3 SOUND AND LIGHT

When a 100 m distant lone person shouts, the intensity of the sound is 55 dB. What will the intensity be if 20 000 people shout together at a distance of 100 m, assuming that each person generates the same sound intensity as the lone person?

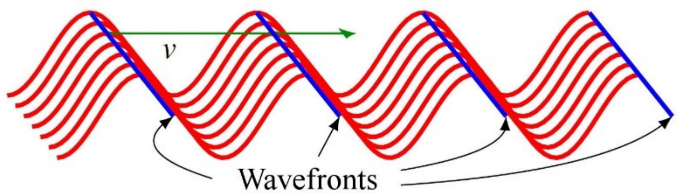
www.boudist.com/archive/2009/03/16/sound-relief-sydney.php

Discover how to solve this problem in this chapter.

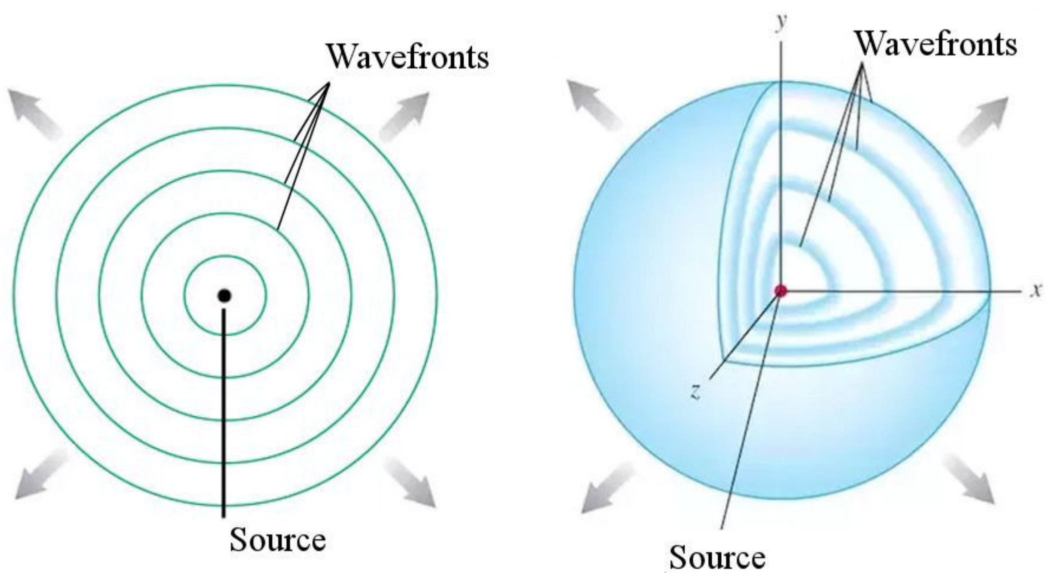
3.1 WAVES IN 2 OR 3 DIMENSIONS

In this chapter, two types of waves propagating in 3 dimensions will be explored. They are sound and light.

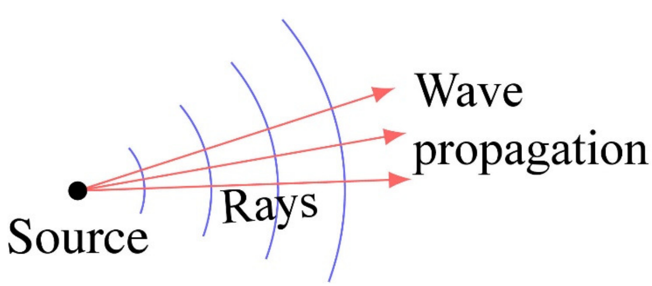
In 2 or 3 dimensions, the wave is represented with ***wavefronts.*** Wavefronts are lines that connect all points of the wave which are at the same phase. To avoid unnecessary complications, wavefronts can often be considered as lines that follow the maxima (or crests) of the wave. (Sometimes, lines that follow the minimums of the wave (trough) will also be used.)



Using wavefronts, here are representations in two and three dimensions of a wave emitted by a point source.



wps.pearsoncustom.com/wps/media/objects/2056/2105451/media_portfolio/10.html and www.quora.com/What-is-a-spherical-wave-front

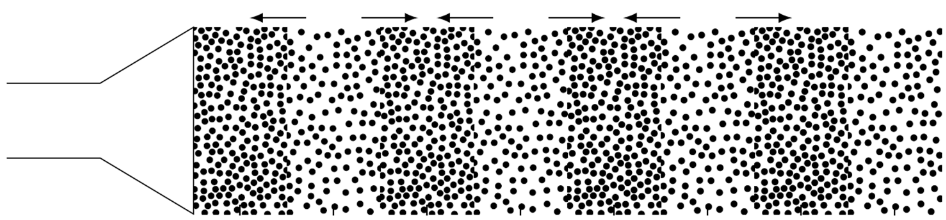


Note that the wavefronts are perpendicular to the direction of propagation of the wave. These lines following the direction of wave propagation are called ***rays***.

3.2 SOUND WAVES

Nature of Sound Waves

When an object vibrates in the air, the vibration disturbs the air molecules around the object. These disturbances propagate and sound waves are thus created. In the following video, you can observe this process in action.
http://www.youtube.com/watch?v=Y84Wc8fs-NA
When the object is moving towards the right in the video, it creates an area where the air is denser and where the pressure is higher. When the object is moving towards the left, it creates an area where the air is less dense and where the pressure is lower. It can be seen that these areas of high and low density propagate towards the right into the tube. A speaker works exactly like that.



When the oscillation is in the same direction as the direction of propagation of the wave, the wave is called a ***longitudinal wave***.

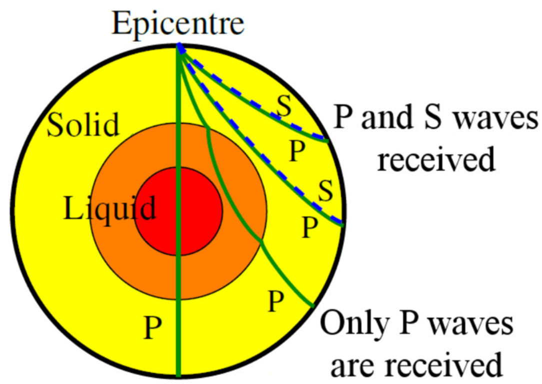
Longitudinal and Transverse Waves in Matter

To have a wave in a medium, there must be a force opposed to the deformations of the medium. If a piece of rope is moved from its equilibrium position, the tension of the rope tries to bring back the rope to its equilibrium position. This is the force that is opposed to the deformations of the medium, thereby allowing waves to propagate on the rope.

Sound is a wave of compression and expansion. When the medium (solid, liquid or gas), is compressed or stretched, the elasticity of matter creates a force opposed to the compression or the expansion. As there is a force opposed to the deformation, sound wave (which are compression waves) can travel in any substance.

Note that transverse waves can also exist in matter, but only in solid. In a transverse wave, the material is moved from one side to the other of the direction of propagation. In a solid, the deformation generates forces that seek to restore the initial position and transverse waves are possible. In fluids (liquids and gases), there is no force opposed to the displacement of matter. If a bit of air is displaced a bit, no force tries to bring back the moved air to the place of departure. Transverse waves are thus impossible in fluids.

