is the round bright spot at the centre). But circular rings around the central spot are also obtained. This succession of maxima and minima is quite characteristic of diffraction.

en.wikipedia.org/wiki/Diffraction



www.arne-lueker.de/Objects/archives/MathcadMatlab/Fresneldiffration/Fresnel.html

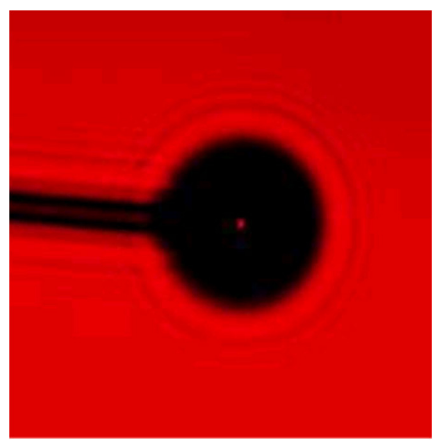
The image to the right shows this sequence of maxima and minima at the edge of the shadow of an object.

The presence of these maxima and minima of intensity remained unexplained for the supporters of both theory (wave and corpuscular) until Augustin Fresnel calculated, in 1815, the intensity of the light according to the wave theory. To his great satisfaction, the wave theory predicted the existence of these minima and maxima of intensity exactly at the places they were observed.

In 1819, Fresnel wanted to present his work to the Academy of Sciences. Siméon-Denis Poisson intended to stand up for the corpuscular theory by studying Fresnel work in detail in order to find questionable elements. Eventually, he was able to deduce from Fresnel's theory there should be a small luminous dot at the centre of the shadow of a circular object! For Poisson, this was totally absurd and was proving that Fresnel was wrong. However, François-Dominique-Jean Arago (who would later become Prime Minister of France) decided to check whether this dot of light exists or not. To the surprise of every supporter of the corpuscular theory, this dot really exists!

This point can be seen in this picture. (Note also the presence of bright and dark fringes around the shadow.) To observe this point, the obstacle must be very small so that the dot is sufficiently bright. Fresnel won the 1819 prize of the Academy.

It was a great victory for the wave theory, but there were still a few supporters of the corpuscular theory.



dusty.physics.uiowa.edu/~goree/teaching/MCAT/

The End of the Corpuscular Theory

The Explanation of Polarization Was Keeping the Corpuscular Theory Alive

Despite Fresnel's success to explain diffraction with the wave theory, the corpuscular theory was not dead. In fact, the corpuscular theory was still surviving because it had a lot more success than the wave theory in explaining polarization. Playing with the shape of light particles, the two refractions obtained in a calcite crystal was thought to depend on the orientation of the particles of light when they enter the substance.

It was not perfect, but it was much better than the explanation given by the wave theory at that time. For a long time, the proponents of the wave theory were completely unable to explain some phenomena related to polarization such a birefringence.

New Observations

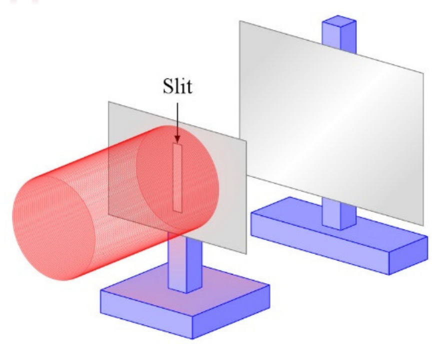
In 1808, Etienne-Louis Malus discovers something special with birefringence. Up to that point, it was believed that two images obtained with double refraction always had the same intensity. Malus discovers that this is not true if light is reflected on a surface before passing through the calcite crystal. By observing the reflection of light on the windows of the Luxembourg Palace in Paris through a crystal of calcite (don't ask me why he started to do that!), he noticed that the two images do not have the same intensity then. The relative intensity of these two images can be changed by rotating the crystal and one of the images can even completely disappear under specific conditions. We now know that there is such a difference because reflected light can be totally or partially polarized. Nobody knew that at this time, but this discovery was the starting point of a series of new experiments on polarization that will bring the victory of the wave theory.

The Victory of the Wave Theory

Then, André-Marie Ampère finally got the wave theory out of the deadlock by proposing, in 1816, that light is a transverse wave. In 1822, Augustin Fresnel further developed this idea of transverse waves. He then got results in perfect harmony with the observations. The last bastion of the corpuscular theory was falling, which meant its death and the triumph of the wave theory. After 1822, there was no longer any significant supporter of the corpuscular theory (until its return in 1905 … see in a further chapter). However, the weird properties of the aether caused a certain discomfort throughout the 19th century. (How could the aether offer no resistance while being rigid at the same time?)

7.2 DIFFRACTION BY A SINGLE SLIT

In this section, the wave spreading will be calculated, with the wave theory, for a wave passing through a very thin slit. Specifically, we want to know what will be seen on a screen a distance L away from the slit.

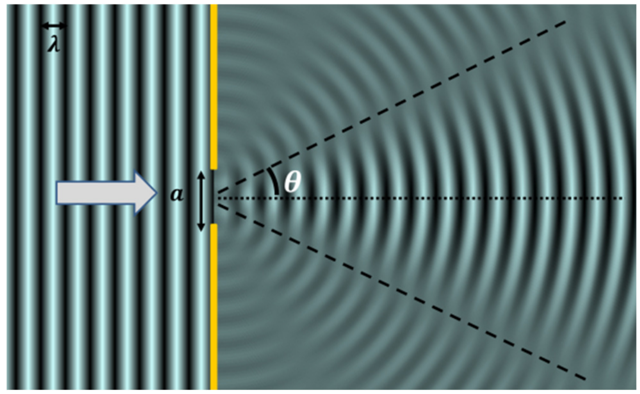


In fact, this case (if L is much larger than the width of the slit) is, by far, the simplest case. Generally, diffraction calculations are quite difficult. Even cases that seem simple at first sight, such as a circular hole, require higher mathematical tools (some knowledge of Bessel functions are required to calculate the result for a circular hole).

Light Intensity

When the wave passes through the slit, it spreads

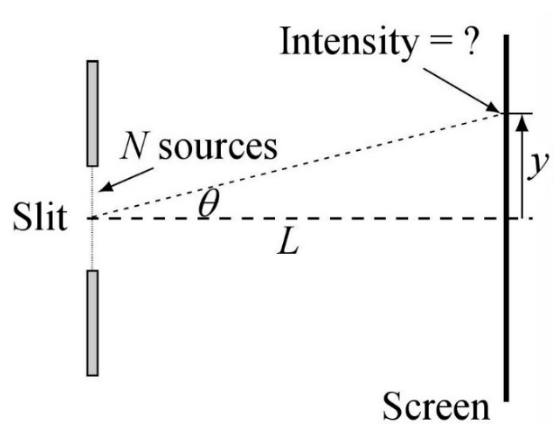
As mentioned before, there is more than just a simple spreading of the wave. The dotted line shows a direction where the intensity is zero. At larger angles, there are other small maxima and other minima.



tsiastnicolas.free.fr/Physique_Chimie/TSI1/Signaux%20Physiques/SP1%20-%20Propagation%20d'un%20signal/sp1_web.publi/web/co/module_SP1_-_Propagation_d_un_signal_34.html

The intensity of the light at the point on the screen shown in the diagram is sought. The position of the point on the screen can be given by y or θ. According to what can be seen in the figure, the link between y and θ is the same as in the previous chapter

$$\tan\theta = \frac{y}{L}$$



To determine the result of the passage of the wave through a slit, we will use the fact that each atom that is not at its equilibrium position in the medium exerts a force on all neighboring atoms and then drags them into its motion. Each atom in the medium can therefore be considered as a wave source. This is Huygens' principle.

Thus, according to Huygens's principle, each point of the wavefront inside the slit can be considered to be a source. As there is an infinite number of such points, we'll suppose that there are N sources in the slit that are sending light towards the point on the screen, and see what happens when N tends towards infinity.

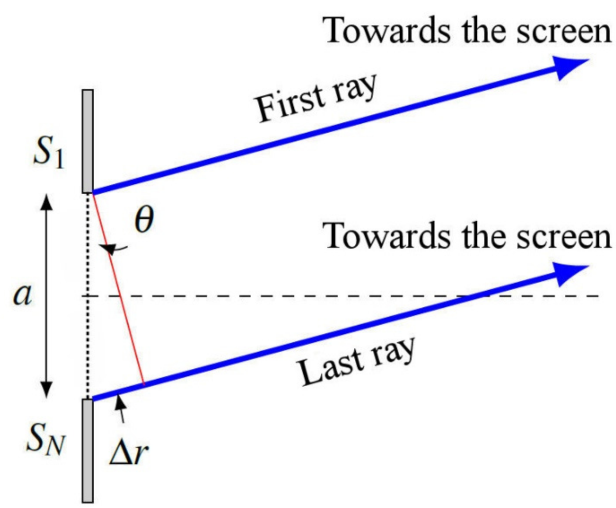
The result of the addition of the waves coming from N sources is already known since it was done when gratings were studied. The amplitude with N sources is

$$A_N = A_1 \frac{\sin\left(\frac{N\Delta\phi}{2}\right)}{\sin\left(\frac{\Delta\phi}{2}\right)}$$

It remains to make N tends towards infinity. However, this will make the distance between the slits tends towards 0 and this will make the phase difference $\Delta\phi$ tends towards 0. This problem is circumvented by using the phase difference between the first and the last rays coming from the slit must be found. This phase difference will be called α.

The Phase Difference α

The phase difference α between the first and the last ray comes from the path length difference that corresponds to the short side of the right triangle (Δr). (It is assumed here that the screen is very far from the slit.)



