Chapter 4 Solutions

1. a) The speed will be found with

$$v = \sqrt{\frac{F_T}{\mu}}$$

We have μ but we need the tension.

The tension in the rope is equal to the weight of the block. This weight is

$$\begin{aligned} F_T &= mg \\ &= 2kg \cdot 9.8\tfrac{N}{kg} \\ &= 19.6N \end{aligned}$$

The wave speed in the aluminum wire is therefore

$$\begin{aligned} v &= \sqrt{\frac{F_T}{\mu}} \\ &= \sqrt{\frac{19.6N}{0.01\frac{kg}{m}}} \\ &= 44.27\tfrac{m}{s} \end{aligned}$$

b) The frequency is

$$\begin{aligned} v &= \lambda f \\ 44.27\tfrac{m}{s} &= 0.2m \cdot f \\ f &= 221.4Hz \end{aligned}$$

c) The wave speed in the steel wire is

$$\begin{aligned} v &= \sqrt{\frac{F_T}{\mu}} \\ &= \sqrt{\frac{19.6N}{0.025\frac{kg}{m}}} \\ &= 28\tfrac{m}{s} \end{aligned}$$

d) As the frequency does not change from one medium to another, it is still 221.4 Hz.

e) The wavelength is

$$v = \lambda f$$
$$28\tfrac{m}{s} = \lambda \cdot 221.4Hz$$
$$\lambda = 0.1265m$$

f) The impedance is

$$\begin{aligned} Z_1 &= \sqrt{F_T \mu} \\ &= \sqrt{19.6N \cdot 0.01\tfrac{kg}{m}} \\ &= 0.4427\tfrac{kg}{s} \end{aligned}$$

g) The impedance is

$$\begin{aligned} Z_2 &= \sqrt{F_T \mu} \\ &= \sqrt{19.6N \cdot 0.025\tfrac{kg}{m}} \\ &= 0.7\tfrac{kg}{s} \end{aligned}$$

h) The amplitude of the reflected wave is

$$\begin{aligned} A_R &= \frac{Z_1 - Z_2}{Z_1 + Z_2} A \\ &= \frac{0.4427\tfrac{kg}{s} - 0.7\tfrac{kg}{s}}{0.4427\tfrac{kg}{s} + 0.7\tfrac{kg}{s}} \cdot 5mm \\ &= -1.126mm \end{aligned}$$

It is therefore an inverted wave with an amplitude of 1.126 mm.

i) The amplitude of the transmitted wave is

$$\begin{aligned} A_T &= \frac{2Z_1}{Z_1 + Z_2} A \\ &= \frac{2 \cdot 0.4427\tfrac{kg}{s}}{0.4427\tfrac{kg}{s} + 0.7\tfrac{kg}{s}} \cdot 5mm \\ &= 3.874mm \end{aligned}$$

j) The power of the incident wave is

$$\begin{aligned} P &= \frac{1}{2}\mu v\omega^2 A^2 \\ &= \frac{1}{2} Z_1 \omega^2 A^2 \end{aligned}$$

The power of the transmitted wave is

$$\begin{aligned} P_T &= \frac{1}{2}\mu v\omega^2 A^2 \\ &= \frac{1}{2} Z_2 \omega^2 A^2 \end{aligned}$$

The ratio of the power transmitted and the initial power is

$$\begin{aligned} \frac{P_T}{P} &= \frac{\frac{1}{2} Z_2 \omega^2 A_T^2}{\frac{1}{2} Z_1 \omega^2 A^2} \\ &= \frac{Z_2 A_T^2}{Z_1 A^2} \\ &= \frac{0.7 \frac{kg}{s} \cdot (0.003874m)^2}{0.4427 \frac{kg}{s} \cdot (0.005m)^2} \\ &= 0.949 \end{aligned}$$

94,9% of the power is transmitted.

2. a)

The transmitted wave is always in the same direction as the original wave. It will be upwards.

b) Since the density of the rope to the right is greater, the reflected wave will be reversed, so downwards.

c) Since the speed is 20 m/s on the rope to the left, the tension is

$$v = \sqrt{\frac{F_T}{\mu}}$$
$$20\tfrac{m}{s} = \sqrt{\frac{F_T}{0.02\frac{kg}{m}}}$$
$$F_T = 8N$$

The speed of the wave on the rope to the right is then

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{8N}{0.05\frac{kg}{m}}} = 12.65\tfrac{m}{s}$$

d) The reflected wave being on the rope to the left, its speed is the same as the initial wave, so 20 m/s.

3. The power of a wave is given by

$$P = \frac{1}{2}\mu v\omega^2 A^2$$

We know that the power of the reflected wave is equal to 50% of the power of the initial wave. This means that

$$\begin{aligned} 0.5 &= \frac{P_R}{P} \\ &= \frac{\frac{1}{2}\mu v\omega^2 A_R^2}{\frac{1}{2}\mu v\omega^2 A^2} \\ &= \left(\frac{A_R}{A}\right)^2 \end{aligned}$$

Since the amplitude of the reflected wave is

$$A_R = \frac{Z_1 - Z_2}{Z_1 + Z_2} A$$

the equation becomes

$$0.5 = \left(\frac{A_R}{A} \right)^2$$

$$= \left(\frac{\frac{Z_1 - Z_2}{Z_1 + Z_2} A}{A} \right)^2$$

$$= \left(\frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2$$

$$= \left(\frac{1 - \frac{Z_2}{Z_1}}{1 + \frac{Z_2}{Z_1}} \right)^2$$

Setting $Z_2/Z_1 = x$ (which is what we're looking for), we arrive at

$$0.5 = \left(\frac{1 - x}{1 + x} \right)^2$$

It only remains to solve for x.