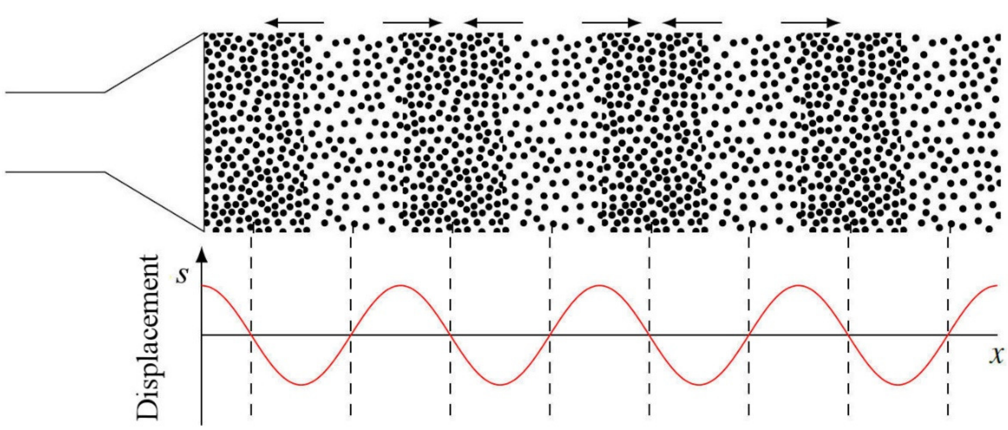
This property allows us to know that the interior of the Earth is liquid. Earthquakes send both longitudinal (noted P in the diagram) and transverse waves (noted S in the diagram) in every direction inside the Earth. Since only longitudinal waves are received on the other side of the Earth, it is possible to conclude that transverse waves cannot pass through the interior of the Earth and that, therefore, the interior of the Earth must be liquid. In locating where transverse waves can be received on the surface of the Earth, the size of the liquid region can even be determined.



In a solid, longitudinal waves and transverse do not necessarily have the same speed. Even if the waves are travelling in the same medium, the speeds can be different since the restoring forces at the origin of the propagation of these waves are different for both types of waves. All longitudinal waves have the same speed and all transverse waves have the same speed, a speed that can be different from the speed of longitudinal waves. For example, earthquakes create longitudinal and transverse waves in the ground. In this case, longitudinal waves are travelling faster than transverse waves. Approximately, longitudinal waves have a speed of 6 km/s near the Earth's surface while transverse waves have a speed of 3 km/s. By measuring the difference in time of arrival between longitudinal waves (called primary waves, because they arrive first) and transverse waves (known as the secondary waves) the distance of the epicentre can be calculated.

Sine Sound Waves

The sound wave can be sine wave. In this case, the displacement of air molecules is described by a sine function.



(The *x-axis* is in the direction of propagation of the wave.)

Since the wave is a sine wave, the wave has a wavelength and frequency. This means that the following formulas, obtained in the previous chapter, are still valid for those sound waves.

$$v = \lambda f = \frac{\omega}{k}$$

where the values of *f* et ω are

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f$$

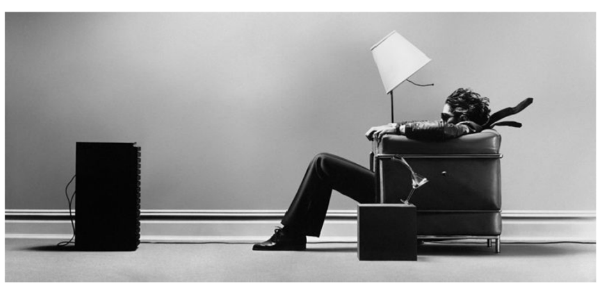
As sound is a longitudinal wave, the displacement is in the same direction as the direction of propagation of the wave (defined as the *x*-axis). Molecules do not move along the *y*-axis but along the *x*-axis. However, *x* is already used to indicate the equilibrium position of the molecule. The letter *s* is, therefore, used to measure the displacement of the molecules of the medium from the equilibrium position. The displacement of the medium for a longitudinal sine sound wave with constant amplitude is given by

Displacement of the Molecules of the Medium When a Sine Sound Wave Is Passing

$$s = A\sin\left(kx \pm \omega t + \phi\right)$$

where *A* is the amplitude of the oscillation motion of the molecules of the medium. (To have a wave with a constant amplitude, it must spread in only one direction, like sound propagating in a tube.)

Again, the air molecules are not transported by the wave. If you focus your gaze on a specific molecule on the animation seen previously
http://www.youtube.com/watch?v=Y84Wc8fs-NA
you can see that the passage of the sine wave simply makes it oscillate around an equilibrium position. This is why this next picture is impossible. The sound cannot push objects in this way.



iconicphotos.wordpress.com/2010/05/31/blown-away-man/

The formula of the position of the molecules $s = A \sin (kx \pm \omega t + \phi)$ means that the velocity and the acceleration of air molecules as a function of time are given by

$$v_x = \pm A\omega\cos\left(kx \pm \omega t + \phi\right)$$
$$a = -A\omega^2 \sin\left(kx \pm \omega t + \phi\right)$$

These formulas, identical to those obtained for a wave on a string in Chapter 2, were obtained by differentiating once and twice the formula of the position. The velocity of the molecules is however noted v_x since this velocity is in the direction of the propagation of the wave which is the x-axis.

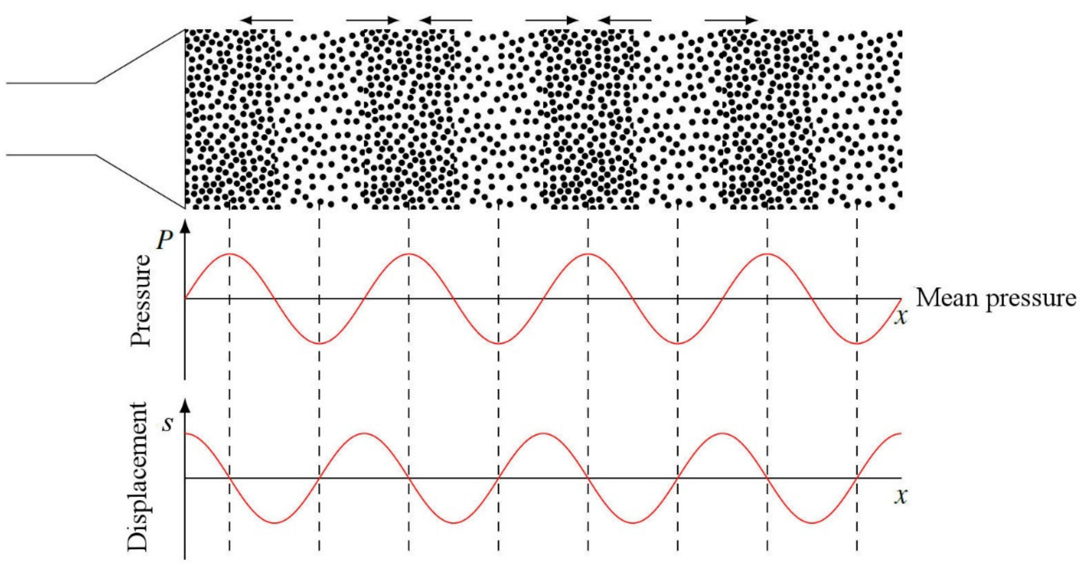
The formula of the position of the molecules $s = A \sin(kx \pm \omega t + \phi)$ also indicates that each air molecule undergoes a harmonic motion. This means that the following formulas

$$s^2 + \left(\frac{v_x}{\omega}\right)^2 = A^2$$

$$\tan\left(kx \pm \omega t + \phi\right) = \frac{s\omega}{\pm v_x}$$

are also valid for sine sound waves.