The value of Δr is

$$\Delta r = a\sin\theta$$

As the phase difference between the rays is

$$\alpha = \frac{\Delta r}{\lambda}2\pi$$

the phase difference is

Phase Difference Between Opposite Side of the Slit

$$\alpha = \frac{a\sin\theta}{\lambda}2\pi$$

The Amplitude

The amplitude is

$$A_N = A_1 \frac{\sin\left(\frac{N\Delta\phi}{2}\right)}{\sin\left(\frac{\Delta\phi}{2}\right)}$$

The phase difference between source 1 and source 2 is $\Delta\phi$, the phase difference between source 1 and source 3 is $2\Delta\phi$, the phase difference between source 1 and source 4 is $3\Delta\phi$, the phase difference between source 1 and source 5 is $4\Delta\phi$, and so on. Thus, the phase difference between source 1 and source N is $(N-1)\Delta\phi$. This last phase difference between the first and the last source is also α.

$$\alpha = (N-1)\Delta\phi$$

When the number of sources becomes very large, this equation becomes

$$\alpha = (N-1)\Delta\phi \approx N\Delta\phi$$

Since $N\Delta\phi = \alpha$, the amplitude can be written as

$$A_N = A_1 \frac{\sin\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2N}\right)}$$

For the denominator, the term inside the sine function becomes very small if N is very large and the approximation $\sin\theta \approx \theta$ can be used to obtain

$$A_N = A_1 \frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2N}\right)}$$

$$A_N = A_1 N \frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}$$

<u>The Intensity</u>

With light, this amplitude is the amplitude of the electric field. Therefore,

$$E_{0tot} = E_{01} N \frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}$$

Thus, the light intensity is

$$\begin{aligned} I_{tot} &= \frac{1}{2} c n \varepsilon_0 E_0^2 \\ &= \frac{1}{2} c n \varepsilon_0 E_{01}^2 N^2 \left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2 \end{aligned}$$

Again, this intensity will be compared to another intensity. Here, it will be compared to the light intensity at the centre of the diffraction pattern on the screen ($\theta = 0$). At this location, the phase difference α is

$$\alpha = \frac{a \sin 0°}{\lambda} 2\pi = 0$$

and the intensity at $\theta = 0$ (called I_0) is thus

$$I_0 = \lim_{\alpha \to 0} \frac{1}{2} c n \varepsilon_0 E_{01}^2 N^2 \left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2$$

$$= \frac{1}{2} cn\varepsilon_0 E_{01}^2 N^2 \left(\lim_{\alpha \to 0} \frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)} \right)^2$$

$$= \frac{1}{2} cn\varepsilon_0 E_{01}^2 N^2 \left(\lim_{x \to 0} \frac{\sin x}{x} \right)^2$$

As this limit is 1, the intensity is

$$I_0 = \frac{1}{2} cn\varepsilon_0 E_{01}^2 N^2$$

The ratio of the intensities is

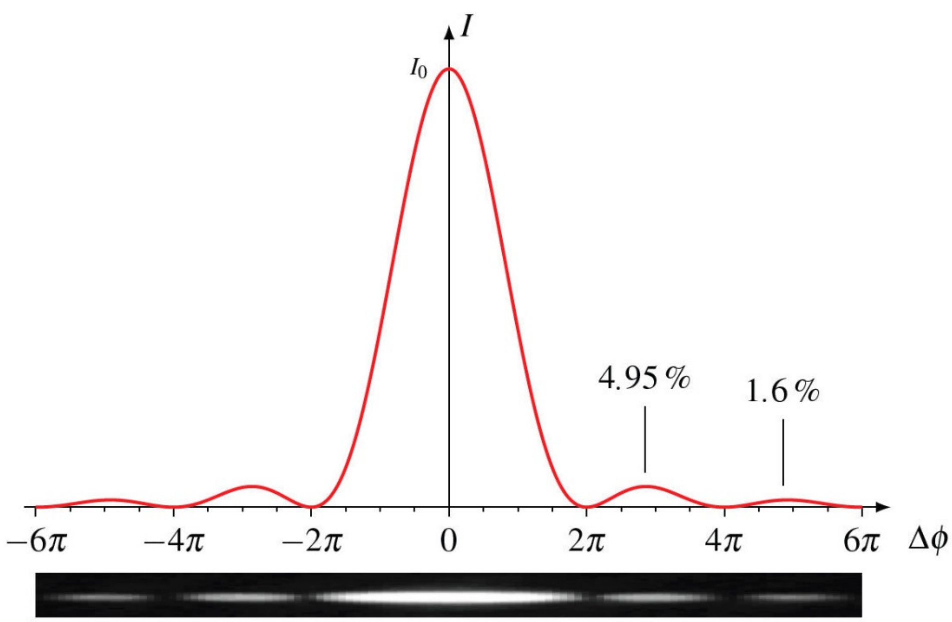
$$\frac{I_{tot}}{I_0} = \left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)} \right)^2$$

Therefore, the result is

Light Intensity for Diffraction by a Slit

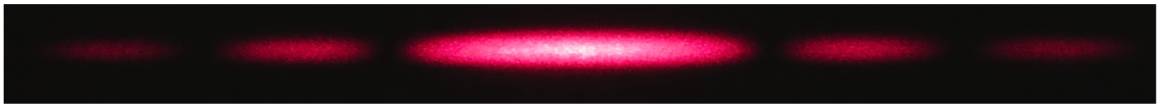
$$I_{tot} = I_0 \left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)} \right)^2$$

The graph of this function is

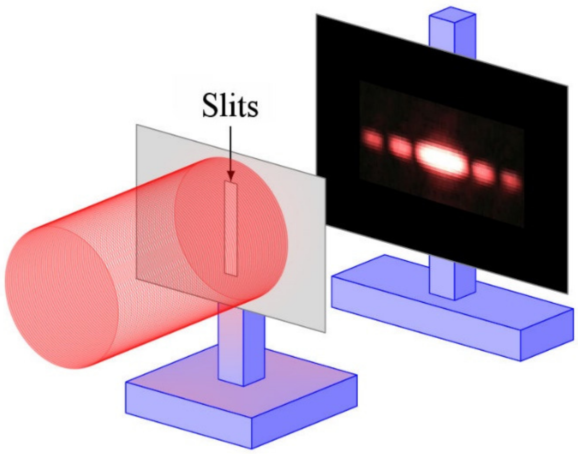


Just underneath the graph, the meaning of this graph can be seen. There is a very bright central maximum surrounded by much less intense maxima. Notice that the central maximum is twice as wide as the other maxima.

Here's what is seen when a laser actually passes through a thin slit.



Without a doubt, that's exactly what is predicted by the theory.



An important note here: a horizontal diffraction pattern, like the one in the diagram, is obtained by using a vertical slit. There is more diffraction if the hole is smaller. The wave is spreading horizontally because the horizontal dimension of the slit is small. It does not extend much in the vertical direction as the slit has a much

Position of the Minima ($I = 0$)

At a minimum, the intensity is $I = 0$. To have this value, the numerator in the formula of the intensity must be zero.

$$\sin\left(\tfrac{\alpha}{2}\right) = 0$$

Therefore, the condition is

$$\frac{\alpha}{2} = \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \ldots$$

$$\frac{\alpha}{2} = M\pi$$

where M is an integer. (However, M cannot be 0 because then the denominator is also zero in the formula of the intensity. We then have a 0/0 division and the limit when α tends towards 0 must be done to find the intensity. This limit was done previously to find I_0 and the resulting intensity was not 0, thereby eliminating $M = 0$).

Using the formula for α, the condition becomes

$$\frac{1}{2}\frac{a\sin\theta}{\lambda}2\pi = M\pi$$

Simplified, this becomes

Angle of the Minima for Diffraction With a Slit

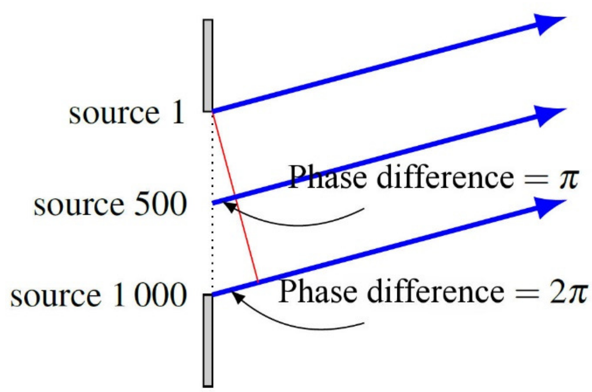
$$a\sin\theta = M\lambda \quad \text{where } M \text{ is a non-zero integer.}$$

Here's the values of M values of the minimums on the screen.

There is a formula for finding the angle of the maxima, but it is quite complex. You might think that they are exactly between the minima, but they aren't because the intensity curve is not symmetrical between two minima. The maximum is always slightly shifted towards the central maximum.

One might wonder why there are minima at these locations. Let's have a look at the waves arriving on the screen at the first minimum. Even if there is an infinite number of sources, let's assume there are 1000 sources to simplify the reasoning.

As this is the first minimum, the phase difference (α) between the first ray (source 1) and the last ray (source 1000) is 2π. This means that the phase difference between source 1 and source 500 is π. Thus, sources 500 and 1000 cancel each other (phase difference of π). The same happens with sources 499 and 999, 498 and 998, 497 and 997 and so on up to sources 1 and 501. As all these waves cancelled each other, there is nothing left and the intensity is zero.



Width of the Central Maximum

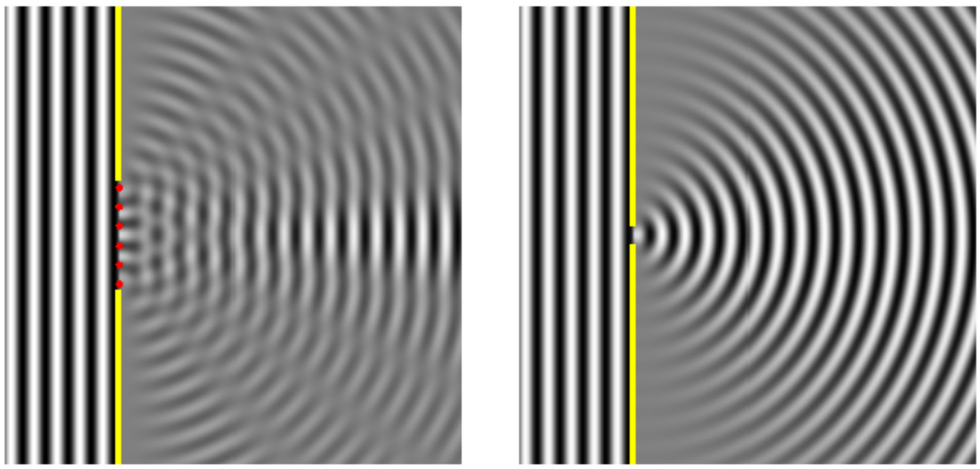
The central maximum ends at the first minimum. Therefore, the angle between the centre of the central maximum and the end of the central maximum, which is called the half-width of the central maximum, is

Half-Width of the Central Maximum

$$\sin\theta = \frac{\lambda}{a}$$

Why Must the Slit Be Thin?

In the first section of this chapter, it was said that diffraction is important only if the slit is not too large compared with the wavelength. This can clearly be seen in the formula of the half-width of the central maximum: a smaller slit implies a greater angle θ. This means that by reducing the width of the slit, the central maximum spreads more.



en.wikipedia.org/wiki/Huygens–Fresnel_principle

The formula for the half-width of the central maximum also indicates that the angle is very small if the width of the slit is much larger than the wavelength. This means that there is not a lot of diffraction in this case. Since visible light has a small wavelength (a few hundreds of nanometres), the slit must be very thin to allow us to observe diffraction. This is why it took a lot of time to highlight the wave nature of light using diffraction.

If the width of the slit becomes smaller than the wavelength, there is no minimum (because there is no solution to the equation of the angle of the minima). This means that the central maximum spreads over 180° and that there is light diffracted in every direction.

Example 7.2.1

Light having a 632 nm wavelength passes through a 0.1 mm wide slit.

a) What is the width of the central maximum on a screen which is 3 m away from the slit?

The half-width of the central maximum is

$$\sin\theta = \frac{\lambda}{a}$$

$$\sin\theta = \frac{632\times10^{-9}m}{0.1\times10^{-3}m}$$
$$\theta = 0.3625°$$

(There's not a lot of diffraction here. This is because the slit is 158 times larger than the wavelength. That's a lot.) On the screen, this angle corresponds to the following distance.

$$\tan\theta = \frac{y}{L}$$
$$\tan 0.3625° = \frac{y}{3m}$$
$$y = 1.9cm$$

So, the total width is 2 · 1.9 cm = 3.8 cm. It's still more than the 0.1 mm wide bright spot we would have obtained without diffraction.

b) What is the light intensity 1 cm from the centre of the central maximum?

