Chapter 3 Solutions

1. We have

$$v = \lambda f$$
$$340\tfrac{m}{s} = \lambda \cdot 50000Hz$$
$$\lambda = 0.0068m = 6.8mm$$

2. The wave speed is

$$\begin{aligned} v &= 331.3\tfrac{m}{s} \cdot \sqrt{\frac{T}{273.15K}} \\ &= 331.3\tfrac{m}{s} \cdot \sqrt{\frac{298.15K}{273.15K}} \\ &= 346.1\tfrac{m}{s} \end{aligned}$$

3. a) The temperature will be found with

$$v = 331.3\tfrac{m}{s} \cdot \sqrt{\frac{T}{273.15K}}$$

but we need the speed of the wave.

The wave speed is found with

$$\begin{aligned} v &= \frac{\omega}{k} \\ &= \frac{560s^{-1}}{1.6m^{-1}} \\ &= 350\tfrac{m}{s} \end{aligned}$$

Thus, the temperature is

$$v = 331.3\tfrac{m}{s} \cdot \sqrt{\frac{T}{273.15K}}$$

$$350\tfrac{m}{s} = 331.3\tfrac{m}{s} \cdot \sqrt{\frac{T}{273.15K}}$$

$$T = 304.9K = 31.7°C$$

b) The maximum speed of the air molecules is

$$\begin{aligned} v_{\max} &= \omega A \\ &= 560s^{-1} \cdot 0.000\,01m \\ &= 0.0056\tfrac{m}{s} \end{aligned}$$

4. The time is

$$\begin{aligned} \Delta t &= \frac{\text{distance}}{\text{speed}} \\ &= \frac{1.496 \times 10^{11} m}{299,792,458\tfrac{m}{s}} \\ &= 499s \\ &= 8\min 19s \end{aligned}$$

5. The distance is

$$\begin{aligned} \Delta x &= v\Delta t \\ &= 299,792,458\tfrac{m}{s} \cdot (365.25 \cdot 24 \cdot 60 \cdot 60s) \\ &= 9.46 \times 10^{15} m \end{aligned}$$

6. The speed of light is

$$\begin{aligned} v &= \frac{c}{n} \\ &= \frac{299,792,458\tfrac{m}{s}}{2.4} \\ &= 1.25 \times 10^{8} \tfrac{m}{s} \end{aligned}$$

7. The wavelength is

$$c = \lambda f$$
$$3\times10^{8}\tfrac{m}{s} = \lambda\cdot10^{15}Hz$$
$$\lambda = 3\times10^{-7}m = 300nm$$

This is a wavelength corresponding to ultraviolet light.

8. The intensity of the sound at this distance is

$$\begin{aligned} I &= \frac{P}{4\pi r^2} \\ &= \frac{50W}{4\pi\cdot(30m)^2} \\ &= 4.42\times10^{-3}\tfrac{W}{m^2} \end{aligned}$$

In decibels, the intensity is

$$\begin{aligned} \beta &= 10dB\cdot\log\left(\frac{I}{10^{-12}\tfrac{W}{m^2}}\right) \\ &= 10dB\cdot\log\left(\frac{4.42\times10^{-3}\tfrac{W}{m^2}}{10^{-12}\tfrac{W}{m^2}}\right) \\ &= 96.5dB \end{aligned}$$

9. The power received is

$$\begin{aligned} P_{received} &= IA_{receiver} \\ &= 0.1\tfrac{W}{m^2}\cdot0.001m^2 \\ &= 0.0001W \end{aligned}$$

The energy received in 2 minutes is

$$\begin{aligned} E &= Pt \\ &= 0.0001\tfrac{W}{m^2}\cdot120s \\ &= 0.012J \end{aligned}$$

10. The power of the source is

$$I = \frac{P}{4\pi r^2}$$
$$0.001\tfrac{W}{m^2} = \frac{P}{4\pi \cdot (10m)^2}$$
$$P = 1.2566W$$

To have 10^{-5} W/m², the distance is

$$I = \frac{P}{4\pi r^2}$$
$$10^{-5}\tfrac{W}{m^2} = \frac{1.2566W}{4\pi r^2}$$
$$r = 100m$$

11. To find the intensity of 1000 firecrackers, then the power of 1 firecracker must be known.

This power can be found from the sound intensity of 1 firecracker at 50 m.