The wave could also have described with a sine for pressure variations.



Note that the pressure maxima correspond to the places where the displacement is zero (the other molecules approach the molecule located at the maximum) and that the pressure minimums also correspond to the places where the displacement is zero (the other molecules move away from the molecule located at the minimum).

For those wanting to see more details concerning the link between the motion of air molecules and the variations of pressure in a sound wave, you can read this document.
http://physique.merici.ca/waves/link-Px.pdf

Speed of Sound in Air

There is a formula giving the speed of sound in air. To obtain this formula, air is considered to be an ideal diatomic gas (i.e. made of molecules composed of two atoms, which is the case for the majority of molecules in air). The proof of this formula will not be shown because some notions of fluid mechanics are required, notions that the vast majority of college students do not have. This formula is

$$v = \sqrt{\frac{1,4RT}{M}}$$

where R = 8.31451 J/mol K is the ideal gas constant
M = 28.9645 g/mol is the average molecular mass of air molecules
T is the air temperature (in kelvin)

If you still want to see proof, here it is
http://physique.merici.ca/waves/proof-vsound.pdf

The following manipulations can then be made.

$$v = \sqrt{\frac{1.4RT}{M}}$$

$$v = \sqrt{\frac{1.4 \cdot R \cdot 273.15K}{M}}\sqrt{\frac{T}{273.15K}}$$

The value of the first square root can then be calculated to obtain

Speed of Sound in Air

$$v = 331.3\tfrac{m}{s} \cdot \sqrt{\frac{T}{273.15K}}$$

where T is the air temperature in kelvin

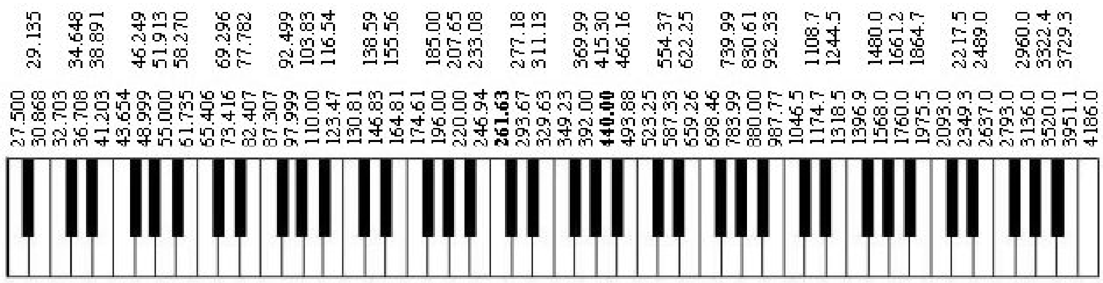
At 0 °C, sound travels at 331.3 m/s while it travels at 343.2 m/s at 20 °C. For temperatures around 0 °C (from about -50 °C to 100 °C), the speed of sound increases by approximately 0.6 m/s per degree Celsius.

There are also formulas giving the speed of sound in fluids and solids. However, these formulas refer to some concepts of fluid physics and solid-state physics. As these concepts are usually unknown to college students, those formulas will not be given. Let's just say that the speed is different when the medium is different.

For example, the speed of sound in water is around 1450 m/s (it varies with temperature) and the speed of sound in steel is around 5000 m/s. A larger speed in steel than in air does not necessarily mean that sound travels more easily in steel than in air. Sound in a steel rod could be absorbed on a much shorter distance than in air, even if the speed of sound is greater in steel.

Sound Frequency

The frequency determines the pitch of the sound. A low frequency corresponds to a low-pitched sound, and a high frequency corresponds to a high-pitched sound. Some frequencies correspond to musical notes as shown in this diagram.

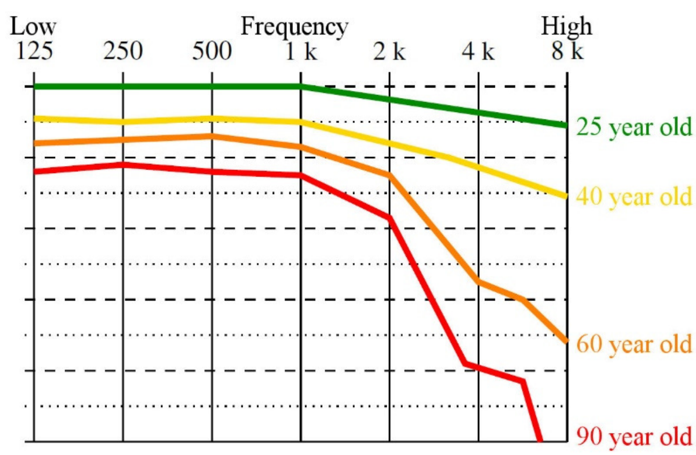


www.sciencebuddies.org/science-fair-projects/project_ideas/Music_p012.shtml

Note that the frequency doubles whenever we go up an octave. The frequency of the third C (called C3) in the diagram is 130.81 Hz, and the frequency of the next C (C4 or middle C) has a frequency of 261.63 Hz, which is double. Here is a fun fact: the frequencies of the notes are not immutable. In fact, they slowly tend to increase over time because orchestras sometimes tune to a higher frequency to get a richer sound. Thus, the note A4 was generally around 425 Hz at the time of Mozart (end of the 18^{th} century) while it is now at 440 Hz. Some orchestras now tune this note to 442 Hz or even 444 Hz.

The lowest frequency that the human ear can hear is a few tens of hertz (Actually, it's a gradual decline in sensitivity from 60 Hz and 20 Hz). Sounds with frequencies lower than that are called *infrasound*.

There is also a maximum pitch that a person can hear but this limit depends on the age of the person. For young adults, the limit is around 20,000 Hz. This limit decreases with age to reach about 5000 Hz for a 90-year-old person (see graph on next page). Sounds with frequencies above 20 000 Hz are called *ultrasound*.



Sound sources producing high-pitch sound (15 000 Hz) were even invented to annoy young people in places where they are not welcome. This sound is very unpleasant for any young person, but old people do not hear it…

Take the test! (Be careful with these tests. Sometimes, sound encoding removes high-pitched sounds. For example, all the sounds with frequencies above 20 000 Hz are eliminated in MP3 encoding, regardless of the selected resolution. In some cases, all sounds with frequencies higher than 16 000 Hz are removed. This does not really matter because it is very rare to have very obvious sounds with frequencies higher than 10 000 Hz in a song, except maybe in *My name is* by Eminem. I checked this test with a spectrum analyzer and there are actual sounds up to 16 000 Hz.)
http://www.youtube.com/watch?v=o810PkmEsOI

Some animals have hearing limits at higher frequencies. Dogs can hear up to around 50 000 Hz, cats up to approximately 60 000 Hz, mice up to approximately 90 000 Hz, bats, which use ultrasounds to detect obstacles, up to 110 000 Hz, and some species of whale hear up to about 150 000 Hz. Don't worry, we're not the worst species. Elephants rarely exceed 12 000 Hz and chickens 2000 Hz.