The intensity is found with

$$I = I_0\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2$$

So, the phase difference at this position is needed. This phase difference is calculated with the angle at this position.

1 cm from the centre of the central maximum, the angle is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{1cm}{300cm}$$
$$\theta = 0.191°$$

At this angle, the phase difference is

$$\alpha = \frac{a\sin\theta}{\lambda}2\pi$$
$$= \frac{0.1\times10^{-3}m\cdot\sin 0.191°}{632\times10^{-9}m}\cdot 2\pi$$
$$= 3.314rad$$

Therefore, the intensity is

$$I = I_0 \left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)} \right)^2$$
$$= I_0 \left(\frac{\sin\left(\frac{3.314 rad}{2}\right)}{\left(\frac{3.314}{2}\right)} \right)^2$$
$$= 0.362 I_0$$

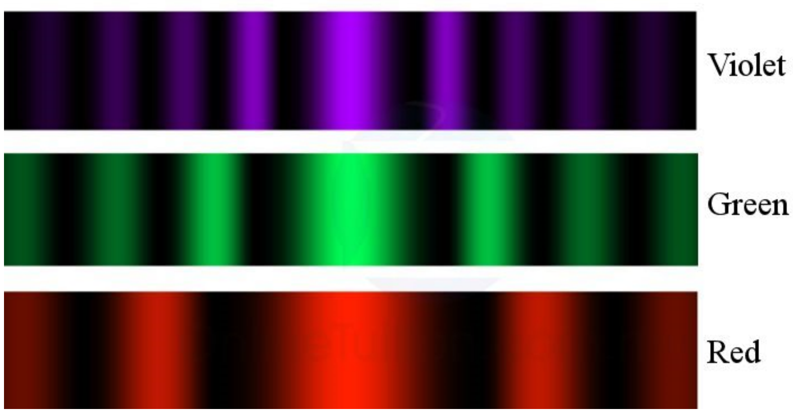
The intensity is thus 36.2% of the intensity at the centre of the central maximum.

Diffraction With Different Colours

In the diffraction formula

$$a \sin\theta = M\lambda$$

it's obvious that the wavelength influences the position of the minima. If the wavelength is smaller, the angles of the minima are smaller. If projected on a screen always at the same distance, this smaller angle results in a smaller distance between the minima if the wavelength is smaller, as can be seen in this image.



spmphysics.onlinetuition.com.my/2013/07/diffraction-of-light-wave.html

Diffraction With White Light

The positions of the minima depend on the wavelength. If the wavelength is smaller, the diffraction minima are closer to each other. So, if white light passes through a slit, each colour will make a different diffraction pattern. This diffraction pattern is then obtained.

www.itp.uni-hannover.de/~zawischa/ITP/diffraction.html

The central maximum for red light is wider than for blue light, making the edges of the central maximum red. The first diffraction maximum is at a smaller angle for violet light than for red light. That's why there is a colour separation at the first maximum. The same thing happens for the other maxima but it is less obvious because there are some overlapping of the maxima. For example, the second maximum of red light is not far from the third maximum of violet light.

7.3 LIGHT INTENSITY WITH MANY SLITS

In the previous chapter, all of the light intensity formulas obtained (with two or more slits) predicted that all the interference maxima should have the same intensity. However, the interference maxima do not all have the same intensity. It is now possible to explain this discrepancy between theory and observations.

Light Intensity Formula With 2 Slits

The intensity formula with 2 slits obtained in the previous chapter was

$$I_{tot} = 4I_1 \cos^2 \frac{\Delta\phi}{2}$$

When this formula was obtained, it was assumed that the intensity of the light emitted by every source is the same in every direction. However, this is not true for slits. The slits emit waves in every direction because diffraction makes the wave spread. Yet, with diffraction, the intensity of the wave is not the same in every direction. Actually, the intensity of the light emitted by a slit is rather

$$I_1 = I_{10}\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2$$

where I_{10} is the intensity made by a single slit at $\theta = 0$ (it's I one-zero, not I ten).

To obtain a correct result, the intensity I_1 in the first formula must then be replaced by this last intensity. Then, the total intensity is

Light Intensity in Young's Experiment

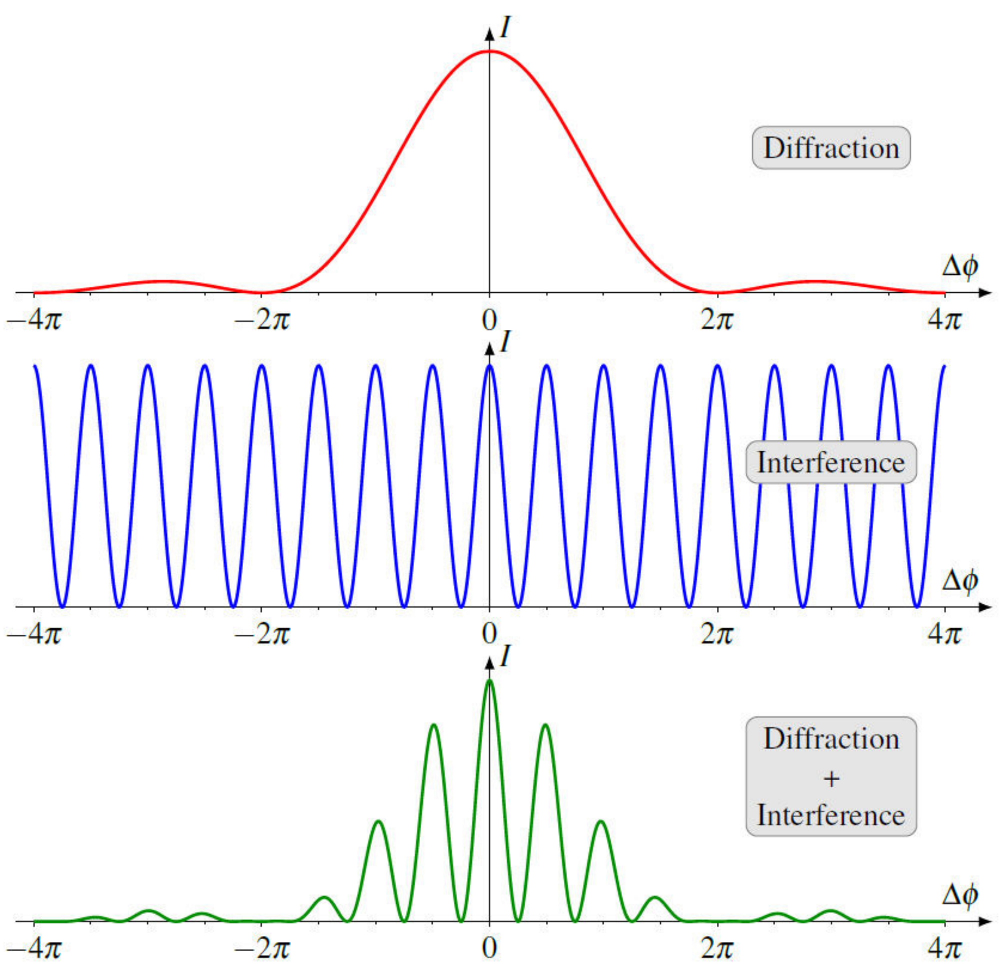
$$I_{tot} = 4I_{10}\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2 \cos^2 \frac{\Delta\phi}{2}$$

In this formula, the phase differences are

$$\Delta\phi = \frac{d\sin\theta}{\lambda}2\pi \qquad \alpha = \frac{a\sin\theta}{\lambda}2\pi$$

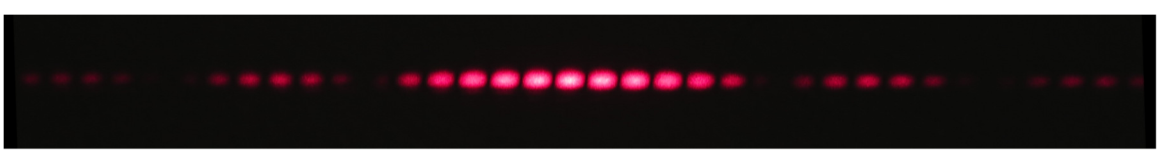
(One of these phase differences depends on the distance between the slits *d* and the other depends on the width of the slits *a*).

Here are the graphs of this intensity for *d* = 4*a*.



The bottom graph, which is the end result, is the multiplication of the two previous graphs. The interference maxima stay at the same place compared to the case where there is only interference but the intensity of the maxima decreases according to the graph of diffraction.

Diffraction then explains why the intensity of the interference maxima in Young's experiment decreases, as can be seen in the following picture.



Example 7.3.1

Light having a 540 nm wavelength passes through two 0.02 mm wide slits 0.1 mm apart. What is the intensity of the light 3 cm from the central maximum if the screen is 5 m away from the slits?

The intensity is found with the following formula.

$$I_{tot} = 4I_{10}\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2 \cos^2\frac{\Delta\phi}{2}$$

The phase difference at 3 cm from the centre must be found. To calculate it, the angle at this position must be known first.

3 cm away from the central maximum, the angle is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{3cm}{500cm}$$
$$\theta = 0.3438°$$

The phase difference is thus

$$\begin{aligned}\Delta\phi &= \frac{d\sin\theta}{\lambda}2\pi \\ &= \frac{0.1\times 10^{-3}m\cdot\sin 0.3438°}{540\times 10^{-9}m}\cdot 2\pi \\ &= 6.981rad\end{aligned}$$

$$\begin{aligned}\alpha &= \frac{a\sin\theta}{\lambda}2\pi \\ &= \frac{0.02\times 10^{-3}m\cdot\sin 0.3438°}{540\times 10^{-9}m}\cdot 2\pi \\ &= 1.396rad\end{aligned}$$

Therefore, the intensity is

$$\begin{aligned}I_{tot} &= 4I_{10}\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2 \cos^2\frac{\Delta\phi}{2} \\ &= 4I_{10}\left(\frac{\sin\left(\frac{1.396rad}{2}\right)}{\left(\frac{1.396rad}{2}\right)}\right)^2 \cos^2\frac{6.981rad}{2} \\ &= 2.994I_{10}\end{aligned}$$

This intensity must be compared to the intensity of the central interference maximum, so at $\theta = 0$. At this point, the intensity is

$$\begin{aligned} I_0 &= 4I_{10}\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2 \cos^2 0 \\ &= 4I_{10} \end{aligned}$$

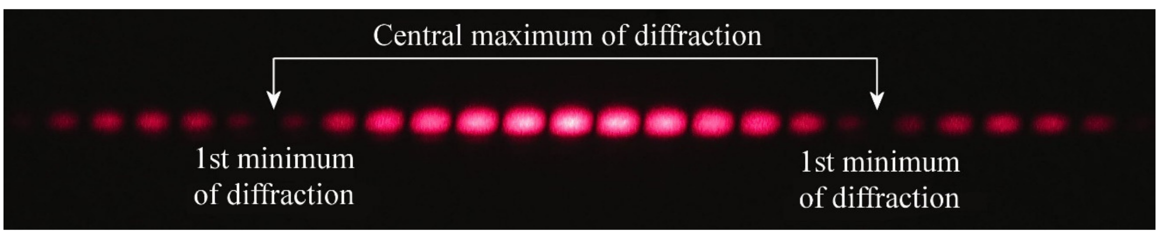
The ratio of intensities is therefore

$$\begin{aligned} \frac{I_{tot}}{I_0} &= \frac{2.994 I_{10}}{4 I_{10}} \\ &= 0.7486 \end{aligned}$$

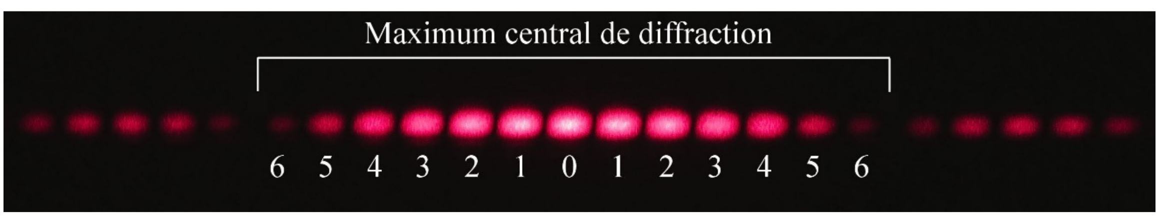
This means that at 3 cm from the center, the intensity is 74.86 % of the intensity in the center of the interference pattern.