At 40 dB, the intensity is

$$\beta = 10dB \cdot \log\left(\frac{I}{10^{-12}\tfrac{W}{m^2}}\right)$$
$$40dB = 10dB \cdot \log\left(\frac{I}{10^{-12}\tfrac{W}{m^2}}\right)$$
$$I = 10^{-8}\tfrac{W}{m^2}$$

The power of 1 firecracker is therefore

$$I = \frac{P}{4\pi r^2}$$
$$10^{-8}\tfrac{W}{m^2} = \frac{P}{4\pi \cdot (50m)^2}$$
$$P = 3.142 \times 10^{-4} W$$

The power of 1000 firecrackers is then

$$P' = 1000 \cdot 3.142 \times 10^{-4} W$$
$$= 0.3142W$$

The intensity at a distance of 200 m is then

$$I = \frac{P}{4\pi r^2}$$
$$= \frac{0.3142W}{4\pi \cdot (200m)^2}$$
$$= 6.25 \times 10^{-7} W$$

In decibels, this intensity is

$$\beta = 10dB \cdot \log\left(\frac{I}{10^{-12} \frac{W}{m^2}}\right)$$
$$= 10dB \cdot \log\left(\frac{6.25 \times 10^{-7} \frac{W}{m^2}}{10^{-12} \frac{W}{m^2}}\right)$$
$$= 57.96dB$$

12. To find the total intensity, the intensity (in W/m²) of each wave must be added. Once this will be done, the result can be changed into decibels.

So, let's calculate the intensity (in W/m²) of each wave.

At 90 dB, the intensity of the first wave is

$$\beta = 10dB \cdot \log\left(\frac{I}{10^{-12} \frac{W}{m^2}}\right)$$
$$90dB = 10dB \cdot \log\left(\frac{I}{10^{-12} \frac{W}{m^2}}\right)$$
$$I = 10^{-3} \frac{W}{m^2}$$

At 95 dB, the intensity of the second sound is

$$\beta = 10dB \cdot \log\left(\frac{I}{10^{-12}\,\frac{W}{m^2}}\right)$$

$$95dB = 10dB \cdot \log\left(\frac{I}{10^{-12}\,\frac{W}{m^2}}\right)$$

$$I = 3.162\times10^{-3}\,\tfrac{W}{m^2}$$

The total intensity is thus

$$\begin{aligned} I_{tot} &= 10^{-3}\,\tfrac{W}{m^2} + 3.162\times10^{-3}\,\tfrac{W}{m^2} \\ &= 4.162\times10^{-3}\,\tfrac{W}{m^2} \end{aligned}$$

In decibels, this intensity is

$$\begin{aligned} \beta &= 10dB \cdot \log\left(\frac{I}{10^{-12}\,\frac{W}{m^2}}\right) \\ &= 10dB \cdot \log\left(\frac{4.162\times10^{-3}\,\frac{W}{m^2}}{10^{-12}\,\frac{W}{m^2}}\right) \\ &= 96.2dB \end{aligned}$$

13. The amplitude will be found with

$$I = \frac{1}{2}\rho v \omega^2 A^2$$

To have the amplitude, the intensity must be known.

At 25 m, the intensity is

$$\begin{aligned} I &= \frac{P}{4\pi r^2} \\ &= \frac{50W}{4\pi \cdot (25m)^2} \\ &= 6.366\times10^{-3}\,\tfrac{W}{m^2} \end{aligned}$$

Therefore, the amplitude is

$$I = \frac{1}{2}\rho v\omega^2 A^2$$

$$6.366\times10^{-3}\,\tfrac{W}{m^2} = \frac{1}{2}\cdot 1.3\tfrac{kg}{m^3}\cdot 340\tfrac{m}{s}\cdot\left(2\pi\cdot 200Hz\right)^2\cdot A^2$$

$$A = 4.271\mu m$$

14. The intensity is

$$\begin{aligned} I &= \frac{1}{2}\rho v\omega^2 A^2 \\ &= \frac{1}{2}\cdot 1.3\tfrac{kg}{m^3}\cdot 340\tfrac{m}{s}\cdot\left(2\pi\cdot 400Hz\right)^2\cdot\left(1\times10^{-7}m\right)^2 \\ &= 1.396\times10^{-5}\,\tfrac{W}{m^2} \end{aligned}$$

In decibel, this intensity is

$$\begin{aligned} \beta &= 10dB\cdot\log\left(\frac{I}{10^{-12}\,\tfrac{W}{m^2}}\right) \\ &= 10dB\cdot\log\left(\frac{1.396\times10^{-5}\,\tfrac{W}{m^2}}{10^{-12}\,\tfrac{W}{m^2}}\right) \\ &= 71.45dB \end{aligned}$$

15. a) 5 km from the source, the intensity is

$$\begin{aligned} I &= \frac{P}{4\pi r^2} \\ &= \frac{20{,}000W}{4\pi\cdot\left(5000m\right)^2} \\ &= 6.366\times10^{-5}\,\tfrac{W}{m^2} \end{aligned}$$

Thus, the intensity is

$$\begin{aligned} \beta &= 10dB\cdot\log\left(\frac{I}{10^{-12}\,\tfrac{W}{m^2}}\right) \\ &= 10dB\cdot\log\left(\frac{6.366\times10^{-5}\,\tfrac{W}{m^2}}{10^{-12}\,\tfrac{W}{m^2}}\right) \\ &= 78.04dB \end{aligned}$$

b) If we lose 7 dB per km, the intensity will be 35 dB lower if the absorption by the air is taken into account absorption air. The intensity is, therefore, 43.04 dB.