Ultrasounds are also used in medicine. During an ultrasound, 2 000 000 Hz sound waves are often used. Using the reflections of this sound on the various organs of the body, it is possible to recreate an image showing the inside of the body, including any babies who may be present.

www.colourbox.com/image/obstetric-ultrasound-of-fetus-at-fourth-month-echography-scan-image-6922639

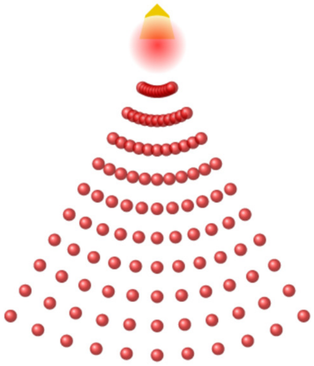
It is important to remember that the sound is a perturbation propagating in matter. This means that there is no sound in space since there is no air in space. This also means that there are serious physics mistake in virtually every science fiction movie. Okay, I admit it would be a bit dull if there was no sound during the scenes happening in the vacuum of space.

3.3 LIGHT

The Nature of Light Waves

The Feud Between the Corpuscular and Wave Theories

It was not at all clear before the start of the 19th century that light is a wave. Before this date, two light theories were opposing each other. These theories were the corpuscular theory and the wave theory (there were other theories as well, but they were less popular.)



According to the proponents of the corpuscular theory, a light source emits light particles.

According to the proponents of the wave theory, a light source emits a wave spreading from the source.

Initially, the corpuscular theory is considerably more popular. Newton was a supporter of the corpuscular theory.

However, a series of observations, made between 1803 and 1822, allowed to conclude that light is a wave. These observations that enabled the wave theory to prevail will be seen in the next chapters.

A Wave in Aether

If light is a wave, there was a problem. How can the wave travel in space where there is no matter? In what matter could the wave travel? Thus, it was postulated that there was a new substance, called *aether* (which has nothing to do with aether in chemistry), and that light was a perturbation propagating in this medium. This substance had to be present everywhere in the universe because light can travel throughout the universe. If light can be received from the Andromeda Galaxy, then aether had to be present everywhere along the path between the Andromeda Galaxy and the Earth. Note that substance was never isolated but it was believed that it had to exist since light waves can propagate.

At the same time, this aether must not exert any frictional force since the Earth is rotating around the Sun without losing any energy because of friction. If aether had exerted only a small frictional force, the Earth would have slowly lost its energy and would have finished his journey in the Sun. This absence of friction had initially suggested that the aether must be a fluid and that light had to be a longitudinal wave (because transverse waves cannot propagate in a fluid).

In 1816, there were still some phenomena (involving polarization) that the wave theory was unable to explain. It was then that André-Marie Ampère discovered that these phenomena could be explained if it was assumed that light is a transverse wave instead of a longitudinal wave. In 1822, Augustin Fresnel further developed this idea of transverse waves, and then obtain results in perfect harmony with the observations. From then on, all optical phenomena known at that time could be explained by the wave theory, and there was no longer any significant supporter of the corpuscular theory (until its return in 1905… see in a further chapter).

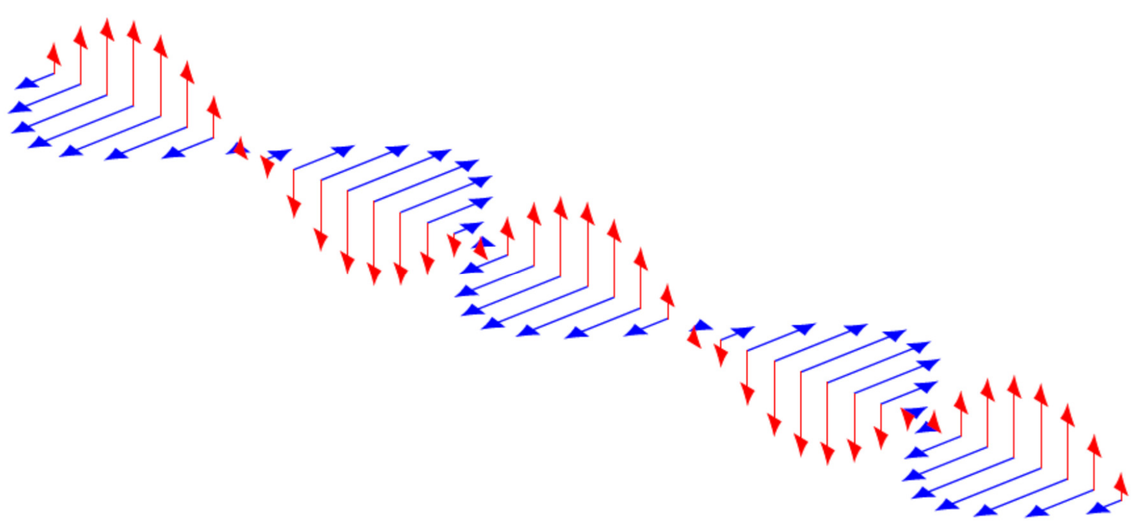
However, if light is a transverse wave, then the aether must be a solid. But then, how could the aether offer no resistance while being rigid at the same time? Throughout the 19th century, many efforts were made (in vain) to try to make rigid ether models in agreement with the observations.

A Transverse Electromagnetic Wave

In 1879, James Clerk Maxwell completed the basic equations of electromagnetism. With these equations, he confirmed that light is an electromagnetic wave and that these waves are really transverse waves.

Attempts were made to combine Maxwell's discoveries with the concept of ether with complications that were sometimes quite spectacular. Despite all these efforts, a coherent theory of the aether was never formulated, and the idea of aether had to be abandoned at the beginning of the 20^{th} century (almost no one talks about it after 1930).

In fact, light does not need a material medium to propagate. Light is a wave of electric and magnetic fields, which are not material things. In this diagram showing a light wave, the electric field is represented by red arrows and the magnetic field by blue arrows.



Here is an animation of the motion of this wave.
http://www.youtube.com/watch?v=4CtnUETLIFs

This is not a mechanical wave since the passage of the wave does not entail any oscillations of a medium. It is said to be a transverse wave because the direction of the fields is always perpendicular to the direction of propagation of the wave. Although there are two fields, only the electric field of the wave will be considered in the sections that follow to simplify.

As the concepts of electric and magnetic fields have not been seen yet, this idea will not be explored further here. (It will be explored in the last chapter in *electricity and magnetism.*) For this course, we can simply say that the electric field of an electromagnetic wave replaces the displacement of the particles in the medium for this type of wave. Therefore, the value of the electric field will be given rather than the displacement of the particles of the medium.

Sine Light Waves

A sine light wave is a wave whose electric field value is given by a sine function.

Electric Field if an Electromagnetic Sine Wave

$$E = E_0 \sin\left(kx \pm \omega t + \phi\right)$$

where E_0 is the amplitude of the electric field, which is measured in N/C (newtons per coulomb). Again, the *x-axis* is in the direction of propagation of the wave. (Note that to have a wave with constant amplitude, the wave must spread in only one direction.)