$$\pm\sqrt{0.5} = \frac{1 - x}{1 + x}$$

To simplify, let's set $C = \pm\sqrt{0.5}$. Therefore,

$$C = \frac{1 - x}{1 + x}$$

$$C(1 + x) = 1 - x$$

$$C + Cx = 1 - x$$

$$x + Cx = 1 - C$$

$$x(1 + C) = 1 - C$$

$$x = \frac{1 - C}{1 + C}$$

If $C = \sqrt{0,5}$, the result is

$$x = \frac{1 - \sqrt{0.5}}{1 + \sqrt{0.5}} = 0.1716$$

This cannot be the right answer because the impedance of the second string is greater than the impedance of the first rope (this means that $Z_2/Z_1 > 1$)

If $C=-\sqrt{0.5}$, the result is

$$x=\frac{1--\sqrt{0.5}}{1+-\sqrt{0.5}}=5.828$$

This is the right answer.

4. a) The impedance is

$$Z=\rho v$$

To calculate it, we need the speed of the wave.

At this temperature, the wave speed is

$$\begin{aligned}v&=331.3\tfrac{m}{s}\cdot\sqrt{\frac{T}{273.15K}}\\&=331.3\tfrac{m}{s}\cdot\sqrt{\frac{288.15K}{273.15K}}\\&=340.3\tfrac{m}{s}\end{aligned}$$

Therefore, the air impedance is

$$\begin{aligned}Z&=\rho v\\&=1.3\tfrac{kg}{m^3}\cdot 340.3\tfrac{m}{s}\\&=442.4\tfrac{kg}{m^2\cdot s}\end{aligned}$$

b) The water impedance is

$$\begin{aligned}Z&=\rho v\\&=1000\tfrac{kg}{m^3}\cdot 1520\tfrac{m}{s}\\&=1,520,000\tfrac{kg}{m^2\cdot s}\end{aligned}$$

c) No, because the water impedance is too different from the air impedance (3435 times greater).

5. a) At 60 dB, the intensity is

$$\beta = 10dB \cdot \log \frac{I}{10^{-12} \frac{W}{m^2}}$$

$$60dB = 10dB \cdot \log \frac{I}{10^{-12} \frac{W}{m^2}}$$

$$I = 10^{-6} \frac{W}{m^2}$$

The wave amplitude is then

$$I = \frac{1}{2} \rho v \omega^2 A^2$$

$$10^{-6} \frac{W}{m^2} = \frac{1}{2} \cdot 1.3 \frac{kg}{m^3} \cdot 330 \frac{m}{s} \cdot \left(2\pi \cdot 200Hz\right)^2 \cdot A^2$$

$$A = 5.433 \times 10^{-8} m$$

To find the amplitude of the wave transmitted in water, the impedances of water and air are needed. These impedances are

$$Z_1 = \rho v = 1.3 \frac{kg}{m^3} \cdot 330 \frac{m}{s} = 429 \frac{kg}{m^2 s}$$

$$Z_2 = \rho v = 1000 \frac{kg}{m^3} \cdot 1450 \frac{m}{s} = 1{,}450{,}000 \frac{kg}{m^2 s}$$

The amplitude of the transmitted wave is therefore

$$\begin{aligned} A_T &= \frac{2Z_1}{Z_1 + Z_2} A \\ &= \frac{2 \cdot 429 \frac{kg}{m^2 s}}{429 \frac{kg}{m^2 s} + 1{,}450{,}000 \frac{kg}{m^2 s}} \cdot 5.4335 \times 10^{-8} m \\ &= 3.214 \times 10^{-11} m \end{aligned}$$

The intensity of this wave in water is

$$\begin{aligned} I &= \frac{1}{2} \rho v \omega^2 A^2 \\ &= \frac{1}{2} \cdot 1000 \frac{kg}{m^3} \cdot 1450 \frac{m}{s} \cdot \left(2\pi \cdot 200Hz\right)^2 \cdot \left(3.214 \times 10^{-11} m\right)^2 \\ &= 1.183 \times 10^{-9} \frac{W}{m^2} \end{aligned}$$

In decibels, this intensity is

$$\beta = 10dB \cdot \log \frac{I}{10^{-12} \frac{W}{m^2}}$$
$$= 10dB \cdot \log \frac{1.183 \times 10^{-9} \frac{W}{m^2}}{10^{-12} \frac{W}{m^2}}$$
$$= 30.7dB$$

The amplitude of the reflected wave is

$$A_R = \frac{Z_1 - Z_2}{Z_1 + Z_2} A$$
$$= \frac{429 \frac{kg}{m^2 s} - 1,450,000 \frac{kg}{m^2 s}}{429 \frac{kg}{m^2 s} + 1,450,000 \frac{kg}{m^2 s}} \cdot 5.4335 \times 10^{-8} m$$
$$= -5.4302 \times 10^{-8} m$$

The intensity of this wave in air is

$$I = \frac{1}{2} \rho v \omega^2 A^2$$
$$= \frac{1}{2} \cdot 1.3 \frac{kg}{m^3} \cdot 330 \frac{m}{s} \cdot (2\pi \cdot 200Hz)^2 \cdot (5.4302 \times 10^{-8} m)^2$$
$$= 9.988 \times 10^{-7} \frac{W}{m^2}$$