The Number of Interference Maxima Within the Central Maximum of Diffraction

It can be seen that there are now some interference maxima inside the central maximum of diffraction (there are 13 such maxima in this picture).



To find the number of interference maxima within the central diffraction maximum, the order of the last interference maximum (m_d) inside the central diffraction maximum. In the image above m_d is 6.



As the diffraction maximum ends at the first minimum of diffraction, the angle of the end of the diffraction maximum is found with

$$a \sin\theta = \lambda$$

At this angle, the order of the interference maximum is

$$d\sin\theta = m\lambda$$

$$d\frac{\lambda}{a} = m\lambda$$

$$m = \frac{d}{a}$$

First, it is possible to obtain an integer, let's say 6. That would mean the 6th interference maximum is exactly at the same place as the first diffraction minimum, and it cannot be seen. Therefore, all the maxima up to $m = 5$ can be seen in the central maximum.

But *d/a* can also be a non-integer number, let's say 8.8. This would mean that the maximum $m = 8$ is before the end of the central maximum and that the maximum $m = 9$ is after the end of the central maximum. Therefore, all the maxima up to $m = 8$ can be seen in the central maximum

Value of *m* for the Last Interference Maximum in the Central Maximum of Diffraction (m_d)

$$\text{Calculate } \frac{d}{a}$$

If an integer is obtained, subtract 1 to get m_d.
If a non-integer is obtained, remove the decimals to get m_d.

The total number of maxima of the interference can then be found. There are m_d interference maxima on one side of the central maximum and m_d interference maxima on the other side. If the central interference maximum ($m = 0$) is added, the number of interference maxima is

Number of Interference Maxima Within the Central Maximum of Diffraction

$$Number = 2m_d + 1$$

With this animation, it is possible to see how the number of maxima within the central maximum of diffraction changes if a changes while d and λ are constant.
http://www.youtube.com/watch?v=sabP2TXDWGs

Example 7.3.2

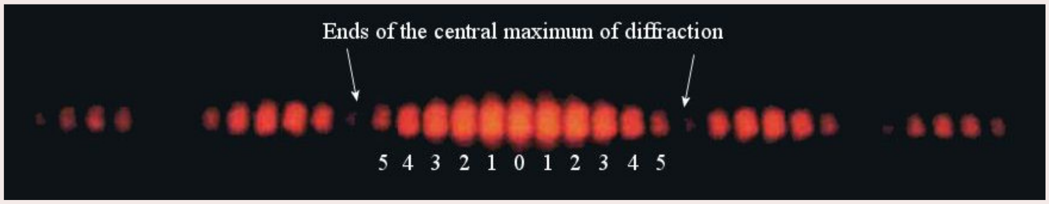
Light having a 632 nm wavelength passes through two 0.01 mm wide slits. The distance between the slits is 0.058 mm, and the screen is 2 m away from the slits.

a) What is the number of interference maxima within the central maximum of diffraction?

We have

$$\frac{d}{a} = \frac{0.058mm}{0.01mm} = 5.8$$

Therefore m_d = 5. The number of maxima is thus 2 · 5 + 1 = 11. The interference pattern would look like this one.



voer.edu.vn/c/youngs-double-slit-experiment/0e60bfc6/24de44ca

b) What is the intensity of the third interference maximum compared to the intensity of the maximum located at the centre of the pattern?

The intensity is calculated with

$$I_{tot} = 4I_{10}\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2 \cos^2\frac{\Delta\phi}{2}$$

To find it, the phase differences are needed. These phase differences can be found from the angle. For the third maximum, we have

$$d\sin\theta = 3\lambda$$
$$0.058\times10^{-3}m\sin\theta = 3\cdot632\times10^{-9}m$$
$$\sin\theta = 0.03269$$

(There is no need to find the angle, the sine will suffice.) At this angle, the phase differences are

$$\begin{aligned}\Delta\phi &= \frac{d\sin\theta}{\lambda}2\pi\\ &= \frac{0.058\times10^{-3}m\cdot0.03269}{632\times10^{-9}m}\cdot2\pi\\ &= 6\pi rad\end{aligned}$$

(We could have predicted this answer for the third maximum.)

$$\begin{aligned}\alpha &= \frac{a\sin\theta}{\lambda}2\pi\\ &= \frac{0.01\times10^{-3}m\cdot0.03269}{632\times10^{-9}m}\cdot2\pi\\ &= 3.2499rad\end{aligned}$$

The intensity is thus

$$\begin{aligned} I_{tot} &= 4I_{10}\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2 \cos^2\frac{\Delta\phi}{2} \\ &= 4I_{10}\cdot\left(\frac{\sin\left(\frac{3.2499}{2}\right)}{\left(\frac{3.2499}{2}\right)}\right)^2 \cdot\cos^2\frac{6\pi}{2} \\ &= 4I_{10}\cdot 0.3776\cdot 1 \\ &= 1.510I_{10} \end{aligned}$$

As the intensity of the central interference maximum is $4I_{10}$, the intensity of the third maximum compared to the intensity of the central maximum is

$$\frac{1.510I_{10}}{4I_{10}} = 0.3776$$

The intensity is thus 37.76% of the intensity of central interference maximum.

Example 7.3.3

The following interference pattern was obtained with slits 0.25 mm appart.

What is, approximately, the width of the slits?

From the image, it can be found that the last maximum of interference within the central maximum of diffraction is the maximum of order 3.

This means that m_d is equal to 3.

This means that *d/a* is between 3 and 4 (3 not included and 4 included).

With the minimum value of *d/a* (3), we have

$$\frac{d}{a} = 3$$

$$\frac{0.25mm}{a} = 3$$
$$a = 0.0833mm$$

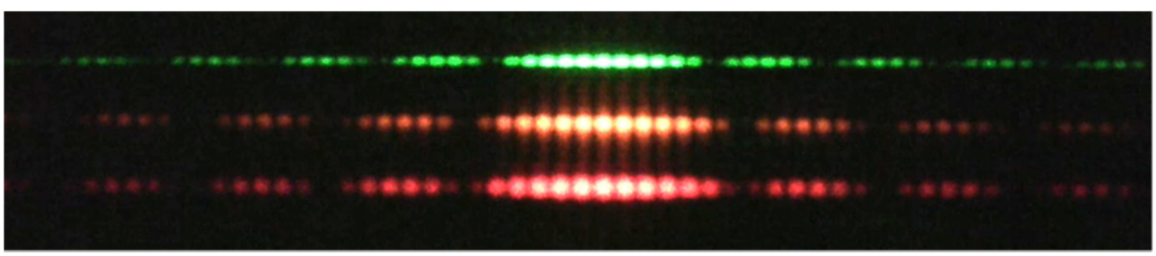
With the maximum value of *d/a* (4), we have

$$\frac{d}{a} = 4$$
$$\frac{0.25mm}{a} = 4$$
$$a = 0.0625mm$$

Therefore, the width of the slits is between 0.0625 mm and 0.0833 mm. Thus, the width is about 0.07 mm (it was actually 0.08 mm).

Young's Experiment With Different Colours

There are some differences if the colour is changed in Young's experiment. The positions of the interference maxima and minima depend on the wavelength. With a small wavelength, the maxima are closer to each other. The same thing happens with diffraction: the diffraction maxima are closer to each other with a smaller wavelength. On the other hand, the number of maxima of interference within the central diffraction maximum does not change since it depends only on the dimensions of the slits and on the distance between the slits which do not change. The following interference patterns are then obtained.

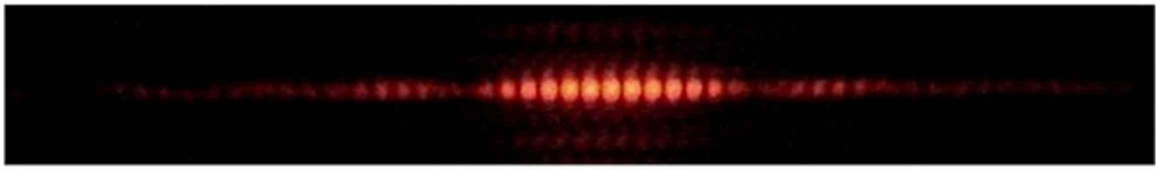


arstechnica.com/science/2012/05/disentangling-the-wave-particle-duality-in-the-double-slit-experiment/

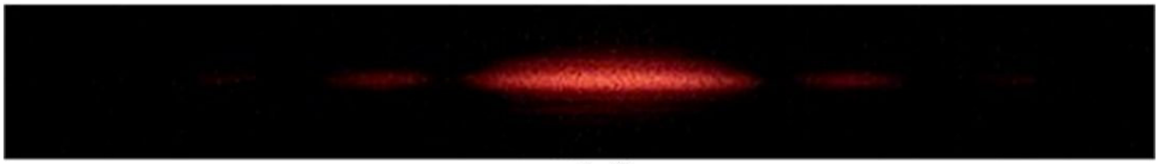
The interference pattern always has the same look for every colour, it is just more "stretched" for larger wavelengths.

What Happens if One Slit Is Blocked?

Initially, with two slits, there are diffraction and interference. If one of the slits is blocked, interference disappears and only diffraction remains. Of course, the intensity also decreases because there is less light arriving at the screen with a single slit.



2 slits



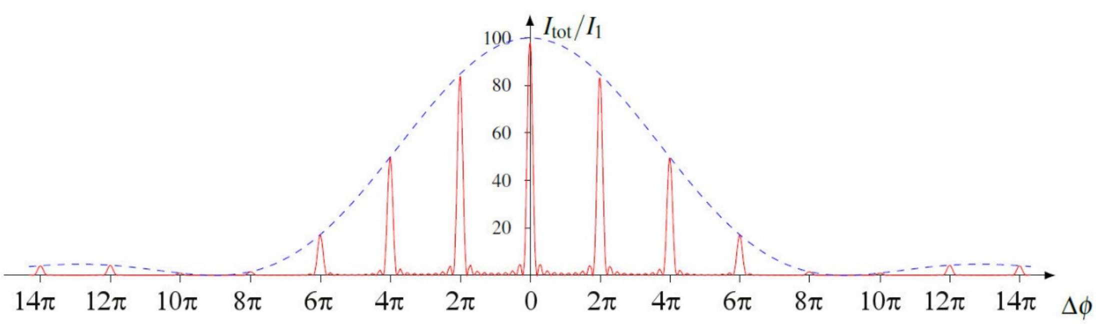
1 slit

en.wikipedia.org/wiki/File:Single_slit_and_double_slit2.jpg

Note that the minima and the maxima of diffraction remain at the same place when one of the slits is blocked.