16. The amplitude will be found with

$$I = \frac{cn\varepsilon_0 E_0^2}{2}$$

To have the amplitude, the intensity must be known.

30 m from the source, the intensity is

$$\begin{aligned} I &= \frac{P}{4\pi r^2} \\ &= \frac{100W}{4\pi \cdot (30m)^2} \\ &= 8.842 \times 10^{-3} \tfrac{W}{m^2} \end{aligned}$$

Then, the amplitude is

$$\begin{aligned} I &= \frac{cn\varepsilon_0 E_0^2}{2} \\ 8.842 \times 10^{-3} \tfrac{W}{m^2} &= \frac{3 \times 10^8 \tfrac{m}{s} \cdot 1 \cdot 8.854 \times 10^{-12} \tfrac{C^2}{Nm^2} \cdot E_0^2}{2} \\ E_0 &= 2.5802 \tfrac{N}{C} \end{aligned}$$

17. Since

$$I = \frac{P}{4\pi r^2}$$

and

$$I = \frac{1}{2}\rho v \omega^2 A^2$$

then

$$\frac{P}{4\pi r^2} = \frac{1}{2}\rho v\omega^2 A^2$$

20 m from the source, we have

$$\frac{P}{4\pi\cdot(20m)^2} = \frac{1}{2}\rho v\omega^2\cdot(20\mu m)^2$$

100 m from the source, we have

$$\frac{P}{4\pi\cdot(100m)^2} = \frac{1}{2}\rho v\omega^2 A^2$$

If we divide the 2nd equation by the 1st equation, we obtain

$$\left(\frac{\frac{P}{4\pi\cdot(100m)^2}}{\frac{P}{4\pi\cdot(20m)^2}}\right) = \left(\frac{\frac{1}{2}\rho v\omega^2 A^2}{\frac{1}{2}\rho v\omega^2\cdot(20\mu m)^2}\right)$$

$$\frac{(20m)^2}{(100m)^2} = \frac{A^2}{(20\mu m)^2}$$

$$\frac{20m}{100m} = \frac{A}{20\mu m}$$

$$A = \frac{20m}{100m}20\mu m$$

$$A = 4\mu m$$

18. The intensity after the first polarizer is

$$\begin{aligned} I &= \frac{1}{2}I_0 \\ &= \frac{1}{2}\cdot 50\,\tfrac{W}{m^2} \\ &= 25\,\tfrac{W}{m^2} \end{aligned}$$

The intensity after the second polarizer is

$$\begin{aligned} I &= I_0 \cos^2 \theta \\ &= 25\tfrac{W}{m^2} \cdot \cos^2 25° \\ &= 20.53\tfrac{W}{m^2} \end{aligned}$$

19. The intensity after the first polarizer is

$$\begin{aligned} I &= \frac{1}{2} I_0 \\ &= \frac{1}{2} \cdot 40\tfrac{W}{m^2} \\ &= 20\tfrac{W}{m^2} \end{aligned}$$

The intensity after the second polarizer is

$$\begin{aligned} I &= I_0 \cos^2 \theta \\ &= 20\tfrac{W}{m^2} \cdot \cos^2 75° \\ &= 1.34\tfrac{W}{m^2} \end{aligned}$$

The intensity after the third polarizer is

$$\begin{aligned} I &= I_0 \cos^2 \theta \\ &= 1.34\tfrac{W}{m^2} \cdot \cos^2 15° \\ &= 1.25\tfrac{W}{m^2} \end{aligned}$$

20. The intensity after the first polarizer is

$$\begin{aligned} I &= \frac{1}{2} I_0 \\ &= \frac{1}{2} \cdot 20\tfrac{W}{m^2} \\ &= 10\tfrac{W}{m^2} \end{aligned}$$

The angle is found with the formula of the intensity after the second polarizer.

$$\begin{aligned} I &= I_0 \cos^2 \theta \\ 4\tfrac{W}{m^2} &= 10\tfrac{W}{m^2} \cdot \cos^2 \theta \\ 0.4 &= \cos^2 \theta \end{aligned}$$