In decibels, this intensity is

$$\beta = 10dB \cdot \log \frac{I}{10^{-12} \frac{W}{m^2}}$$
$$= 10dB \cdot \log \frac{9.988 \times 10^{-7} \frac{W}{m^2}}{10^{-12} \frac{W}{m^2}}$$
$$= 59.99dB$$

b) The difference of intensity (in decibel) is

$$\beta_T - \beta = 10dB \cdot \log \frac{I_T}{10^{-12} \frac{W}{m^2}} - 10dB \cdot \log \frac{I}{10^{-12} \frac{W}{m^2}}$$
$$= 10dB \left(\log \frac{I_T}{10^{-12} \frac{W}{m^2}} - \log \frac{I}{10^{-12} \frac{W}{m^2}} \right)$$

As a subtraction of two logarithms is equal to the logarithm of the division, we have

$$\begin{aligned}\beta_T - \beta &= 10dB\left(\log\frac{I_T}{10^{-12}\frac{W}{m^2}} - \log\frac{I}{10^{-12}\frac{W}{m^2}}\right)\\ &= 10dB\cdot\log\frac{\left(\frac{I_T}{10^{-12}\frac{W}{m^2}}\right)}{\left(\frac{I}{10^{-12}\frac{W}{m^2}}\right)}\\ &= 10dB\cdot\log\frac{I_T}{I}\end{aligned}$$

Thus, we need the intensity ratio. This ratio is

$$\frac{I_T}{I} = \frac{\frac{1}{2}Z_2\omega^2 A_T^2}{\frac{1}{2}Z_1\omega^2 A^2}$$

$$\frac{I_T}{I} = \frac{Z_2 A_T^2}{Z_1 A^2}$$

Since the amplitude of the transmitted sound is

$$A_T = \frac{2Z_1}{Z_1 + Z_2}A$$

we have

$$\frac{I_T}{I} = \frac{Z_2}{Z_1 A^2}\left(\frac{2Z_1}{Z_1+Z_2}\right)^2 A^2$$

$$\frac{I_T}{I} = \frac{Z_2}{Z_1}\left(\frac{2Z_1}{Z_1+Z_2}\right)^2$$

$$\frac{I_T}{I} = \frac{4Z_1Z_2}{\left(Z_1+Z_2\right)^2}$$

Using the values of the impedances, this ratio is

$$\begin{aligned}\frac{I_T}{I} &= \frac{4\cdot 429\frac{kg}{m^2s}\cdot 1\,450\,000\frac{kg}{m^2s}}{\left(429\frac{kg}{m^2s}+1\,450\,000\frac{kg}{m^2s}\right)^2}\\ &= 1.1827\times 10^{-3}\end{aligned}$$

Thus,

$$\begin{aligned}\beta_T - \beta &= 10dB \cdot \log \frac{I_T}{I} \\ &= 10dB \cdot \log\left(1.1827\times 10^{-3}\right) \\ &= -29.27dB\end{aligned}$$

This means that the intensity always decreases by 29.3 dB.

6. The wavelength is

$$\begin{aligned}\lambda_{\text{substance}} &= \frac{\lambda_{\text{vacuum}}}{n} \\ &= \frac{500nm}{1.33} \\ &= 375.9nm\end{aligned}$$

7. The speed of light is

$$v = \frac{c}{n}$$

We need the refractive index to obtain the speed.

The refractive index is found with

$$\begin{aligned}\lambda_{\text{substance}} &= \frac{\lambda_{\text{vacuum}}}{n} \\ 480nm &= \frac{600nm}{n} \\ n &= 1.25\end{aligned}$$

Therefore, the speed of light is

$$\begin{aligned}v &= \frac{c}{n} \\ &= \frac{299,792,458 \frac{m}{s}}{1.25} \\ &= 2.4\times 10^{8} \frac{m}{s}\end{aligned}$$

8. To calculate the intensity, we need the amplitude of the waves. These amplitudes can be found from the amplitude of the incident wave and the index of refraction of the media.

The amplitude of the incident wave is

$$I = \frac{cn\varepsilon_0 E_0^2}{2}$$

$$5\tfrac{W}{m^2} = \frac{3\times10^8 \tfrac{m}{s}\cdot 1\cdot 8.854\times10^{-12}\tfrac{C^2}{Nm^2}\cdot E_0^2}{2}$$

$$E_0 = 61.36\tfrac{N}{C}$$

Thus, the amplitude of the reflected and transmitted waves are

$$\begin{aligned} E_{0R} &= \frac{n_1 - n_2}{n_1 + n_2} E_0 \\ &= \frac{1-1.33}{1+1.33}\cdot 61.36\tfrac{N}{C} \\ &= -8.690\tfrac{N}{C} \end{aligned}$$

$$\begin{aligned} E_{0T} &= \frac{2n_1}{n_1 + n_2} E_0 \\ &= \frac{2\cdot 1}{1+1.33}\cdot 61.36\tfrac{N}{C} \\ &= 52.67\tfrac{N}{C} \end{aligned}$$

This means that the intensity of the transmitted and reflected waves are

$$\begin{aligned} I_R &= \frac{cn\varepsilon_0 E_{0R}^2}{2} \\ &= \frac{3\times10^8 \tfrac{m}{s}\cdot 1\cdot 8.854\times10^{-12} \tfrac{C^2}{Nm^2}\cdot\left(-8.690\tfrac{N}{C}\right)^2}{2} \\ &= 0.100\tfrac{W}{m^2} \end{aligned}$$

$$\begin{aligned} I_T &= \frac{cn\varepsilon_0 E_{0T}^2}{2} \\ &= \frac{3\times10^8 \tfrac{m}{s}\cdot 1.33\cdot 8{,}854\times10^{-12} \tfrac{C^2}{Nm^2}\cdot\left(52.67\tfrac{N}{C}\right)^2}{2} \\ &= 4.900\tfrac{W}{m^2} \end{aligned}$$

Therefore, 98% of the energy is transmitted.