Multiple Slits and Gratings

With several slits, the result is similar to that obtained with two slits: the maxima remain in the same place, but their intensity is modulated by diffraction. For example, here is the graph of the intensity for 10 slits, when the distance between the slits is 4.5 times the width of the slits.



It is easy to see that there are 10 slits since there are 8 small maxima between the large maxima. It can also be seen that the intensity of the large maxima now decreases following the diffraction curve (dash line). As the distance between the slits is 4.5 times larger than the width of the slits, we have

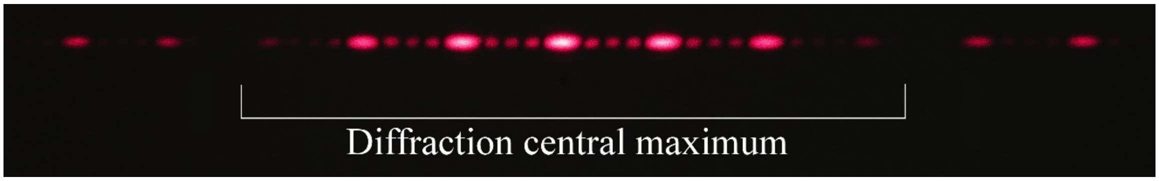
$$\frac{d}{a} = 4.5$$

This means that

$$m_d = 4$$

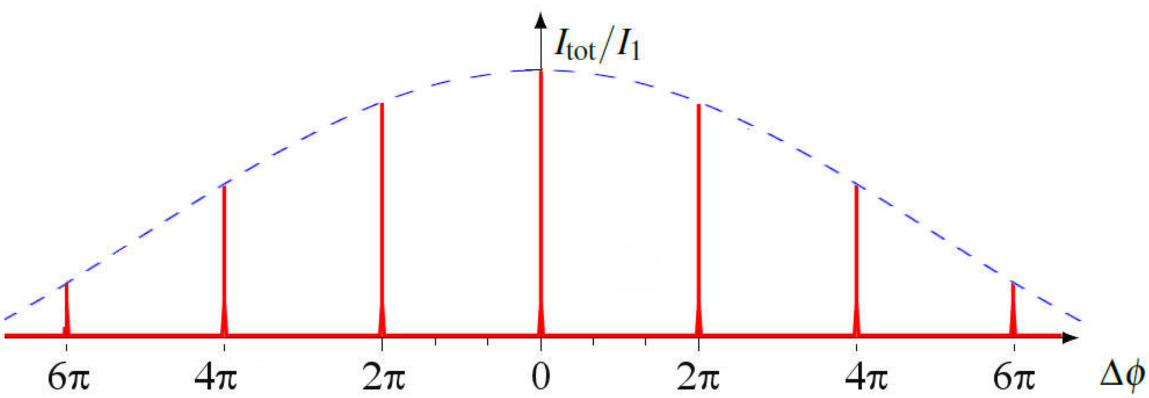
On the graph, we can see that the maximum of order 4 is indeed the last maximum of interference in the central maximum of diffraction.

It is clear from this image that the intensity of the maximums obtained with 5 slits is modulated by the diffraction.

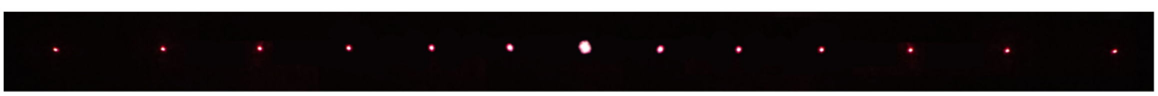


(In this image, we notice that m_d = 3, which means that the distance between the slits is between 3 and 4 times greater than the width of the slits (which was the case since d was 0.125 mm and a was 0.04 mm in this case, leading to the ratio d/a = 3.125).

The same thing happens with gratings. The intensity of the light is modulated by the diffraction in each slit. Thus, the real graph of the intensity of light looks like this.



It is clear from this image that the intensity of the maxima obtained with a grating is modulated by the diffraction.

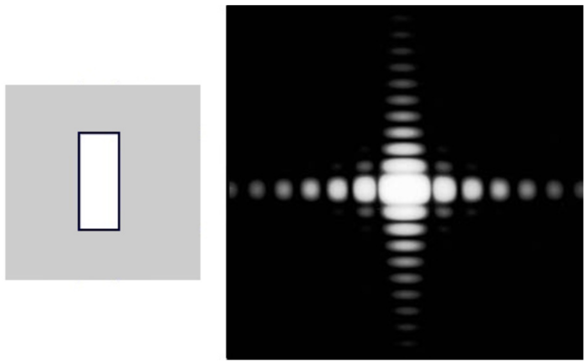


A gradual decrease in intensity can be seen as the maxima are farther away from the center of the interference pattern. Generally, the slits of a grating are so thin (you don't really have a choice when there are several hundred slits per millimeter) that the first minimum diffraction is very far from the center on the interference pattern.

7.4 DIFFRACTION PATTERNS FOR HOLES OR OBSTACLES HAVING OTHER SHAPES

Rectangular Hole

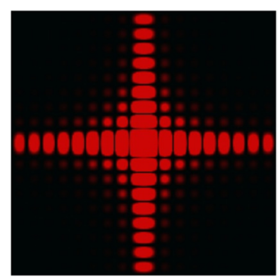
The diffraction pattern for a rectangular hole is



minerva.union.edu/jonesc/scientific_photos%202010.htm

It's actually a combination, in two dimensions, of what is known about a thin slit. Here, the two dimensions of the slit create diffraction, and there is more diffraction in the direction for which the slit is the thinnest. For the pattern obtained here, the horizontal dimension of the slit is smaller, and the wave spreads out more in the horizontal direction than in the vertical direction. In the previous sections, we were dealing with the extreme version of this case: the hole was very long in the vertical direction so that there was no diffraction in that direction.

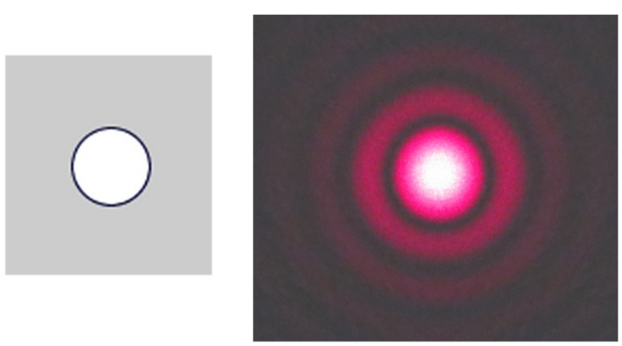
With a square hole, the horizontal diffraction is identical to the vertical diffraction, and the interference pattern looks like the pattern shown in the image to the right.



en.academic.ru/dic.nsf/enwiki/4998

Circular Hole

With a circular hole, the following diffraction pattern is obtained.



It can be shown, with fairly complex calculations that the first diffraction minimum is at the angle given by this formula.

Angle of the First Diffraction Minimum for a Circular Hole

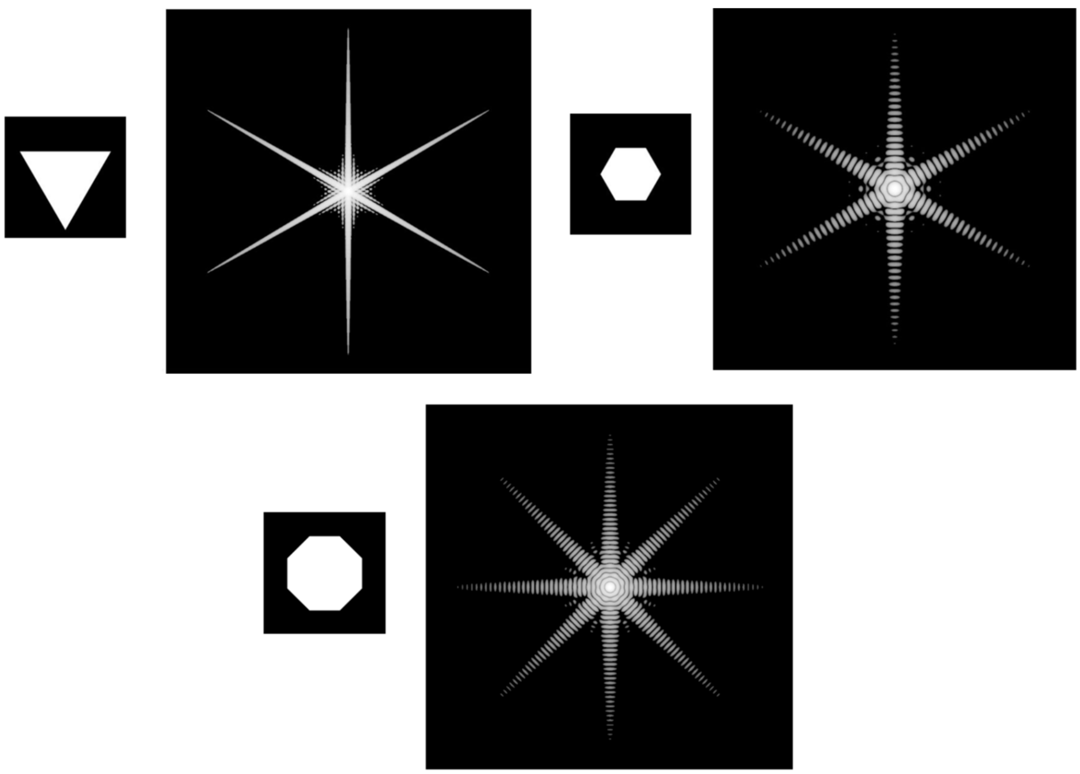
$$\sin\theta = 1.22\frac{\lambda}{a}$$

where *a* is the diameter of the hole. The radius of the central diffraction maximum can be found with this formula since it ends at the first minimum.

This is an experiment with a laser passing through circular holes of various sizes.
http://www.youtube.com/watch?v=BD27GXVHg2c

Triangular, Hexagonal and Octagonal Holes

Here are the diffraction patterns obtained with triangular, hexagonal and octagonal holes.