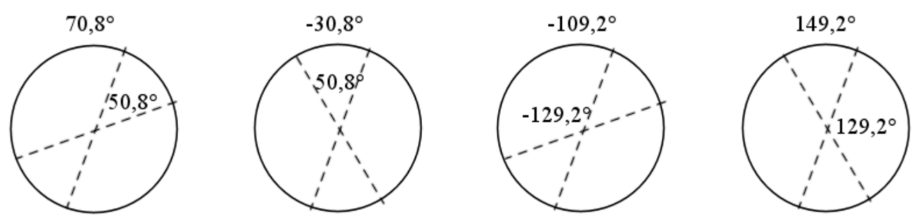
There are 2 solutions to the square root:

$$\sqrt{0.4} = \cos\theta \qquad \text{and} \qquad -\sqrt{0.4} = \cos\theta$$

As there are two solutions to inverse cosine function, the answers are

$$\sqrt{0.4} = \cos\theta \qquad \text{and} \qquad -\sqrt{0.4} = \cos\theta$$
$$\theta = \pm 50.8° \qquad \text{and} \qquad \theta = \pm 129.2°$$

These angles are the angles between the current polarizer and the previous polarizer. As the angle of the previous polarizer is 20°, these solutions are



The first solution is identical to the third, and the second solution is identical to the fourth. The two solutions are therefore 70.8° and 149.2°.

21. After the passage through the polarizer at an angle θ, we have

$$5\tfrac{W}{m^2} = I_0 \cos^2\theta$$

Since the intensity decreases to 3 W/m² if the polarizer is turned by 20°, we have

$$3\tfrac{W}{m^2} = I_0 \cos^2(\theta + 20°)$$

So we have 2 equations and 2 unknowns. If the second equation is divided by the first, we have

$$\frac{3}{5} = \frac{I_0 \cos^2(\theta + 20°)}{I_0 \cos^2\theta}$$
$$\sqrt{\frac{3}{5}} = \frac{\cos(\theta + 20°)}{\cos\theta}$$

Since $\cos(a+b) = \cos a \cos b - \sin a \sin b$, this equation is

$$\sqrt{\frac{3}{5}} = \frac{\cos\theta \cdot \cos 20° - \sin\theta \cdot \sin 20°}{\cos\theta}$$

$$\sqrt{\frac{3}{5}} = \cos 20° - \tan\theta \cdot \sin 20°$$

$$\theta = 25,767°$$

Thus, the initial intensity is

$$5\tfrac{W}{m^2} = I_0 \cdot \cos^2 25,767°$$

$$I_0 = 6,165\tfrac{W}{m^2}$$

22. a) The frequency is

$$\begin{aligned} f' &= f\,\frac{v - v_o}{v - v_s} \\ &= 350Hz \cdot \frac{340\tfrac{m}{s} - 0\tfrac{m}{s}}{340\tfrac{m}{s} - 35\tfrac{m}{s}} \\ &= 390.2Hz \end{aligned}$$

b) The wavelength is

$$\begin{aligned} \lambda' &= \lambda\left(1 - \frac{v_s}{v}\right) \\ &= \frac{v}{f}\left(1 - \frac{v_s}{v}\right) \\ &= \frac{340\tfrac{m}{s}}{350Hz} \cdot \left(1 - \frac{35\tfrac{m}{s}}{340\tfrac{m}{s}}\right) \\ &= 0.8714m \end{aligned}$$

c) The frequency is

$$\begin{aligned} f' &= f\,\frac{v - v_o}{v - v_s} \\ &= 350Hz \cdot \frac{340\tfrac{m}{s} - 0\tfrac{m}{s}}{340\tfrac{m}{s} - \left(-35\tfrac{m}{s}\right)} \\ &= 317.3Hz \end{aligned}$$

d) The wavelength is

$$\begin{aligned}\lambda' &= \lambda\left(1-\frac{v_s}{v}\right)\\ &= \frac{v}{f}\left(1-\frac{v_s}{v}\right)\\ &= \frac{340\frac{m}{s}}{350Hz}\cdot\left(1-\frac{-35\frac{m}{s}}{340\frac{m}{s}}\right)\\ &= 1.0714m\end{aligned}$$

23. a) The frequency is

$$\begin{aligned}f' &= f\,\frac{v-v_o}{v-v_s}\\ &= 400Hz\cdot\frac{340\frac{m}{s}-10\frac{m}{s}}{340\frac{m}{s}-0\frac{m}{s}}\\ &= 388.2Hz\end{aligned}$$

b) The wavelength is

$$\begin{aligned}\lambda' &= \lambda\left(1-\frac{v_s}{v}\right)\\ &= \frac{v}{f}\left(1-\frac{v_s}{v}\right)\\ &= \frac{340\frac{m}{s}}{400Hz}\cdot\left(1-\frac{0\frac{m}{s}}{340\frac{m}{s}}\right)\\ &= 0.85m\end{aligned}$$

c) The frequency is

$$\begin{aligned}f' &= f\,\frac{v-v_o}{v-v_s}\\ &= 400Hz\cdot\frac{340\frac{m}{s}-\left(-12\frac{m}{s}\right)}{340\frac{m}{s}-0\frac{m}{s}}\\ &= 414.1Hz\end{aligned}$$

d) The wavelength is

$$\begin{aligned}\lambda' &= \lambda\left(1-\frac{v_s}{v}\right)\\ &= \frac{v}{f}\left(1-\frac{v_s}{v}\right)\\ &= \frac{340\frac{m}{s}}{400Hz}\cdot\left(1-\frac{0\frac{m}{s}}{340\frac{m}{s}}\right)\\ &= 0.85m\end{aligned}$$