9. The ratio of the intensities is

$$\begin{aligned} \frac{I_T}{I} &= \frac{\frac{1}{2}cn_2\varepsilon_0 E_{0T}^2}{\frac{1}{2}cn_1\varepsilon_0 E_0^2} \\ &= \frac{n_2 E_{0T}^2}{n_1 E_0^2} \end{aligned}$$

However, the amplitude of the transmitted wavelength is

$$E_{0T} = \frac{2n_1}{n_1+n_2}E_0$$

Thus,

$$\begin{aligned} \frac{I_T}{I} &= \frac{n_2 E_{0T}^2}{n_1 E_0^2} \\ &= \frac{n_2}{n_1 E_0^2}\left(\frac{2n_1}{n_1+n_2}\right)^2 E_0^2 \\ &= \frac{4n_2 n_1}{\left(n_1+n_2\right)^2} \end{aligned}$$

Since the intensity ratio is 0.92 and $n_1 = 1$, this equation becomes

$$\frac{I_T}{I} = \frac{4n_2 n_1}{(n_1 + n_2)^2}$$
$$0.92 = \frac{4n_2 \cdot 1}{(1 + n_2)^2}$$

It only remains to solve this equation for n_2.

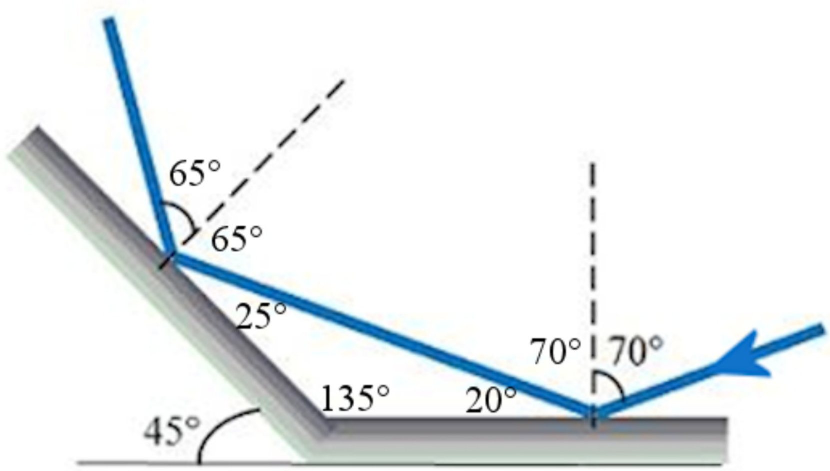
$$0.92 = \frac{4n_2 \cdot 1}{(1 + n_2)^2}$$
$$0.92(1 + n_2)^2 = 4n_2$$
$$0.92 + 1.84n_2 + 0.92n^2 = 4n_2$$
$$0.92 - 2.16n_2 + 0.92n^2 = 0$$

The solutions of this quadratic equation are

1.7888 et 0.5590

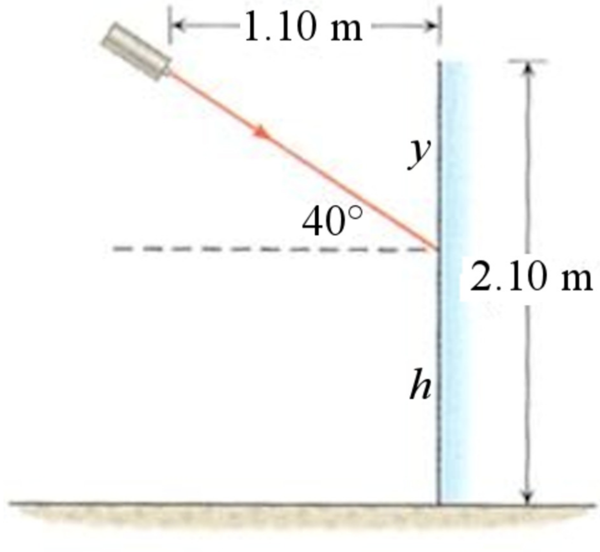
Only the first solution is possible.

10. We have the following angles.



The 25° angle was obtained from the fact that the sum of the angles of a triangle must be 180°.

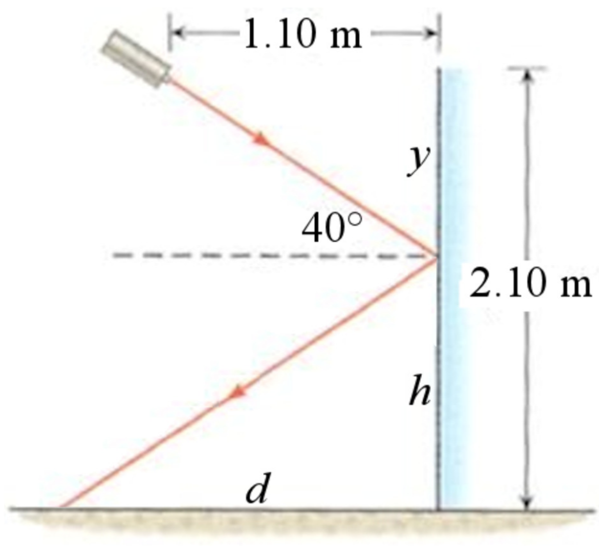
11. The height h at which the laser strikes the mirror can be found.