Since this is a sine wave, the wave has a wavelength and frequency. This means that the following formulas, obtained in the previous chapter, are still valid for light waves.

$$v = \lambda f = \frac{\omega}{k}$$

where f and ω are

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f$$

Speed of Light

In vacuum, electromagnetic waves always travel at the same speed. This speed is

Speed of Light in Vacuum

$$c = 299\ 792\ 458 \text{ m/s}$$

(This will be proven in *electricity and magnetism*.)

In a transparent medium, light waves travel at a slower speed. The speed depends on the substance in which light propagates. The speed is then given by

Speed of Light in a Transparent Substance

$$v = \frac{c}{n}$$

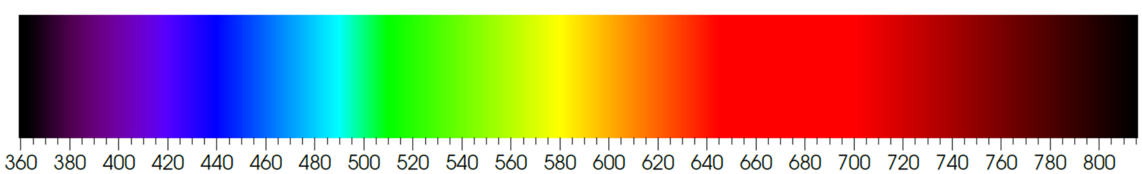
where n is a quantity called the *refractive index* that depends on the substance. Here are some values of this index for different substances.

Substance	Refractive Index
Air	1.000 293
Water	1.333
Glass	About 1.5
Ethanol	1.361
Diamond	2.409
Gallium Phosphide	3.50

The explanation for the slowdown of the wave in a substance is a bit more complicated than you might think. When the wave enters a material, the electric and magnetic fields make the charged particles composing the material oscillate with the same frequency as the wave. However, charged oscillating particles emit electromagnetic waves. The resulting wave in the material is the superposition of the initial wave, which goes at speed c and of the waves emitted by the oscillating particles of matter. Curiously, the result of this superposition is a wave of the same frequency but with a smaller wavelength than the initial wave. This change implies a smaller speed.

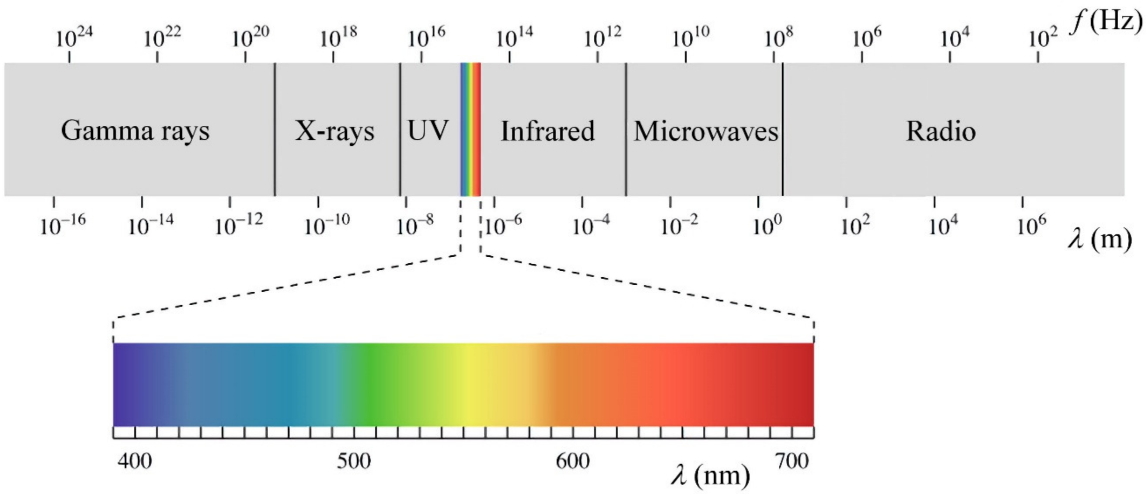
The Electromagnetic Spectrum

The wavelength of the light is associated with its colour. This image shows the colours associated with the different wavelengths (written at the bottom of the diagram).



There is no colour associated with wavelengths greater than 750 nm (approximately) or below 350 nm (approximately) because the human eye is not sensitive to these waves. As for sound, our senses cannot detect every kind of electromagnetic waves. Only waves with wavelengths between 400 nm and 700 nm can be perceived by a human. This is the ***visible part of the electromagnetic spectrum***.

In reality, this spectrum extends on both sides of the visible spectrum and a good part of the spectrum is composed of electromagnetic waves that our eyes cannot perceive. These waves were categorized as follows.



fr.khanacademy.org/science/physics/light-waves/introduction-to-light-waves/a/light-and-the-electromagnetic-spectrum

Let's have a look at each of these different categories.

Radio Waves and Microwaves (Hertz 1888)

Radio Waves: λ larger than 3 m
Microwaves: λ between 1 mm and 3 m

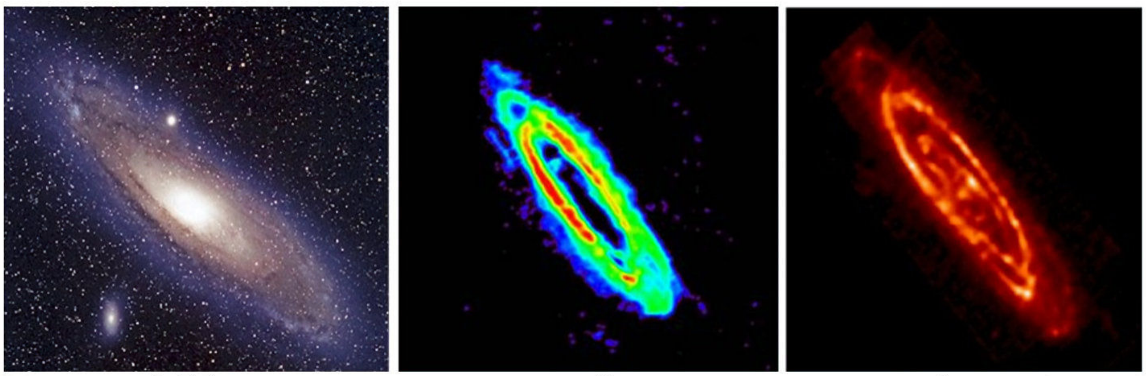
These waves are mostly used for communications. There is a very detailed allocation of each wavelength according to its use: television, commercial radio, cell phones, aeronautical radio navigation, maritime navigation, meteorology, satellite communications, radio astronomy, space exploration and other. The allocations for Canada can be seen on this site.
http://www.ic.gc.ca/eic/site/smt-gst.nsf/vwapj/spectallocation-08.pdf/$FILE/spectallocation-08.pdf

These allocations are necessary to avoid unwanted waves to be received by the wrong device. It would be really annoying if, while speaking on the cell phone, you were constantly hearing radio or police communications at the same time.

Microwave ovens use microwaves at 2450 MHz (wavelength equal to 12.2 cm). Here is a demonstration of a dangerous use of microwave ovens.

https://www.youtube.com/watch?v=8j5_iaIrWeE

In radio astronomy, radio waves and microwaves emitted by celestial objects are studied. Images that correspond to what would be seen if the human eyes were sensitive to these wavelengths are created. There are things that can be seen on these pictures that cannot be seen with visible light. For example, here are three images of the Andromeda Galaxy. The first is in visible light, the second in radio waves and the last in microwaves (λ = 24 µm).