Made by Jonathan Ruel

These types of diffraction can often be seen in pictures or movies. The hole in a camera diaphragm often has a hexagonal, octagonal or even triangular shape, as seen in this picture.

www.fredmiranda.com/forum/topic/839374

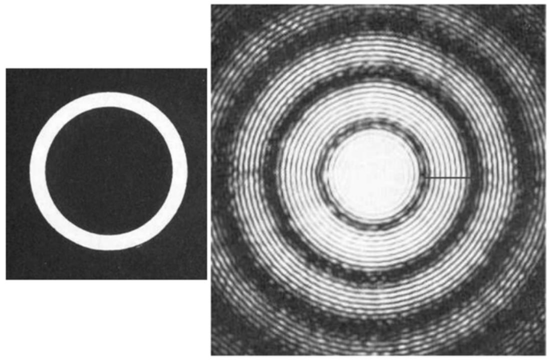
Diffraction patterns can then be seen on the pictures.

www.slrlounge.com/school/diffraction-aperture-and-starburst-effects

The hole in the diaphragm of this device has an octagonal shape. On these images, the diameter of the aperture is different from one image to the next. On the image identified *f*/22 (meaning that the aperture is 22 times smaller than the focal length), the diameter of the opening is very small, and there is a lot of diffraction. As less light can pass through the small opening, the exposure time must be longer. At the other extreme (image identified *f*/2.8), the aperture is larger and there is virtually no diffraction. With a diaphragm letting more light pass, the exposure time can be shorter.

Holes Having the Shape of a Ring

With a hole having the shape of a ring, the following diffraction pattern is obtained.

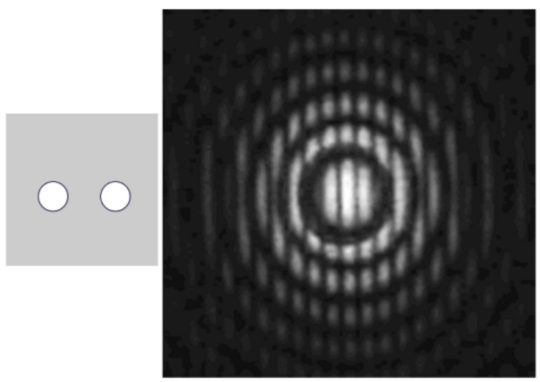


personal.ee.surrey.ac.uk/Personal/D.Jefferies/aperture.html

This looks a little like the pattern with two slits, i.e. the interference and diffraction, but with a circular shape.

Two Circular Hole Side by Side

With two circular holes side by side, the following diffraction pattern is obtained.

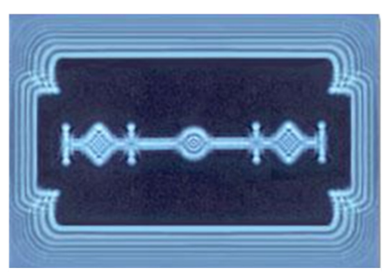


personal.ee.surrey.ac.uk/Personal/D.Jefferies/antennas.html

This looks like the diffraction pattern made by a circular hole superimposed on the interference fringes made by two sources.

A Razor Blade

With a razor blade, the following diffraction pattern is obtained.

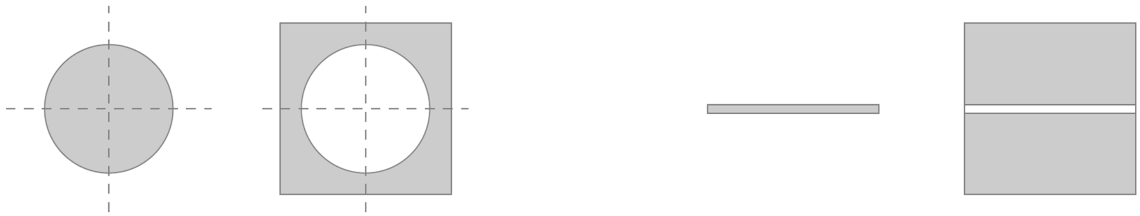


micro.magnet.fsu.edu/primer/lightandcolor/diffractionintro.html

A very characteristic feature of diffraction can be observed: the edges of the shadow are fuzzy. The shade area and the lit area are separated by many maxima and minima. There are also a series of maximum and minimum inside the lit areas in the centre.

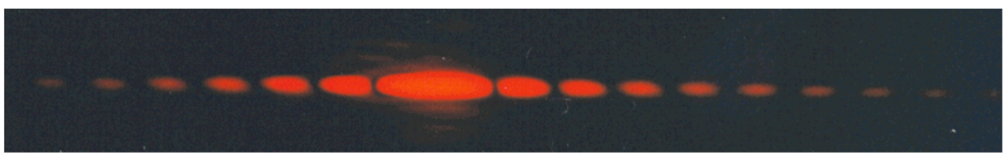
Babinet's Principle

This principle states that the diffraction pattern is the same if it is obtained from an opaque object or from a hole in a plate with the same shape as the object, called the conjugate. Here is an example of an object and its conjugate.



In fact, there are a few restrictions. It applies to the diffraction pattern obtained on a screen very far compared to the dimensions of the object and elsewhere than at $\theta = 0°$.

This means that the diffraction pattern obtained with a hair is identical to the diffraction pattern obtained with a slit, except exactly at the centre of the pattern. Here is a diffraction pattern obtained with a hair in an upright position. It is identical to the pattern obtained with a vertical slit.



chem.lapeer.org/PhysicsDocs/Goals2000/Laser2.html

Here's how this principle can be justified. If there is no object or its conjugate, there is no diffraction at all and the intensity would be zero everywhere, except at $\theta = 0$. However, this situation is equivalent to what is obtained by adding the light coming from the diffraction made by the object and the light coming from the diffraction made by its conjugate. Therefore, this addition must give a vanishing intensity. This means that the two diffraction patterns must have the same intensity, but be out of phase by π.

Coronas

When light coming from the Sun or the Moon passes through a region of the atmosphere where there are water droplets, the droplets act as circular obstacles and there is diffraction. According to Babinet's principle, the diffraction made by a droplet is identical to the diffraction made by a circular hole. With a lot of randomly positioned droplets, the effects of interference disappear and only the diffraction pattern remains. When this happens, a diffraction pattern can be seen around the Sun or the Moon.

en.wikipedia.org/wiki/Corona_(optical_phenomenon)

In this case, the central maximum of diffraction (called the aureole) is, most of the time, the only thing that can be seen. The size of the corona depends on the diameter of the droplets. There is also some colour separation since the central maximum width depends on the wavelength. Since the angle is smaller for smaller wavelengths, blue light makes a smaller central maximum. Thus, the middle of the corona seems rather bluish because blue light is more concentrated there than the other colours. On the other hand, the edges of the corona are red since only red light can be diffracted at such a large angle. With the Sun, the other diffraction maximum can sometimes be seen. To have a clear corona, the size of water droplets must be fairly uniform.

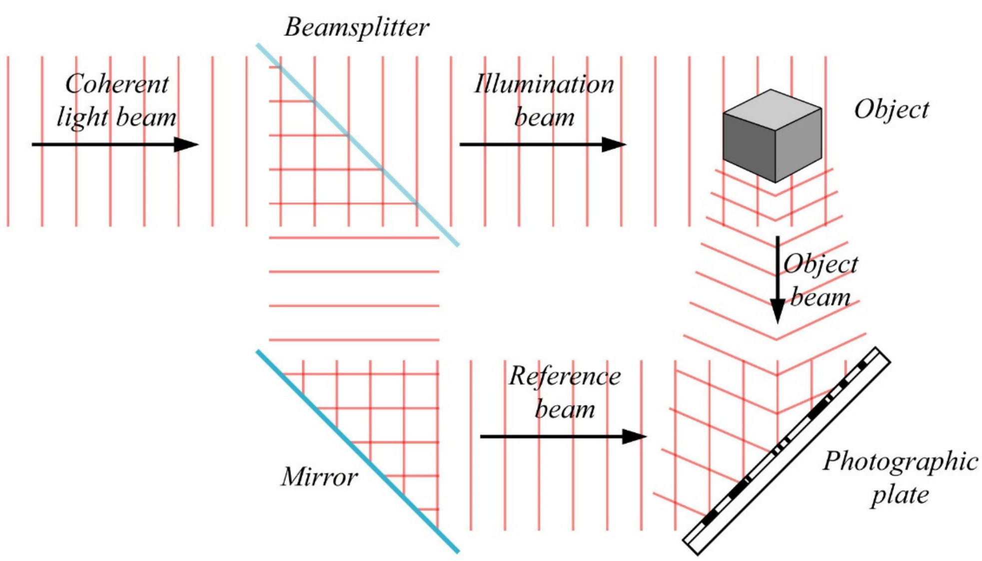
Coronas must not be confused with halos. Halos are formed when light is refracted by ice crystals present in the atmosphere. In its simplest version, the halo is a circle at 11° from the Moon or the Sun.

meteo-alpes.org/drupal/samedi-25-septembre-2010-halo-lunaire-lors-dun-bivouac-au-bord-du-joekulsarlon-islande

Holograms

A hologram is made with interference and diffraction. It's actually a diffraction pattern. It is obtained by letting light pass through a peculiar photographic plate.

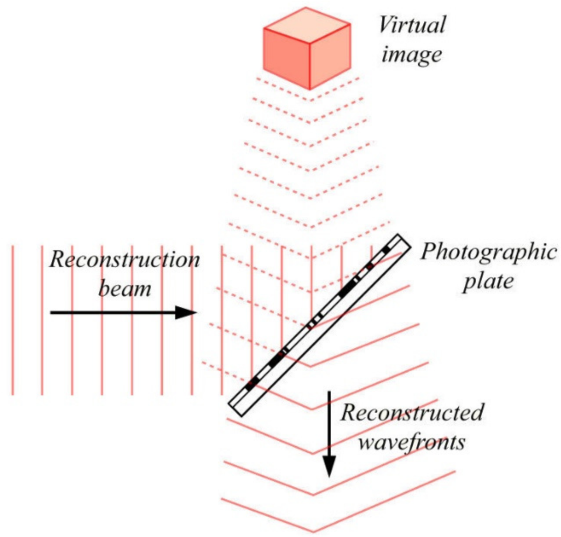
The image on the photographic plate is made in the following way: a monochromatic (a single wavelength, so a single colour) and coherent (that keeps the same phase constant) beam of light passes through a beam splitter that separates the beam into two parts. Some of this light goes, after reflection on a mirror, directly to the photographic plate (it is the reference beam) and the other part illuminates an object (the illumination beam). The photographic plate then captures the waves reflected by the mirror and the waves reflected by the object (the object beam).



en.wikipedia.org/wiki/Holography#How_holography_works

Then, there is interference between the two waves and there is an interference pattern on the plate. As this plate is a photographic plate, a picture of the interference pattern, which is quite complex, is taken. This photographic plate is transparent. It is a glass plate on which the interference pattern is recorded (the plate darkens where there is a maximum).

The plate is then removed and a beam of light (same wavelength as used initially), called the reconstruction beam, is sent through this glass plate. The light can then pass in some places and is blocked in some other places depending on what is engraved on the plate. The light is literally passing through many small misshapen holes. As the light passes through these holes, there are some diffraction and some interference.