In the diagram, we have

$$\tan 50° = \frac{1.10m}{y}$$
$$y = 0.923m$$

The height is therefore h = 2.100 m – 0.923 m = 1.177 m. Then the distance to the ground is found with



$$\tan 50° = \frac{d}{1.117m}$$
$$d = 1.403m$$

12. a) According to the law of refraction, the index is

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$n_X \cdot \sin(25°) = 1.33 \cdot \sin(48°)$$
$$n_X = 2.34$$

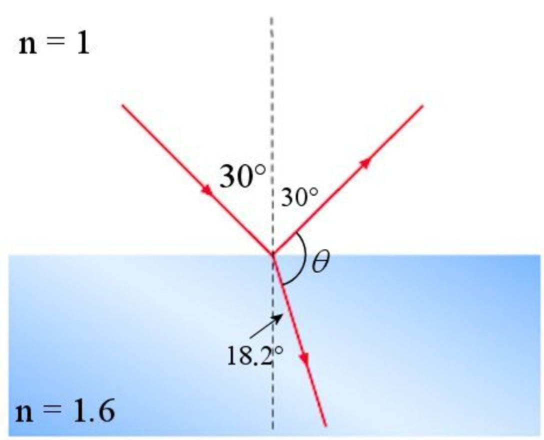
b) The speed of light in the unknown substance is

$$v = \frac{c}{n}$$
$$= \frac{3\times10^8 \frac{m}{s}}{2.34}$$
$$= 1.28\times10^8 \frac{m}{s}$$

13. The angle of refraction is

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1 \cdot \sin(30°) = 1.6 \cdot \sin\theta_2$$
$$\theta_2 = 18.21°$$

Therefore, we have



The angle is thus

$$30^\circ + \theta + 18.21^\circ = 180^\circ$$
$$\theta = 131.79^\circ$$

14. The temperature will be found with

$$v = 331.3\tfrac{m}{s} \cdot \sqrt{\frac{T}{273.15K}}$$

We need the speed to obtain the temperature. The speed will be found with the law of refraction

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

We know both angles but we need the speed of the wave in the first medium. This speed is

$$\begin{aligned} v &= 331.3\tfrac{m}{s} \cdot \sqrt{\frac{T}{273.15K}} \\ &= 331.3\tfrac{m}{s} \cdot \sqrt{\frac{293.15K}{273.15K}} \\ &= 343.2\tfrac{m}{s} \end{aligned}$$

Then, the speed of the wave in the 2^{nd} region can be found.

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$
$$\frac{\sin 45^\circ}{\sin 50^\circ} = \frac{343.2\tfrac{m}{s}}{v_2}$$
$$v_2 = 371.8\tfrac{m}{s}$$

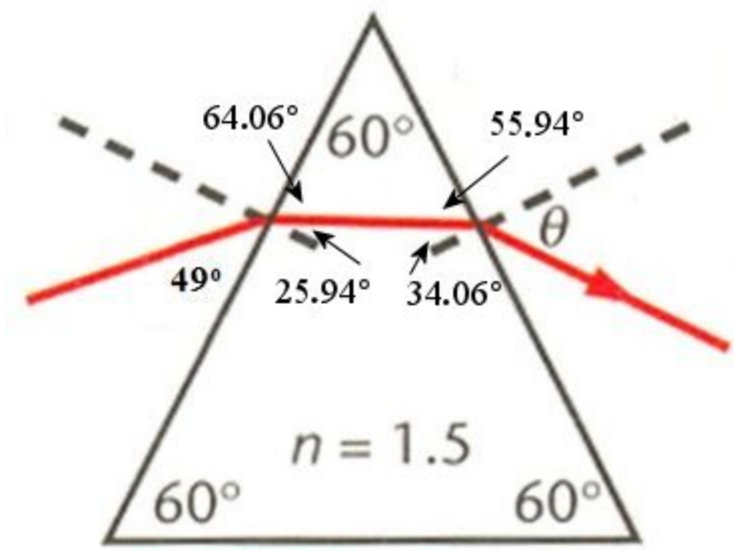
Thus, the temperature is

$$v = 331.3\tfrac{m}{s} \cdot \sqrt{\frac{T}{273.15K}}$$
$$371.8\tfrac{m}{s} = 331.3\tfrac{m}{s} \cdot \sqrt{\frac{T}{273.15K}}$$
$$T = 344.1K$$
$$T = 70.9°C$$

15. The angle θ_2 is found with the law of refraction.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1 \cdot \sin(41°) = 1.5 \cdot \sin\theta_2$$
$$\theta_2 = 25.94°$$

Therefore, we have



This is how these angles were found.

64.06°: We have this angle because 25.94° and 64.06° must be equal to 90° when added.

55.94°: We have this angle because the sum of the angles of a triangle is 180°. Thus, we must have 64.06°+60°+55.94° = 180°.

34.06°: We have this angle because 55.94° and 34.06° must be equal to 90° when added.

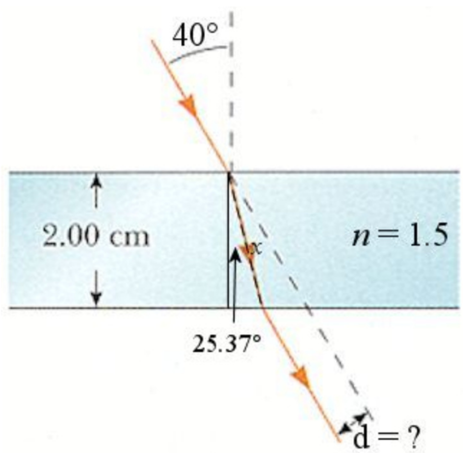
Finally, the angle θ is found with the law of refraction.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1.5\sin(34.06°) = 1\sin\theta$$
$$\theta = 57.16°$$

16. First, the angle of refraction in the glass is found with the law of refraction.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1\cdot\sin(40°) = 1.5\cdot\sin\theta_2$$
$$\theta = 25.37°$$