24. The frequency is

$$\begin{aligned}f' &= f\,\frac{v-v_o}{v-v_s}\\ &= 400Hz\cdot\frac{340\frac{m}{s}-\left(-41.67\frac{m}{s}\right)}{340\frac{m}{s}-22.22\frac{m}{s}}\\ &= 480.4Hz\end{aligned}$$

25. Removing the wind speed (by adding 15 km/h towards the left to all velocities), the speed of the red car is 135 km/h and the speed of the police car is 95 km/h. Therefore, the frequency is

$$\begin{aligned}f' &= f\,\frac{v-v_o}{v-v_s}\\ &= 400Hz\cdot\frac{340\frac{m}{s}-\left(-37.5\frac{m}{s}\right)}{340\frac{m}{s}-26.39\frac{m}{s}}\\ &= 481.4Hz\end{aligned}$$

26. a) When the train is moving towards the stationary observer, the frequency is

$$\begin{aligned}f' &= f\,\frac{v-v_o}{v-v_s}\\ 150Hz &= f\cdot\frac{335\frac{m}{s}-0\frac{m}{s}}{335\frac{m}{s}-v}\\ 150Hz &= f\cdot\frac{335\frac{m}{s}}{335\frac{m}{s}-v}\end{aligned}$$

When the train is moving away from the stationary observer, the frequency is

$$f' = f\frac{v - v_o}{v - v_s}$$

$$125Hz = f \cdot \frac{335\frac{m}{s} - 0\frac{m}{s}}{335\frac{m}{s} - -v}$$

$$125Hz = f \cdot \frac{335\frac{m}{s}}{335\frac{m}{s} + v}$$

We then have 2 equations and 2 unknowns. By dividing the equations one by the other, we obtain

$$\frac{150Hz}{125Hz} = \frac{f \cdot \frac{335\frac{m}{s}}{335\frac{m}{s} - v}}{f \cdot \frac{335\frac{m}{s}}{335\frac{m}{s} + v}}$$

$$1.2 = \frac{335\frac{m}{s} + v}{335\frac{m}{s} - v}$$

$$1.2 \cdot \left(335\frac{m}{s} - v\right) = 335\frac{m}{s} + v$$

$$402\frac{m}{s} - 1.2 \cdot v = 335\frac{m}{s} + v$$

$$67\frac{m}{s} = 2.2 \cdot v$$

$$v = 30.45\frac{m}{s}$$

b) Using the speed, the frequency is easily calculated

$$125Hz = f \cdot \frac{335\frac{m}{s}}{335\frac{m}{s} + v}$$

$$125Hz = f \cdot \frac{335\frac{m}{s}}{335\frac{m}{s} + 30.45\frac{m}{s}}$$

$$f = 136.4Hz$$

27. We have

$$f' = f\frac{v - v_o}{v - v_s}$$

$$420Hz = 400Hz \cdot \frac{345\frac{m}{s} - v}{345\frac{m}{s} - \left(v + 15\frac{m}{s}\right)}$$

The solution is

$$420\cdot\left(345\tfrac{m}{s}-\left(v+15\tfrac{m}{s}\right)\right)=400\cdot\left(345\tfrac{m}{s}-v\right)$$
$$420\cdot\left(330\tfrac{m}{s}-v\right)=400\cdot\left(345\tfrac{m}{s}-v\right)$$
$$21\cdot\left(330\tfrac{m}{s}-v\right)=20\cdot\left(345\tfrac{m}{s}-v\right)$$
$$6930\tfrac{m}{s}-21\cdot v=6900\tfrac{m}{s}-20\cdot v$$
$$30\tfrac{m}{s}=v$$

The speed of the red car is therefore 30 m/s (108 km/h) and the speed of the police car is 45 m/s (162 km/h).

28. The frequency will be given by

$$f'=f\frac{v-v_o}{v-v_s}$$

However, speed of sound is needed to calculate the frequency.