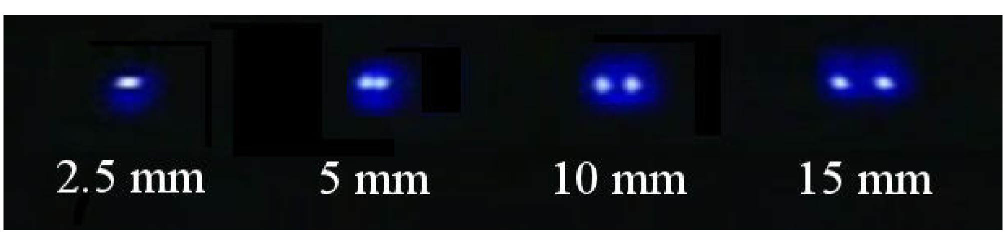


Then something really surprising happens: the wave coming out of the photographic plate due to diffraction and interference is absolutely identical to the wave we would have if the light were coming directly from the object (the object beam). As the wave is identical, there is no difference for the eye between this wave coming from diffraction and interference and the wave coming from the object. Therefore, the viewer sees an object as if it were there.

You can see how a hologram looks on this video.
http://www.youtube.com/watch?v=cKlxsEd7p0w

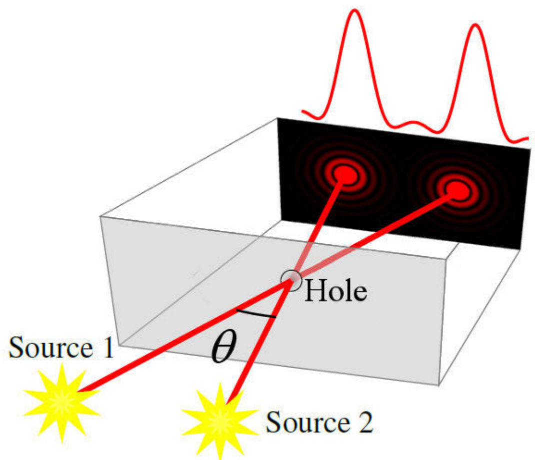
7.5 ANGULAR RESOLUTION

If a person looks at two light sources, he sees them separately provided that they are sufficiently spaced from each other. The following image illustrates this. It is possible to see both light sources when they are separated by 10 mm and 15 mm but it is more difficult if the sources are separated by less than 5 mm.

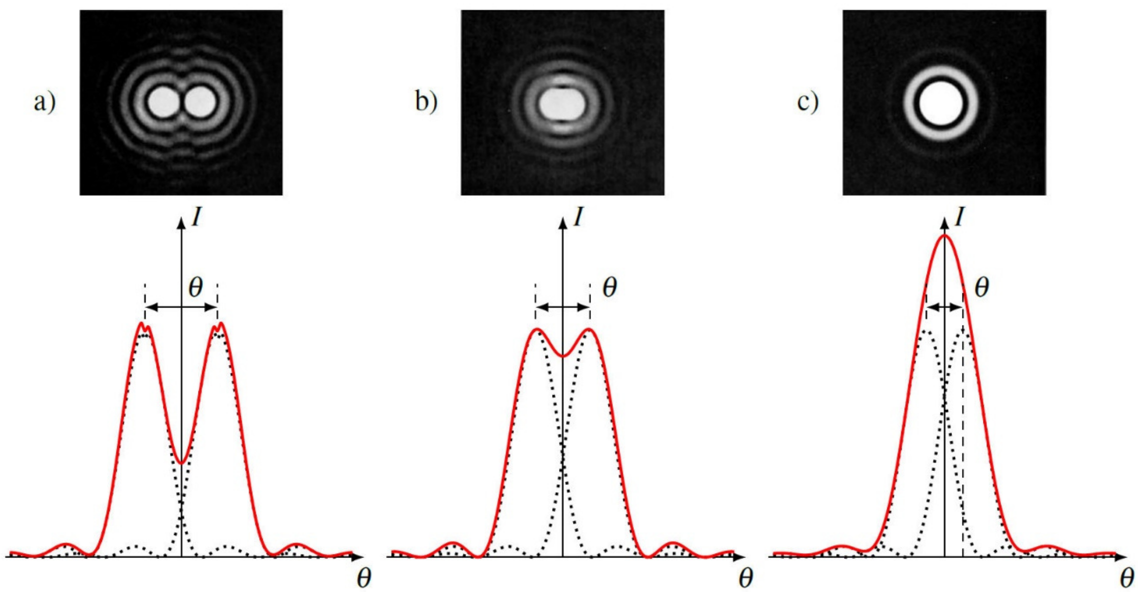


tsgphysics.mit.edu/front/?page=demo.php&letnum=Q%206&show=0

This limit to the resolution of the eye or the optical device comes from light diffraction. When light enters the eye through the pupil or passes through the opening of the optical device, there is some diffraction. So, the light forms a diffraction pattern on the retina or on the photographic plate instead of a single point of light. With two sources, two overlapping diffraction patterns are seen.



The following diagram shows what can be seen with different source separation.



If the angle between the two sources is sufficiently large (case a), then the two diffraction patterns do not overlap much and there are two distinct peaks of intensity in the graph of the resulting intensity. Clearly, two sources can be seen.

If the angle is smaller (case c), then the two diffraction patterns overlap and there is a single peak on the graph of the intensity when the intensities of each diffraction pattern are added. It is then impossible to see two sources.

It is quite difficult to determine exactly the minimum separation angle required in order to be able to distinguish the presence of two sources. A somewhat arbitrary criterion is used: the Rayleigh criterion. According to this criterion, two sources are seen separately if the centre of the central diffraction maximum of one of the sources is at the same place as the first minimum of diffraction of the other source. This corresponds to case b in the diagram. Then, it is barely possible to distinguish the presence of two sources.

The minimum angular separation, called the *critical angle* (θ_c), is thus equal to the angle between the centre of the central maximum and the first minimum. For a circular hole (which is what is found in the eye and several optical instruments), this angle is

Minimum Angular Separation Between Two Sources Required to See Them as Two Distinct Sources

$$\sin\theta_c = 1.22\frac{\lambda}{a}$$

where a is the diameter of the hole.

If the angular separation between the sources is greater than the critical angle, two distinct sources are seen, and if the angular separation is smaller than the critical angle, a single source is seen. Details whose size is smaller than the critical angle cannot be seen.

Example 7.5.1

How far is a car when it is possible to distinguish, with the naked eye, the two front lights of the car? The lights are 1.5 m apart.

To solve this problem, the critical angle must be known. This angle is found with

$$\sin\theta_c = 1.22\frac{\lambda}{a}$$

The wavelength and the diameter of the hole are thus needed.

The lights emit a white light, whose wavelength ranges from 400 nm to 700 nm. An average of 550 nm will be used for the calculation.

But beware, when the light passes through the pupil, it is already in the eye. The wavelength is, therefore, different because the eye is full of fluid whose refractive index is about equal to the refractive index of water. Therefore, the wavelength of light in the eye is

$$\lambda = \frac{\lambda_0}{n} = \frac{550nm}{1.33} = 413.5nm$$

It remains to determine the diameter of the pupil. This diameter may vary depending on the light intensity and can be, at most, about 0.8 cm. (The maximum value is used to have less diffraction. The headlights of the car are easier to distinguish by night because the pupil is very large then). Therefore, the critical angle is

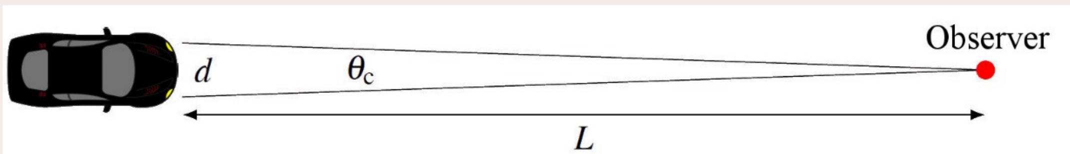
$$\sin\theta_c = 1.22\frac{\lambda}{a}$$

$$\sin\theta_c = 1.22\cdot\frac{413.5\times10^{-9}m}{0.8\times10^{-2}m}$$

$$\sin\theta_c = 6.306\times10^{-5}$$

$$\theta_c = 0.0036°$$

That 0.2′ angle is not very big! (One minute of arc, 1′, is 1/60 of a degree; 1′ is approximately the angle made by a $1 coin at a distance of 90 m.) The distance of the car is then found with this triangle.



As the angle is small, the distance between the headlights is almost equal to the arc of the circle and the angle, in radians, is

$$\theta_{c(rad)} = \frac{d}{L}$$

This leads to

$$L = \frac{d}{\theta_{c(rad)}}$$

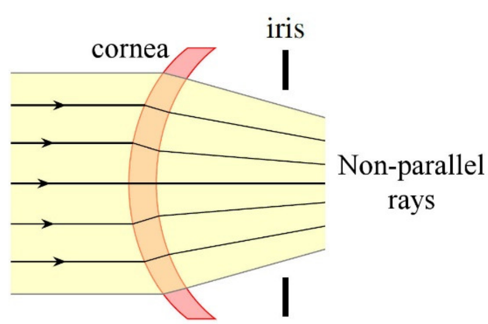
$$= \frac{1,5m}{6.306\times10^{-5}rad}$$

$$= 23\,780m = 23.8km$$

So, we can say that we are able to see that there are two headlights when the car is closer than about 20 kilometres. This is an approximate value because of the approximations made and the somewhat arbitrary definition of the Rayleigh criterion.

This seems like a lot. In reality, the distance is less than this distance because of defects in the eye and air turbulence.

To be honest, there was a bit of cheating in this last example. The diffraction formula obtained in this chapter is valid if the rays are virtually all parallel to each other when they pass through the hole. (This condition was set when it was assumed that the phase difference between each source in the slit was the same.) This means that the distance of the light sources must be large compared to the diameter of the hole. Here, with the sources tens of



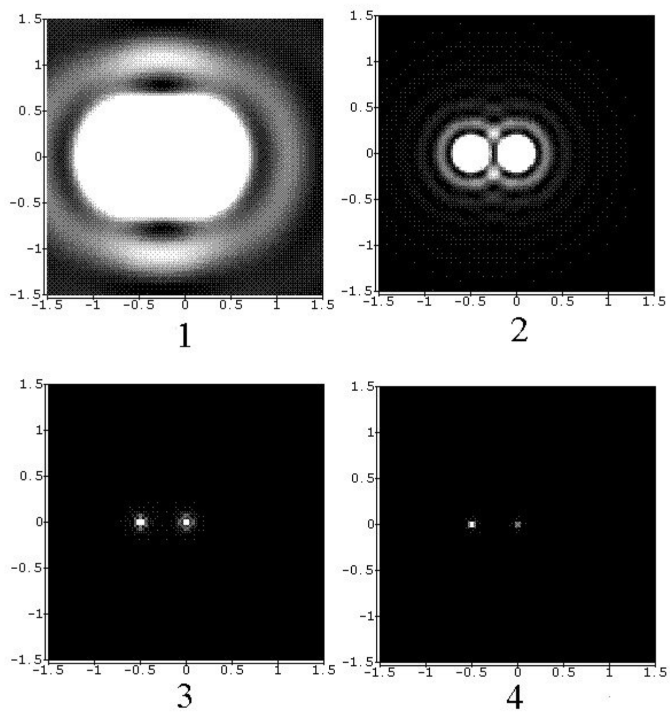
kilometres away, the rays arriving at the eye are indeed almost parallel, and it seems that the condition is met. However, the situation changes somewhat when the rays enter the eye. When they are refracted by the cornea, the rays are no longer parallel when they pass through the pupil.