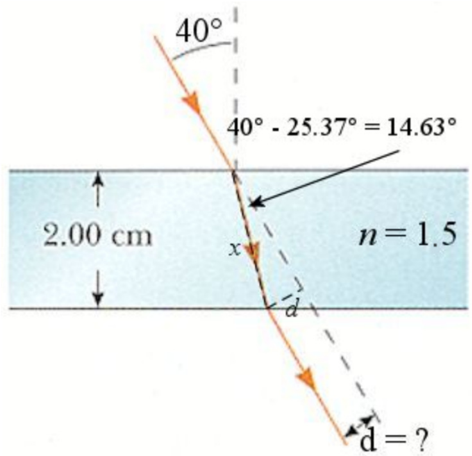
Therefore, we have



Then, the length of the ray of light in the glass (identified by the dotted line x) is found. It is found with

$$\cos 25.37 = \frac{2cm}{x}$$
$$x = 2.214cm$$

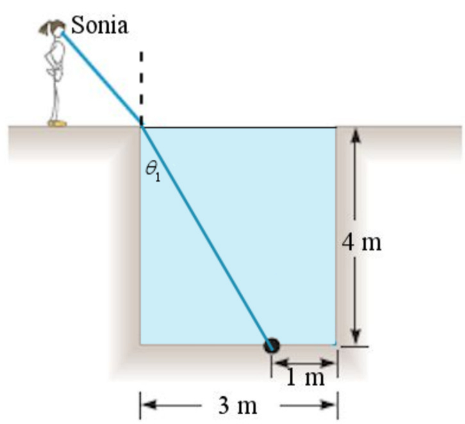
Finally, we have another right triangle with the sides x and d.



With this triangle, *d* is found.

$$\sin(14.63°) = \frac{d}{x}$$
$$\sin(14.63°) = \frac{d}{2.21cm}$$
$$d = 0.559cm$$

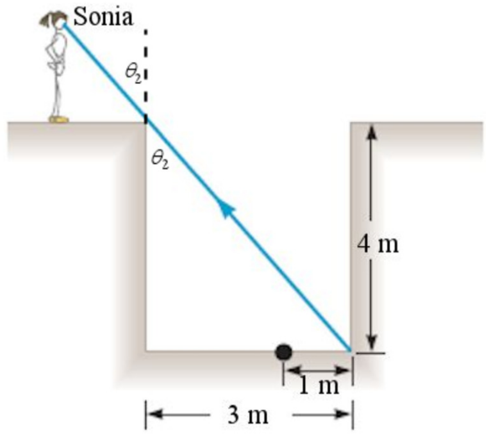
17. To see the point, we have, at worst, the following situation.



The angle in this figure is

$$\tan\theta_1 = \frac{2m}{4m}$$
$$\theta_1 = 26.57°$$

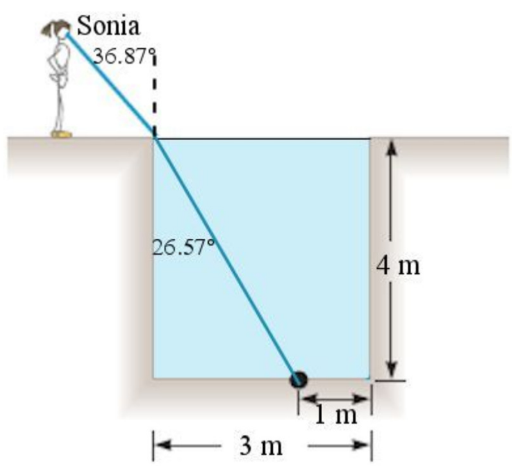
Only the angle on the outside the liquid is missing. This angle is found with the initial situation.



The angle in this figure is

$$\tan\theta_2 = \frac{3m}{4m}$$
$$\theta_2 = 36.87°$$

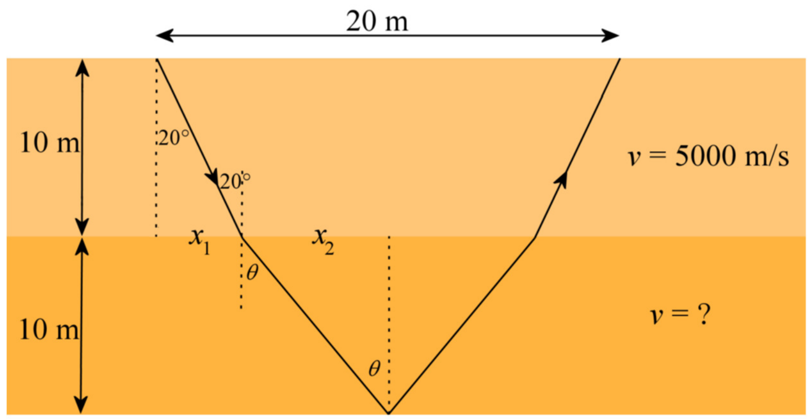
We then have the following situation.



The index of refraction must then be

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$n \cdot \sin(26.57°) = 1 \cdot \sin 36.87°$$
$$n = 1.342$$

18. According to this diagram, we have



$$10m = x_1 + x_2$$

As

$$\frac{x_1}{10m} = \tan 20°$$

and

$$\frac{x_2}{10m} = \tan\theta$$

we have

$$10m = 10m \cdot \tan 20° + 10m \cdot \tan\theta$$

The value of θ can now be calculated.

$$1 = \tan 20° + \tan\theta$$
$$\theta = 32.46°$$

Thus, the law of refraction gives

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$
$$\frac{\sin 20°}{\sin 32.46°} = \frac{5000 \frac{m}{s}}{v_2}$$
$$v_2 = 7846 \frac{m}{s}$$

19. The speed of light will be calculated with

$$v = \frac{c}{n}$$

The refractive index is needed.