At this temperature, the speed of sound is

$$\begin{aligned} v&=331.3\tfrac{m}{s}\cdot\sqrt{\frac{T}{273.15K}}\\ &=331.3\tfrac{m}{s}\cdot\sqrt{\frac{298.15K}{273.15K}}\\ &=346.13\tfrac{m}{s}\end{aligned}$$

The frequency of the sound arriving directly from the car is

$$\begin{aligned} f'&=f\frac{v-v_o}{v-v_s}\\ &=400Hz\cdot\frac{346.13\tfrac{m}{s}-5\tfrac{m}{s}}{346.13\tfrac{m}{x}-25\tfrac{m}{s}}\\ &=424.91Hz\end{aligned}$$

Now let's find the frequency of the sound reflected on the wall.

For the first step, a wall at rest received the sound coming from a moving car. The frequency received by the wall is

$$\begin{aligned} f' &= f\frac{v - v_o}{v - v_s} \\ &= 400Hz \cdot \frac{346.13\frac{m}{s} - 0\frac{m}{s}}{346.13\frac{m}{x} - 25\frac{m}{s}} \\ &= 431.14Hz \end{aligned}$$

The wall now becomes a source emitting at this frequency. The frequency of this sound received by the person in the car is

$$\begin{aligned} f' &= f\frac{v - v_o}{v - v_s} \\ &= 431.14Hz \cdot \frac{346.13\frac{m}{s} - \left(-5\frac{m}{s}\right)}{346.13\frac{m}{x} - 0\frac{m}{s}} \\ &= 437.37Hz \end{aligned}$$

The person in the car therefore receives a sound at 424.91 Hz and sound at 437.37 Hz.

29. First, let's find the frequency of the sound arriving at the wall. In this case, the observer (the wall) is stationary and the car has a positive velocity. The received frequency is, therefore,

$$\begin{aligned} f' &= 400Hz \cdot \frac{v - v_o}{v - v_s} \\ &= 400Hz \cdot \frac{v}{v - v_{car}} \end{aligned}$$

Then, the wall becomes a source emitting that frequency. We then have a stationary source (the wall) and an observer who has a negative velocity (the car). Thus, the sound received by the person in the car has the following frequency

$$\begin{aligned} f'' &= f'\frac{v - v_o}{v - v_s} \\ &= f'\frac{v - -v_{car}}{v} \\ &= f'\frac{v + v_{car}}{v} \end{aligned}$$

Substituting f' by the value found earlier, the equation becomes

$$f'' = 400Hz \cdot \frac{v}{v - v_{car}} \cdot \frac{v + v_{car}}{v}$$
$$= 400Hz \cdot \frac{v + v_{car}}{v - v_{car}}$$

Since f'' is 415 Hz, we have

$$415Hz = 400Hz \cdot \frac{v + v_{car}}{v - v_{car}}$$

The speed of the car is thus

$$415 \cdot (v - v_{car}) = 400 \cdot (v + v_{car})$$
$$415 \cdot v - 415 \cdot v_{car} = 400 \cdot v + 400 \cdot v_{car}$$
$$15 \cdot v = 815 \cdot v_{car}$$
$$v_{car} = \frac{3}{163} v$$

We need the speed of sound.

At this temperature, the speed of sound is

$$v = 331.3 \tfrac{m}{s} \cdot \sqrt{\frac{T}{273.15K}}$$
$$= 331.3 \tfrac{m}{s} \cdot \sqrt{\frac{293.15K}{273.15K}}$$
$$= 343.21 \tfrac{m}{s}$$

Therefore, the speed of the car is

$$v_{voiture} = \frac{3}{163} \cdot 343.21 \tfrac{m}{s}$$
$$= 6,317 \tfrac{m}{s}$$
$$= 22,74 \tfrac{km}{h}$$

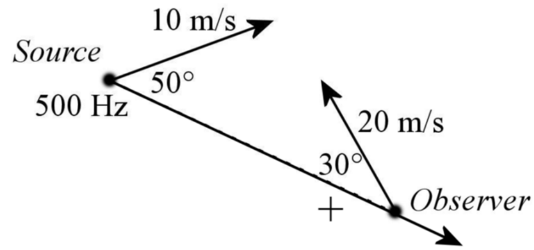
30. The frequency heard is

$$\begin{aligned} f' &= f\frac{v - v_o\cos\theta_o}{v - v_s\cos\theta_s} \\ &= 500Hz\cdot\frac{340\frac{m}{s}}{340\frac{m}{s} - 10\frac{m}{s}\cdot\cos 50°} \\ &= 509.6Hz \end{aligned}$$

31. The frequency heard is

$$\begin{aligned} f' &= f\frac{v - v_o\cos\theta_o}{v - v_s\cos\theta_s} \\ &= 500Hz\cdot\frac{340\frac{m}{s} - 20\frac{m}{s}\cdot\cos 150°}{340\frac{m}{s} - 10\frac{m}{s}\cdot\cos 50°} \\ &= 535.6Hz \end{aligned}$$

Attention: the angle is the angle between the velocity and the positive axis that goes from the source to the observer. That's why the angle is 150° for the observer.