Visible Radio Microwave

www.wired.com/wiredscience/2011/04/jill-tarter-qa/
thebeautifulstars.blogspot.ca/2011/08/multi-wavelength-views-of-stuff.html

Common Mistake: Using the Speed of Sound for the Speed of Radio Waves

Radio waves passing from the transmitting antenna to your home radio or your car radio travel at the speed of light. They cannot be heard in this form. They will only become sound wave once your receiver has transformed them into sound waves.

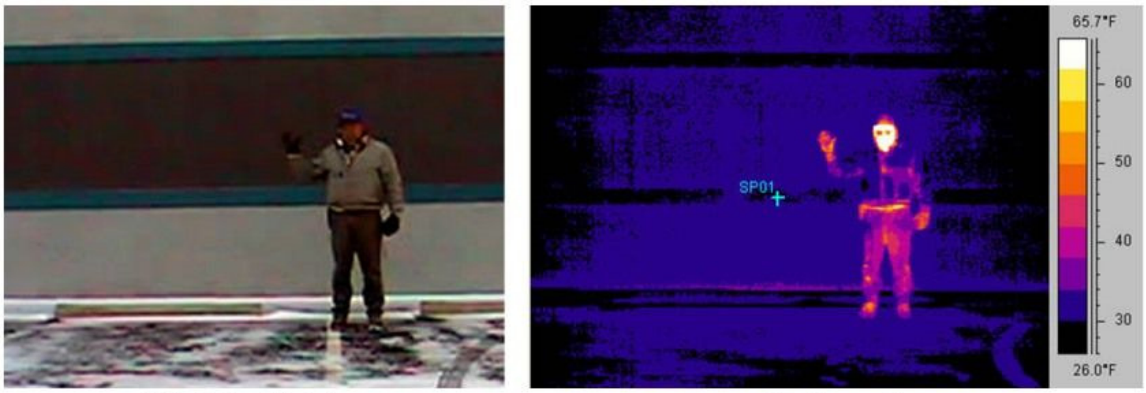
Infrared (Herschell 1800)

Infrared: λ between 700 nm and 1 mm

Infrared rays are absorbed by many kinds of molecules. The absorbed energy will then be transformed into vibration and rotation of molecules, which means that their temperature will rise. This is why infrared (at least a part thereof) are often perceived as a heat sensation.

It will be seen in chapter 11 that objects emit electromagnetic radiation and that the nature of this radiation changes depending on the temperature of the object. For objects with temperatures ranging from 3 K to 4000 K (this obviously includes most of the ordinary life objects), this radiation is composed primarily of infrared radiation. If the object is hotter, more radiation is emitted per unit area. If humans could see infrared radiation, all objects around us would be light sources. According to the intensity of the radiation emitted, we would be able to know the temperature of the object.

Even if our eyes are not sensitive to infrared light, some cameras are. With them, images that show what would be seen with infrared radiation are obtained.



Visible light Infrared

www.solarcrete.com/solarcrete-insulated-concrete-wall-buildings.php

In this picture, the most important source of infrared is the face of this man, the hottest element of this picture.

Ordinary digital cameras are sensitive to a part of the infrared spectrum. An infrared image can be obtained by using a filter that blocks all visible light. The image to the right was obtained in this way. This picture is an image in the near infrared (700 to 900 nm). This picture was taken in summer even if everything seems to be frozen.

There are many others on this site.
http://www.smashingmagazine.com/2009/01/11/40-incredible-near-infrared-photos/
This is essentially a black and white photography that measures the intensity of infrared radiation emitted by objects. At these wavelengths, infrared rays are not emitted by hot objects but are rather coming from the reflection of sunlight on the objects, as with visible light.

Be warned, some video cameras capture infrared radiation. They, therefore, capture the radiation emitted by hot objects. In this clip, a source of infrared suddenly appears behind the person. This is emitted by a hot gas coming from the anus of the person (yes, a fart). This gas stands out from the cold surrounding air because it emits much more infrared radiation than the cold ambient air.
https://www.youtube.com/watch?v=NccsJ4MxCW0

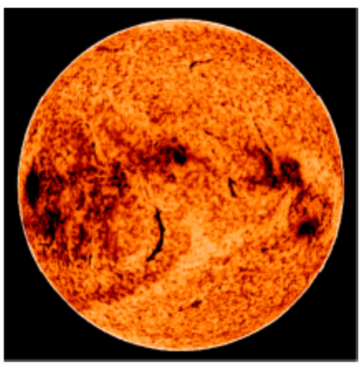
Some materials are opaque in visible light but transparent to other wavelengths (or vice versa). This is the case here with this image showing that a black garbage bag is opaque in visible light but transparent in infrared.

Visible light

Infrared

coolcosmos.ipac.caltech.edu/cosmic_kids/learn_ir/

Note, however, that his glasses were transparent for visible light, but are opaque for the infrared.



Astronomers also use infrared light to study celestial objects. Here is an image of the Sun in infrared ($\lambda = 1083$ µm). It has a very different look than with visible light.

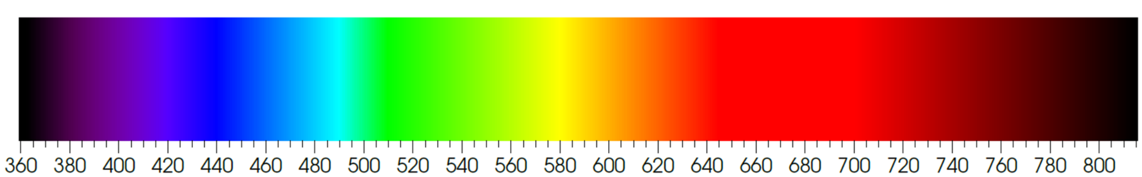
solar.physics.montana.edu/ypop/Spotlight/Today/infrared.html

Visible light

Visible Light: λ between 400 nm and 700 nm

(Actually, it can go from 380 nm to 750 nm.)

For visible light, each wavelength corresponds to a specific colour. Here is an image showing the colours depending on the wavelength.



Colour	Wavelength (nm)	Colour	Wavelength
Red	700 to 625	Green-Blue	530 to 492
Orange	625 to 590	Cyan	492 to 487
Yellow	590 to 580	Blue-Green	487 to 482
Yellow-Green	580 to 575	Blue	482 to 465
Green-Yellow	575 to 560	Indigo	465 to 435
Green	560 to 530	Violet	435 to 400

These boundaries between the colours are somewhat approximate.

Ultraviolet (Ritter 1801)

Ultraviolet: λ between 10 nm and 400 nm

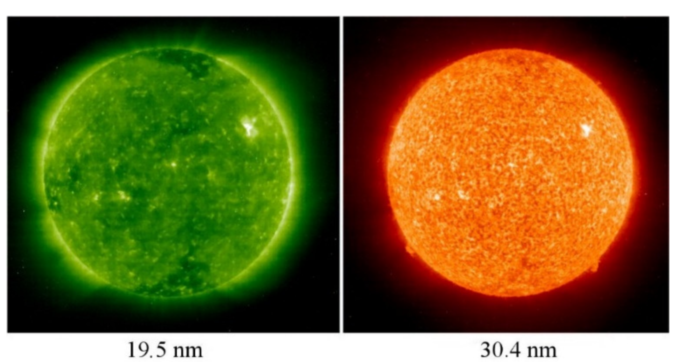
(For wavelengths smaller than the wavelength of visible light, the boundaries between categories are fuzzy so that certain wavelengths fit into two categories.)