If the critical angle is 60°, then the refractive index is

$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\sin 60° = \frac{n}{1.33}$$
$$n = 1.152$$

Therefore, the speed of light is

$$\begin{aligned} v &= \frac{c}{n} \\ &= \frac{3\times10^8 \frac{m}{s}}{1.152} \\ &= 2.60\times10^8 \frac{m}{s} \end{aligned}$$

20. The critical angle is

$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\sin\theta_c = \frac{1}{1.5}$$
$$\theta_c = 41.81°$$

As the angle of incidence is 52° (90° – 38°), there is a total reflection since the angle of incidence is greater than the critical angle.

21. The critical angle is

$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\sin\theta_c = \frac{1.5}{1}$$
$$\theta_c \text{ does not exist}$$

As there is no critical angle, a total reflection is impossible.

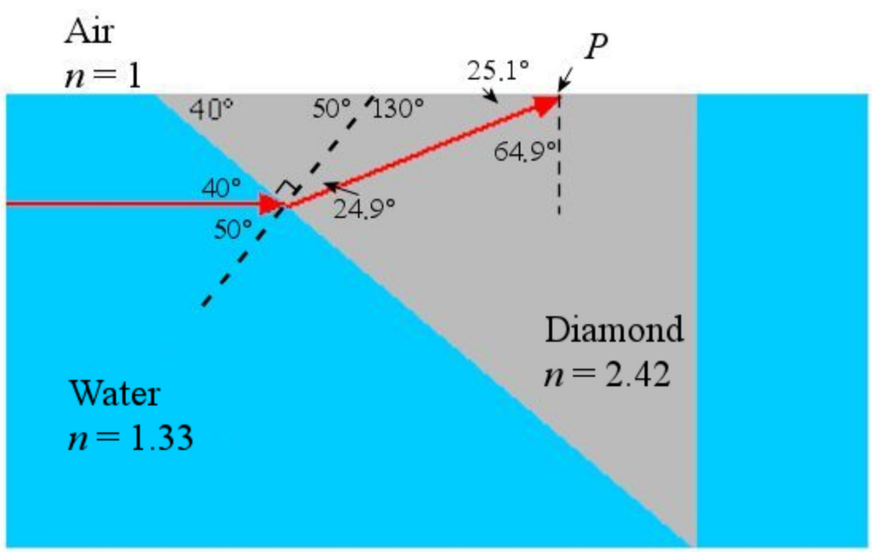
22. The critical angle is

$$\sin\theta_c = \frac{v_1}{v_2}$$
$$\sin\theta_c = \frac{340\frac{m}{s}}{5000\frac{m}{s}}$$
$$\theta_c = 3.899°$$

Thus, the distance is

$$\tan\theta_c = \frac{x}{3m}$$
$$x = 0.2045m$$

23. We have the following angles.



This is how those angles were found.

40° between the red beam and the interface between water and diamond. Alternate internal of the 40° at the end of the piece of diamond

50° angle incidence of the ray at the water-diamond interface.
40° and 50° must be equal to 90° when added.

24.9° angle of refraction
Comes from the law of refraction

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1.33\sin(50°) = 2.42\sin\theta_2$$
$$\theta_2 = 24.9°$$

50° angle (at the end of the dotted line, near the interface between air and diamond).
The sum of the angles of a triangle must be 180°.
We must then have 40° + 90° + 50° = 180°.

130° angle
The sum of the 50° angle and the 130° angle must give 180° (supplementary angles).

25.1° angle
The sum of the angles of a triangle must be 180°.
We must then have 130° + 24.9° + 25.1° = 180°.

64.9° angle
25.1° and 64.9° must be equal to 90° when added.

The angle of incidence of the ray is 64.9°. Is it greater than the critical angle? The critical angle is

$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\sin\theta_c = \frac{1}{2.42}$$
$$\theta_c = 24.4°$$

As the angle of incidence is greater than the critical angle, there is a total reflection at point P.

24. The angle of refraction for the red light is

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1 \cdot \sin(80°) = 1.62 \cdot \sin\theta_2$$
$$\theta_2 = 37.44°$$

The angle of refraction for the violet light is

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1 \cdot \sin(80°) = 1.66 \cdot \sin\theta_2$$
$$\theta_2 = 36.39°$$

The difference between those two angles is 37.44° - 36.39° = 1.05°.

25. The angle of polarization is

$$\tan\theta_p = \frac{n_2}{n_1}$$
$$\tan\theta = \frac{1.55}{1}$$
$$\theta_p = 57.2°$$

26. The angle of polarization is

$$\tan\theta_p = \frac{n_2}{n_1}$$
$$\tan\theta = \frac{1.2}{1.6}$$
$$\theta_p = 36.9°$$

27. If the reflected ray is polarized, then the angle of incidence is equal to the angle of polarization. This angle is

$$\tan\theta_p = \frac{n_2}{n_1}$$
$$\tan\theta = \frac{1.7}{1}$$
$$\theta_p = 59.53°$$

The angle of refraction is thus

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1 \cdot \sin 59.53° = 1.7 \cdot \sin\theta_2$$
$$\theta_2 = 30.46°$$

28. a) With a 48° critical angle, we have

$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\sin 48° = \frac{n_2}{n_1}$$

The individual values of the indices of refraction cannot be found, but the value of n_2/n_1 can be found. Its value is

$$\sin 48° = \frac{n_2}{n_1}$$
$$\frac{n_2}{n_1} = 0.743$$

The angle of polarization is therefore

$$\tan\theta_p = \frac{n_2}{n_1}$$
$$\tan\theta = 0.743$$
$$\theta_p = 36.6°$$

b) No, since the angle of polarization is 36.6° and the total reflection begins at 48°.