32. a) The frequency is

$$\begin{aligned} c &= \lambda f \\ 3\times10^8\tfrac{m}{s} &= 585\times10^{-9}m\cdot f \\ f &= 5.128\times10^{14}Hz \end{aligned}$$

The frequency received by the observer is

$$f' = f\frac{c - v_o}{c - v_s}$$
$$= 5.128\times10^{14}Hz \cdot \frac{3\times10^8 \frac{m}{s} - 0\frac{m}{s}}{3\times10^8 \frac{m}{s} - 3\times10^7 \frac{m}{s}}$$
$$= 5.698\times10^{14}Hz$$

This frequency corresponds to the wavelength

$$c = \lambda f$$
$$3\times10^8 \tfrac{m}{s} = \lambda \cdot 5.698\times10^{14}Hz$$
$$\lambda = 526.5nm$$

Which corresponds to bluish-green light.

b) The frequency received by the observer is

$$f' = f\frac{c - v_o}{c - v_s}$$
$$= 5.128\times10^{14}Hz \cdot \frac{3\times10^8 \frac{m}{s} - 0\frac{m}{s}}{3\times10^8 \frac{m}{s} + 3\times10^7 \frac{m}{s}}$$
$$= 4.662\times10^{14}Hz$$

This frequency corresponds to the wavelength

$$c = \lambda f$$
$$3\times10^8 \tfrac{m}{s} = \lambda \cdot 4.662\times10^{14}Hz$$
$$\lambda = 643.5nm$$

Which corresponds to red light.

33. The emitted frequency is

$$c = \lambda f$$
$$3\times10^8 \tfrac{m}{s} = 600\times10^{-9}m \cdot f$$
$$f = 5\times10^{14}Hz$$

The frequency received is

$$c = \lambda f'$$
$$3\times10^8 \tfrac{m}{s} = 470\times10^{-9} m \cdot f'$$
$$f' = 6.383\times10^{14} Hz$$

Therefore, the speed is found with

$$f' = f\frac{c - v_o}{c - v_s}$$
$$6.383\times10^{14} Hz = 5\times10^{14} Hz \cdot \frac{3\times10^8 \tfrac{m}{s} - v_0}{3\times10^8 \tfrac{m}{s} - 0\tfrac{m}{s}}$$
$$v_0 = -8.3\times10^7 \tfrac{m}{s}$$

The negative sign indicates that the observer must move towards the source. The speed is 8.3 x 10^7 m/s.

34. a) With a 10 dB intensity, the intensity *I* is

$$\beta = 10dB \cdot \log\frac{I}{10^{-12} \tfrac{W}{m^2}}$$
$$10dB = 10dB \cdot \log\frac{I}{10^{-12} \tfrac{W}{m^2}}$$
$$I = 10^{-11} \tfrac{W}{m^2}$$

Therefore, the distance is

$$I = \frac{P}{4\pi r^2}$$
$$10^{-11} \tfrac{W}{m^2} = \frac{20,000W}{4\pi r^2}$$
$$r = 12,616km$$

This answer seems much too high...This noise would be heard on almost half of the surface of the Earth. Yet, this is sound source 10 times less powerful than a jet taking off. It’s normal to get such a high distance since the fact that air absorbs sound was ignored.

b) If the air absorbs the sound at the rate of 7 dB/km, then the number of decibels will be

$$\beta' = \beta - 0,007\tfrac{dB}{m}r$$

where β is the intensity that we would have without absorption. With a source at a distance *r*, this intensity without absorption is

$$\beta = 10dB\log\frac{\frac{P}{4\pi r^2}}{10^{-12}\frac{W}{m^2}}$$

$$\beta = 10dB\cdot\log\frac{P}{4\pi\times10^{-12}\frac{W}{m^2}r^2}$$

$$\beta = 10dB\cdot\log\frac{P}{4\pi\times10^{-12}\frac{W}{m^2}(1m)^2} - 10dB\cdot\log\left(\frac{r}{1m}\right)^2$$

$$\beta = 10dB\cdot\log\frac{20,000W}{4\pi\times10^{-12}\frac{W}{m^2}} - 20dB\cdot\log\left(\frac{r}{1m}\right)$$

$$\beta = 152.02dB - 20dB\cdot\log\left(\frac{r}{1m}\right)$$

Thus, the intensity with absorption is

$$\beta' = 152.02dB - 20dB\cdot\log\left(\frac{r}{1m}\right) - 0.007\tfrac{dB}{m}\cdot r$$