As for infrared radiation and visible light, ultraviolet rays are absorbed by many molecules. Melanin in the skin absorbs ultraviolet radiation from the Sun to give you a beautiful tan. The molecules in white clothes absorb ultraviolet light coming from *black lights* in bars and re-emit it in visible light. They then seem to be much brighter than what they are supposed to be according to the ambient lighting.

As with infrared, ultraviolet photography can be done with a camera sensitive to these wavelengths (or a part of these wavelengths). Here is a sample image of ultraviolet photography. Some features that were invisible in visible light can now be seen.

loveplantlife.blogspot.ca/2012/08/a-bees-eye-view-of-flowers.html

As always, astronomy studies can be done with ultraviolet radiation. Here are two images of the Sun in ultraviolet.



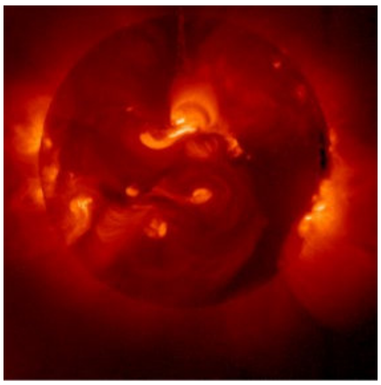
www.thesuntoday.org/the-sun-now/

X-Rays (Röntgen 1895)

X-Rays: λ between 0.001 nm and 100 nm

At these wavelengths, electromagnetic waves have a significant penetration power. They may go through several substances with varying degrees of absorption depending on the material. Images like this can then be obtained.

lupusadventurebetweenthelines.wordpress.com/2011/03/20/lupus-arthritis-on-a-cloudy-day/



The Sun has a very different look in X-rays (wavelength between 0.3 and 4.5 nm for this picture).

www.thesuntoday.org

The Andromeda Galaxy also has a very different look in X-rays.

Visible light X-rays

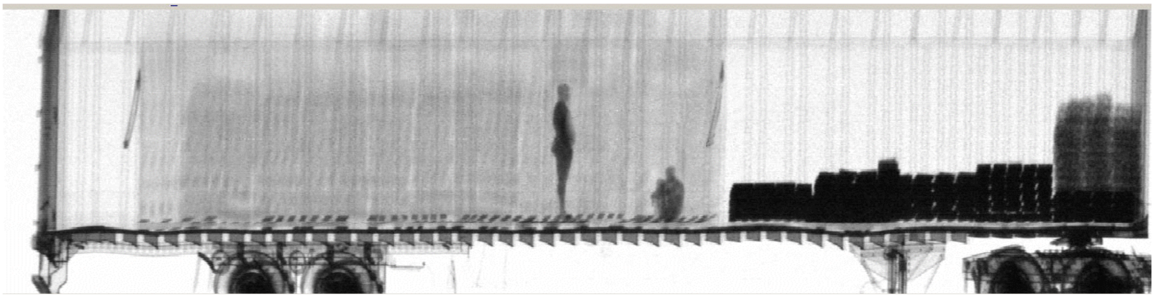
chandra.harvard.edu/photo/2007/m31/

Gamma Rays (Villard 1900)

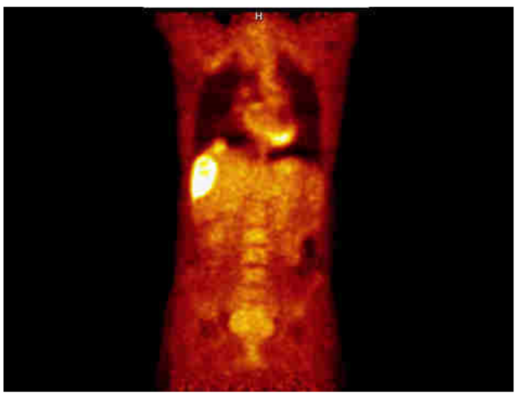
Gamma Rays: λ smaller than 0.01 nm

There is a significant overlap between X-rays and gamma rays. Traditionally, X-rays are small wavelength waves obtained in atoms transitions while gamma rays are the result of radioactive decay. It is the origin of the radiation rather than the wavelength that determines the type of wave.

At these wavelengths, electromagnetic waves have an even greater penetrating power than X-rays, and they can be used somewhat similarly. The following image was obtained with gamma rays. People hiding in this truck were detected in this way.



teachnuclear.ca/contents/cna_radiation/gamma_rays/



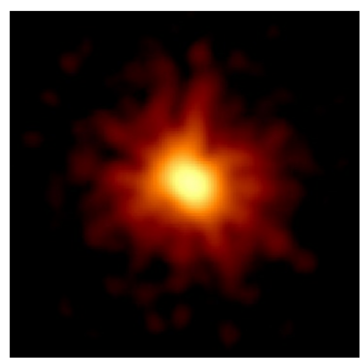
www.nucmedinfo.com/Pages/petbase.html

Gamma rays are also used in medical imaging. This image to the left was obtained with gamma rays.

Earth is constantly bombarded by gamma rays, which are then part of cosmic rays. In these cosmic rays, waves having a wavelength as small as 10^{-20} m were observed.

Gamma-ray bursts are also observed in the universe. The cause of these gigantic explosions, the largest in the universe, is not known at the moment. They can free up to 10^{44} J, or about all the energy that the Sun will release during his lifetime (10 billion years). A typical gamma-ray burst lasts tens of seconds.

The image on the right shows a gamma-ray burst visible to the naked eye for a few seconds on March 19, 2008. The object that generated this burst is located 10.6 billion light-years from Earth, which means that it would take 10.6 billion years to get to this object traveling at the speed of light (if the universe was not expanding).



en.wikipedia.org/wiki/GRB_080319B

Note on Lasers

Lasers emit electromagnetic waves that are not different in nature from other electromagnetic waves.

The only difference that is of interest here is the fact that the light from a laser is monochromatic. The wave emitted by a laser is an electromagnetic wave with a very precise wavelength. For example, helium-neon lasers emit red light with a 632.8 nm wavelength. As only one colour is present, the light is called *monochromatic*. It's really rare to have monochromatic natural light. For example, the light from a red object is often a superposition of several sine waves with wavelengths around 650 nm rather than composed of a single wave having a very specific wavelength. (In fact, even laser light spans a range of wavelength, but this range is very small.)

Otherwise, laser light is not very different from ordinary light. Some feats realized with lasers have nothing to do with these two characteristics. For example, lasers are used to cut metal plates, but they are able to cut like this simply because lasers that emit a very bright light can be manufactured. A metal plate could also be cut with ordinary light if enough of it can be focused on the place where the cut must be made. Laser light has no specific cutting or explosive properties that ordinary light does not have.

Note that the light emitted by lasers is not necessarily in the visible part of the spectrum. There are, for example, laser emitting microwave (in this case, the laser is rather called a maser) or infrared (like lasers in CD players, which emit light having a 780 nm wavelength).