29. Let's start by finding the percentages of the reflected and transmitted wave at each surface.

When the light comes from air to glass, the transmitted and reflected amplitudes are

$$\begin{aligned} E_{0R} &= \frac{n_1 - n_2}{n_1 + n_2} E_0 \\ &= \frac{1-1,5}{1+1,5} \cdot E_0 \\ &= -0.2E_0 \end{aligned}$$

$$\begin{aligned} E_{0T} &= \frac{2n_1}{n_1 + n_2} E_0 \\ &= \frac{2 \cdot 1}{1+1,5} \cdot E_0 \\ &= 0.8E_0 \end{aligned}$$

This means that the ratio of the intensities of the transmitted and reflected wave to the intensity of the initial wave are

$$\begin{aligned} \frac{I_R}{I_0} &= \frac{\left(\frac{cn_1\varepsilon_0 E_{0R}^2}{2}\right)}{\left(\frac{cn_1\varepsilon_0 E_0^2}{2}\right)} \\ &= \frac{E_{0R}^2}{E_0^2} \\ &= \frac{(0.2E_0)^2}{E_0^2} \\ &= (0.2)^2 \\ &= 0.04 \end{aligned}$$

$$I_T = \frac{\left(\frac{cn_2\varepsilon_0 E_{0T}^2}{2}\right)}{\left(\frac{cn_1\varepsilon_0 E_0^2}{2}\right)}$$
$$= \frac{n_2 E_{0T}^2}{n_1 E_0^2}$$
$$= \frac{n_2 (0.8E_0)^2}{n_1 E_0^2}$$
$$= \frac{n_2 (0.8)^2}{n_1}$$
$$= \frac{1.5(0.8)^2}{1}$$
$$= 0.96$$

Thus, 96% of the energy enters the glass.

Once inside the glass, the light reaches the glass air interface. In this interface, the amplitudes of reflected and transmitted waves are

$$E_{0R} = \frac{n_1 - n_2}{n_1 + n_2} E_0$$
$$= \frac{1.5 - 1}{1 + 1.5} \cdot E_0$$
$$= 0.2E_0$$

$$E_{0T} = \frac{2n_1}{n_1 + n_2} E_0$$
$$= \frac{2 \cdot 1.5}{1 + 1.5} \cdot E_0$$
$$= 1.2E_0$$

This means that the ratio of the intensities of the transmitted and reflected wave to the intensity of the initial wave are

$$\frac{I_R}{I_0} = \frac{\left(\frac{cn_2\varepsilon_0 E_{0R}^2}{2}\right)}{\left(\frac{cn_2\varepsilon_0 E_0^2}{2}\right)}$$

$$= \frac{E_{0R}^2}{E_0^2}$$

$$= \frac{\left(0.2E_0\right)^2}{E_0^2}$$

$$= \left(0.2\right)^2$$

$$= 0.04$$

$$I_T = \frac{\left(\frac{cn_1\varepsilon_0 E_{0T}^2}{2}\right)}{\left(\frac{cn_2\varepsilon_0 E_0^2}{2}\right)}$$

$$= \frac{n_1 E_{0T}^2}{n_2 E_0^2}$$

$$= \frac{n_1\left(1.2E_0\right)^2}{n_2 E_0^2}$$

$$= \frac{n_1\left(1.2\right)^2}{n_2}$$

$$= \frac{1\left(1.2\right)^2}{1.5}$$

$$= 0.96$$

The intensity of the transmitted light can now be found.

Ray with no reflection

In this case, the light passes directly through both surfaces. The percentage of light doing this is

$$0.96 \cdot 0.96 = 0.9216 = 92.16\%$$

Ray that makes 1 round trip

In this case, the light enters glass, reflects 2 times and comes out of the glass. The percentage of light doing this is

$$0.96 \cdot 0.04 \cdot 0.04 \cdot 0.96 = 0.96^2 \cdot 0.04^2$$

Ray that makes 2 round trips

In this case, the light enters glass, reflects 4 times and comes out of the glass. The percentage of light doing this is

$$0.96 \cdot 0.04 \cdot 0.04 \cdot 0.04 \cdot 0.04 \cdot 0.96 = 0.96^2 \cdot 0.04^4$$

Ray that makes 3 round trips

In this case, the light enters glass, reflects 6 times and comes out of the glass. The percentage of light doing this is

$$0.96^2 \cdot 0.04^6$$

And so on...

Summing all these intensities, the result is

$$\begin{aligned} I_{tot} &= (0.96)^2 + (0.96)^2 (0.04)^2 + (0.96)^2 (0.04)^4 + (0.96)^2 (0.04)^6 + ... \\ &= (0.96)^2 \left(1 + 0.0016 + 0.0016^2 + 0.0016^3 + ...\right) \end{aligned}$$

Such a sum is a geometric series. You can calculate it directly or you can take a formula that gives the sum. Let's find this formula. If you have a geometric series

$$S = a + ar + ar^2 + ar^3 + ...$$

Then

$$\begin{gathered} Sr = ar + ar^2 + ar^3 + ar^4 + ... \\ Sr = S - a \\ S - Sr = a \\ S(1 - r) = a \\ S = \frac{a}{1-r} \end{gathered}$$

Therefore, the sum here is

$$\begin{aligned} I_{tot} &= (0.96)^2 \cdot \frac{1}{1-0.0016} \\ &= 0.9231 \end{aligned}$$

Thus, 92.31% of the light is transmitted. This means that 7.69% of the light is reflected.