To hear a 10 dB sound, the equation gives

$$10dB = 152.02dB - 20dB\cdot\log\left(\frac{r}{1m}\right) - 0.007\tfrac{dB}{m}\cdot r$$

$$1 = 15.202 - 2\log\left(\frac{r}{1m}\right) - 0.0007m^{-1}\cdot r$$

$$0.0007m^{-1}\cdot r = 15.202 - 1 - 2\log\left(\frac{r}{1m}\right)$$

$$0.0007m^{-1}\cdot r = 14.202 - 2\log\left(\frac{r}{1m}\right)$$

$$r = 20,288m - \tfrac{2m}{0.0007}\log\left(\frac{r}{1m}\right)$$

It only remains to solve for *r*, but it's not easy. Of course, Maple can be used to solve. An iterative method can also be used. In this method, a value of *r* is taken at random and *r* is calculated with the formula. The value obtained is then used to calculate *r* again with the formula. This new value is then used again to determine

r and so on until the value does not change anymore. If this happens, the answer is obtained. Let's try it here by using 20 km as the starting value (the equation indicates that r is smaller than 20,288 m). Then, the results are

1st iteration

$$\begin{aligned} r &= 20,288m - \tfrac{2m}{0.0007}\log(20,000) \\ &= 8000m \end{aligned}$$

2nd iteration

$$\begin{aligned} r &= 20\,288m - \tfrac{2m}{0.0007}\log(8000) \\ &= 9137m \end{aligned}$$

3rd iteration

$$\begin{aligned} r &= 20\,288m - \tfrac{2m}{0.0007}\log(9137) \\ &= 8972m \end{aligned}$$

4th iteration

$$\begin{aligned} r &= 20\,288m - \tfrac{2m}{0.0007}\log(8972) \\ &= 8994m \end{aligned}$$

5th iteration

$$\begin{aligned} r &= 20\,288m - \tfrac{2m}{0.0007}\log(8994) \\ &= 8991m \end{aligned}$$

6th iteration

$$\begin{aligned} r &= 20\,288m - \tfrac{2m}{0.0007}\log(8991) \\ &= 8992m \end{aligned}$$

7th iteration

$$\begin{aligned} r &= 20\,288m - \tfrac{2m}{0.0007}\log(8992) \\ &= 8992m \end{aligned}$$

Et voilà! The distance is 8992 m.

35. Let's separate this beam into two parts. The first part is the beam with intensity I_{max} after its passage through the polarizer. To have this intensity, the axis of the polarizer must be aligned with the direction of the most intense polarization. If the polarizer is turned by an angle θ, then the intensity that passes for this most intense wave is

$$I_{max} = I_{max} \cos^2 \theta$$

The second is the beam with intensity I_{min} after its passage through the polarizer. This component is perpendicular to the most intense component. Thus, the angle between this component and the polarization axis is 90° – θ. The intensity of the light of this component is, therefore,

$$I_{min} = I_{min} \cos^2 (90 - \theta)$$

The total intensity is the sum of these two intensities.

$$I = I_{max} \cos^2 \theta + I_{min} \cos^2 (90 - \theta)$$

But since

$$p = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

The intensity I_{min} is

$$\begin{aligned}
(I_{max} + I_{min}) p &= I_{max} - I_{min} \\
I_{max} p + I_{min} p &= I_{max} - I_{min} \\
I_{min} p + I_{min} &= I_{max} - I_{max} p \\
I_{min} (p + 1) &= (1 - p) I_{max} \\
I_{min} &= \frac{1 - p}{1 + p} I_{max}
\end{aligned}$$

Thus, the total intensity becomes

$$\begin{aligned}
I &= I_{max} \cos^2 \theta + I_{min} \cos^2 (90 - \theta) \\
&= I_{max} \left(\cos^2 \theta + \frac{1 - p}{1 + p} \cos^2 (90 - \theta) \right) \\
&= \frac{I_{max}}{1 + p} \left((1 + p) \cos^2 \theta + (1 - p) \cos^2 (90 - \theta) \right)
\end{aligned}$$

But since $\cos(90-\theta)=\sin\theta$, the intensity becomes

$$\begin{aligned}
I &= \frac{I_{max}}{1+p}\left((1+p)\cos^2\theta+(1-p)\sin^2\theta\right) \\
&= \frac{I_{max}}{1+p}\left(\cos^2\theta+p\cos^2\theta+\sin^2\theta-p\sin^2\theta\right) \\
&= \frac{I_{max}}{1+p}\left(\cos^2\theta+\sin^2\theta+p\left(\cos^2\theta-\sin^2\theta\right)\right) \\
&= \frac{I_{max}}{1+p}\left(1+p\left(\cos^2\theta-\sin^2\theta\right)\right)
\end{aligned}$$

Finally, since $\cos^2\theta-\sin^2\theta=\cos 2\theta$, we obtain

$$\begin{aligned}
I &= \frac{I_{max}}{1+p}\left(1+p\cos 2\theta\right) \\
&= \frac{1+p\cos 2\theta}{1+p} I_{max}
\end{aligned}$$

This is what we were supposed to demonstrate.