So, the diffraction formula used is not really valid. Actually, there is more diffraction than what was calculated, and what was done can be considered as just an approximation. To obtain an exact value of the distance, a calculation of the diffraction with non-parallel rays is needed. This calculation is way much more difficult (and not really college-level). Considering this effect, the theoretical angular resolution of the eye does not increase much and is about 0.35′ (10^{-4} rad).

The actual angular resolution of the eye, coming from diffraction and other defects of the eye, is rather about 1.7′ (5×10^{-4} rad). (Under optimal conditions, some people can go up to 0,7′, i.e. 2×10^{-4} rad.) It's still pretty remarkable that the actual angular resolution of the eye is very close to the theoretical angular resolution limit. Note that the angular resolution needed to have a 20/20 vision is 4′ (12×10^{-4} rad).

Let's take a look at what this means for a telescope. Here are four pictures of two stars separated by 0.5″ (1″ is 1°/3600) with different telescopes.

The first image was obtained with a 15 cm diameter telescope, for which the critical angle is 1″. It is then difficult to see that there are two stars. The second image was made with a 50 cm diameter telescope for which the critical angle is 0.3″. It is now possible to see that there are two stars, but a lot of diffraction is still present. The third image was made with a 2.4 m diameter telescope having a critical angle of 0.06″. The two stars are now seen as separated sources and the effect of diffraction has diminished.



www.astro.ljmu.ac.uk/courses/phys134/scopes.html

The last image was made with a 5.1 m diameter telescope having a critical angle of 0.03″. There is almost no diffraction at all.

A large diameter is also advantageous: a lot more light is captured. The first image was obtained with an exposure time of 30 minutes while the last image was obtained with an exposure time of 1.6 seconds.

Example 7.5.2

What is the size of the smallest object that can be seen on the Moon, from the surface of the Earth, with a 114 mm diameter telescope?

As white light has wavelengths ranging from 400 nm to 700 nm, an average of 550 nm will be used for the calculation.

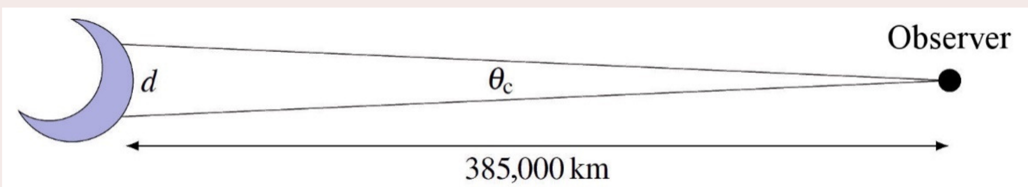
The critical angle of the telescope is

$$\sin\theta_c = 1.22\frac{\lambda}{a}$$

$$\sin\theta_c = 1.22\cdot\frac{550\times10^{-9}m}{114\times10^{-3}m}$$

$$\theta_c = 0.00034°$$

The distance between objects on the Moon is then found with this triangle.



Since the angle is small, we can say that the distance between objects on the Moon is an arc of a circle so that the angle, in radians, is

$$\theta_{c(rad)} = \frac{d}{L}$$

Therefore,

$$\begin{aligned} d &= L\theta_{c(rad)} \\ &= 3.85\times10^{8} m\cdot 5.89\times10^{-6} rad \\ &= 2.27 km \end{aligned}$$

It is thus impossible to see the footprints left by the astronauts on the Moon.

Diffraction also limits the resolution of microscopes. Although it will not be proven, the maximum resolution of microscopes is equal to about half the wavelength of the light used. So, details whose size is less than about 200 nm cannot be seen with a microscope. Therefore, it is impossible to see individual atoms, whose size is about 1 nm, with a microscope.

SUMMARY OF EQUATIONS

Phase Difference Between Opposite Side of the Slit

$$\alpha = \frac{a\sin\theta}{\lambda}2\pi$$

Light Intensity for Diffraction by a Slit

$$I = I_0\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2$$

Angle of the Minima for Diffraction With a Slit

$a\sin\theta = M\lambda$ where M is a non-zero integer.

Half Width of the Central Maximum

$$\sin\theta = \frac{\lambda}{a}$$

Light Intensity in Young's Experiment

$$I_{tot} = 4I_{10}\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2\cos^2\frac{\Delta\phi}{2}$$

Number of Interference Maxima Within the Central Maximum of Diffraction

Calculate $\frac{d}{a}$

If an integer is obtained, subtract 1 to get m_d.
If a non-integer is obtained, remove the decimals to get m_d.

$$Number = 2m_d + 1$$

Angle of the First Diffraction Minimum for a Circular Hole

$$\sin\theta = 1.22\frac{\lambda}{a}$$

Minimum Angular Separation between Two Sources Required to See Them as Two Distinct Sources

$$\sin\theta_c = 1.22\frac{\lambda}{a}$$

EXERCISES

7.2 Diffraction by a Single Slit

1. Light having a 500 nm wavelength passes through a 0.01 mm wide slit. The diffraction pattern is on a screen located 2 m away from the slit.

   a) What is the distance between the centre of the central maximum and the first minimum?
   b) What is the distance between the centre of the central maximum and the second minimum?

2. The central maximum is 4 cm wide when the diffraction pattern is observed on a screen located 5 m away from a slit. What is the width of the slit if the wavelength of the light is 560 nm?

3. Microwaves pass through a 1 cm wide slit. On a screen located 1.6 m away from the slit, the central maximum is 50 cm wide. What is the wavelength of the microwaves?

4. Light having a 600 nm wavelength passes through a 0.1 mm wide slit. The diffraction pattern is observed on a screen located 2 m away from the slit. What is the intensity 0.5 cm away from the centre of the central maximum?

5. When violet light, whose wavelength is 450 nm, passes through a slit, the central maximum is 4 cm wide on a screen located 3 m away from the slit. What will the width of the central maximum be if the light is changed for red light, whose wavelength is 650 nm?

6. The central maximum is 10 cm wide on a screen located 4 m away from a slit. What is the distance between the first minimum and the second minimum?

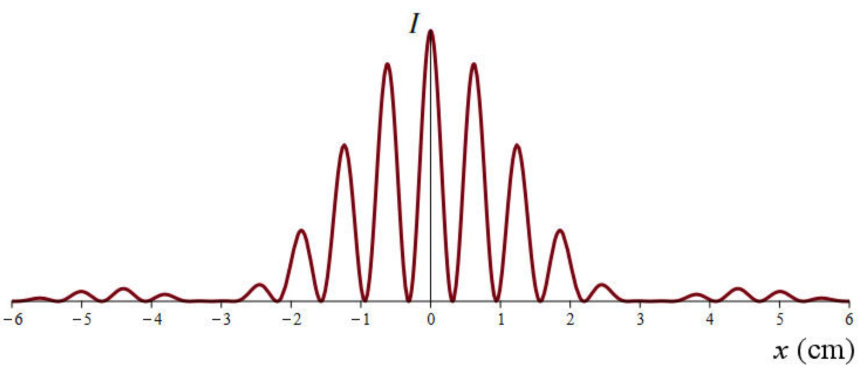
7. Waves pass through a slit. Then, the angle of the first minimum is $\theta = 20°$. What are the angles of the other minima?

7.3 Light Intensity with Many Slits

8. In Young's experiment, light having a wavelength of 600 nm passes through two 0.04 mm wide slits 0.2 mm apart. The interference pattern is on a screen 2.4 m away from the slits.

   a) How many interference maxima are there inside the central diffraction maximum?
   b) What is the intensity of the light 3 cm away from the centre of the central maximum of interference?
   c) What is the intensity of the first order interference maximum compared to the intensity of the central maximum of interference?

9. This is the graph of the light intensity obtained on a screen when light having a wavelength of 650 nm passes through two slits. The distance between the screen and the slits is 2 m.



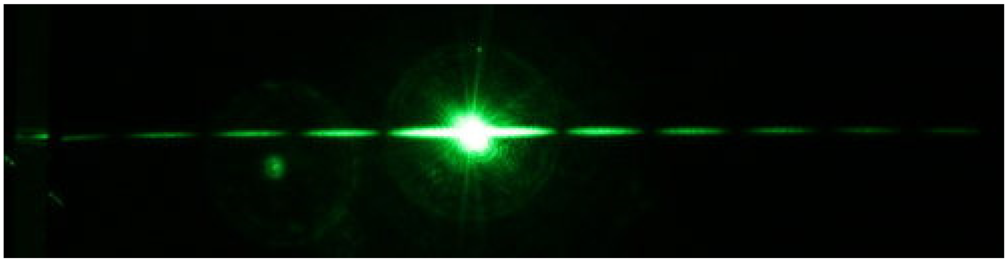
   a) What is the distance between the slits?
   b) What is the width of the slits?
   (The answers are approximations because the values must be estimated on the graph.)

7.4 Diffraction Patterns for Holes or Obstacles Having Other Shapes

10. Light having a 560 nm wavelength passes through a circular hole whose diameter is 0.1 mm. The diffraction pattern is observed on a screen located 2 m away from the hole. What is the distance between the centre of the central maximum and the first minimum?

11. Light having a 620 nm wavelength passes through a circular hole. On a screen located 1.8 m away from the hole, the central maximum has a diameter of 6 mm. What is the diameter of the hole?

12. This is the diffraction pattern obtained when a hair is placed in the path of a laser with a 523 nm wavelength. The first diffraction minimum is 6.5 cm from the centre of the diffraction pattern. The screen is 9.67 m away from the hair. What is the diameter of the hair?



ujap.de/index.php/view/HairMeasurement

7.5 Angular Resolution

13. Two objects are 2 cm apart. From what maximum distance can these two items be seen separately with the naked eye if the diameter of the pupil is 3 mm? (Use the circular hole formula as an approximation.)

14. A spy satellite is at an altitude of 200 km. This satellite observes the surface of the Earth with a 25 cm diameter telescope. What is the maximum distance that there can be between two objects on the surface of the Earth so that the satellite can perceive that there are two objects?

15. Two stars are located 5 light years from Earth (so at 4.73×10^{13} km). What is the minimum telescope diameter required to distinguish the two stars if the distance between the stars is 80 million km?

Challenges

(Questions more difficult than the exam questions.)

16. Light having a 600 nm wavelength passes through a 0.1 mm wide slit. At what distance from the centre of the central maximum is located the first diffraction maximum if the screen is 2 m away from the slit? (The exact distance is sought, not an approximation.)

    If all goes according to plan, you will come upon an equation that you are not able to solve. Maybe this internet site can help you.
    http://www.wolframalpha.com/input/?i=x%3D2*sinx

ANSWERS

7.2 Diffraction by a Single Slit

1. a) 10.0 cm b) 20.1 cm
2. 0.14 mm
3. 1.544 mm
4. $0.5445I_0$
5. 5.778 cm
6. 5.002 cm
7. The second minimum is at 43.16°. There is no other minimum.

7.3 Light Intensity with Many Slits

8. a) 9 b) $0.1097I_0$ c) 0.875
9. a) 0.2 mm b) 0.04 mm

7.4 Diffraction Patterns for Holes or Obstacles Having Other Shapes

10. 1.366 cm
11. 0.4538 mm
12. 77.8 μm

7.5 Angular Resolution

13. 118.9 m
14. 53.7 cm
15. 39.7 cm

Challenges

16. 1.716 cm