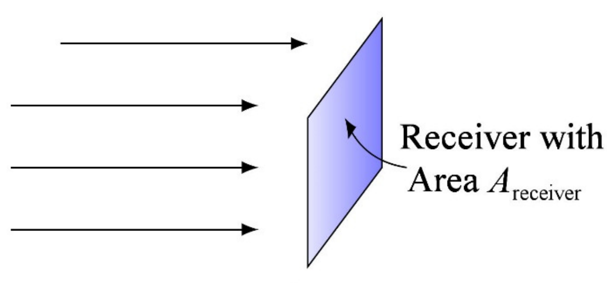
3.4 INTENSITY

Sound and light are waves propagating in three dimensions. This caused a change in how the energy is measured compared to the energy of a wave on a rope, which is a wave in one dimension. With three dimensional waves, the energy is linked to the intensity of the wave instead of the power.

Definition of Intensity

For a wave on a rope, a receiver located at the end of the rope receives all the power of the wave because the wave has no other place to go. For a wave in three dimensions, a receiver captures only a part of the wave, as it is possible for the rest of the wave to pass beside the receiver.

In the following situation, a wave heading towards the right arrives on a receiver with an area $A_{receiver}$.



Of course, more energy is received if the energy of the wave is received for a longer time. The energy received must then be proportional to the time during which the energy is received.

Also, more energy is received if the area of the receiver is greater. The energy received must then be proportional to the area of the receiver.

Finally, there must be a factor that depends on the energy of the wave. This factor will be called the *wave intensity*. Not a lot of energy is received with a low-intensity wave and a lot is received with a high-intensity wave. The energy received must be proportional to the intensity of the wave.

This means that

$$E_{received} = IA_{receiver}t$$

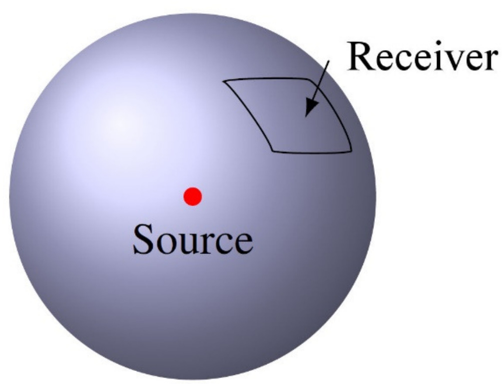
where I is the intensity of the wave. Power is energy divided by time. Thus, the previous formula can be divided by t to obtain a formula of the received power.

Received Power

$$P_{received} = IA_{receiver}$$

(This equation is, in fact, the definition of intensity.) To obtain a power in watts, the unit of intensity must be W/m².

Intensity at a Distance *r* From an Isotropic Source



Imagine being at a distance r from a source that emits an energy E during time t. Here, the energy is equally distributed in every direction. This is called an isotropic source. Thus, at a distance r, the emitted energy is uniformly distributed on a sphere surrounding the source.

At a certain distance from the source, there is a receiver with an area $A_{receiver}$. This receiver captures only a part of the energy emitted. The captured proportion is simply given by the ratio between the area of the sensor ($A_{receiver}$) and the total area on which the energy of the wave is spread.

$$\frac{E_{received}}{E} = \frac{A_{receiver}}{A_{sphere}}$$

$$E_{received} = \frac{E}{A_{sphere}} A_{receiver}$$

$$E_{received} = \frac{E}{4\pi r^2} A_{receiver}$$

Dividing the two sides of this equation by time transforms energy into power since $P = E/t$, the result is

$$P_{received} = \frac{P}{4\pi r^2} A_{receiver}$$

Since $P_{received} = IA_{receiver}$, the intensity is

Intensity of a Wave Emitted by an Isotropic Source

$$I = \frac{P}{4\pi r^2}$$

This formula is valid for sound and light waves.

Example 3.4.1

An isotropic light source has a power of 100 W.

a) What is the intensity of the wave 120 m from the source?

The intensity is

$$\begin{aligned} I &= \frac{P}{4\pi r^2} \\ &= \frac{100W}{4\pi \cdot (120m)^2} \\ &= 5.526\times10^{-4} \tfrac{W}{m^2} \end{aligned}$$

b) What is the energy received in 10 seconds with a sensor with an area of 2 m² and at 120 m from the source?

The energy received is $E_{received} = P_{received}t$. Thus, the power received must be known. This received power is

$$\begin{aligned} P_{received} &= IA_{receiver} \\ &= 5.526\times10^{-4} \tfrac{W}{m^2} \cdot 2m^2 \\ &= 1.1052\times10^{-3} W \end{aligned}$$

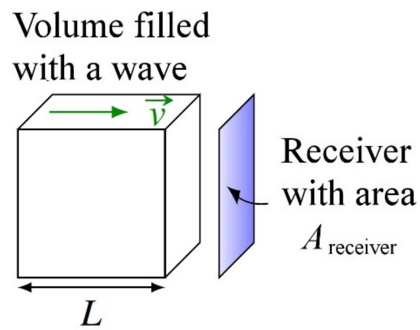
Since power is energy per unit of time, the energy received is

$$\begin{aligned} E_{received} &= P_{received}t \\ &= 1.1052\times10^{-3}W\cdot10s \\ &= 0.011052J \end{aligned}$$

Sound Intensity

The Intensity From the Amplitude of the Wave

Imagine that there is a sinusoidal sound wave moving towards the right and that this wave arrives on a receiver. Let's find the energy there is inside an imaginary cubic volume travelling with the wave and arriving on the receiver. This imaginary cube is moving at the speed of sound towards the receiver.



Inside the cube, the molecules are making a harmonic motion. This means that the energy of one molecule is

$$E_{1\,molecule} = \frac{1}{2}m\omega^2A^2$$

If the energies of all the molecules inside the cube are added, the total energy received is obtained.

$$\begin{aligned} E_{received} &= \sum\frac{1}{2}m\omega^2A^2 \\ &= \frac{1}{2}\left(\sum m\right)\omega^2A^2 \\ &= \frac{1}{2}m_{total}\omega^2A^2 \end{aligned}$$

The angular frequency and the amplitude were factored out of the sum because it is assumed that the wave has the same frequency and the same amplitude everywhere in the cube. Using the density of the medium (ρ) the energy can be written as

$$\begin{aligned} E_{received} &= \frac{1}{2}\rho\left(Volume\right)\omega^2A^2 \\ &= \frac{1}{2}\rho LA_{receiver}\omega^2A^2 \end{aligned}$$

The received energy is also $E_{received} = IA_{receiver}t$. Thus, the equation becomes

$$IA_{receiver}t = \frac{1}{2}\rho LA_{receiver}\omega^2A^2$$

$$It = \frac{1}{2}\rho L\omega^2 A^2$$

It only remains to find the time taken to receive the energy. The reception of the wave begins when the front of the cube arrives at the sensor and finishes when the back of the cube arrives at the sensor. The receiving time, therefore, corresponds to the time that it takes for the back of the cube to arrive at the sensor. As the back of the cube is at a distance L from the sensor and is moving at speed v, the time is L/v. Therefore,

$$It = \frac{1}{2}\rho L\omega^2 A^2$$

$$I\frac{L}{v} = \frac{1}{2}\rho L\omega^2 A^2$$

Solving for I, the following result is obtained.

$$I = \frac{1}{2}\rho v\omega^2 A^2$$

Impedance

Again, the part of this formula that only depends on the medium (i.e. the density ρ and speed v) is the impedance of the medium.

Impedance of the Medium for Sound Waves

$$Z = \rho v$$

The units of the impedance are kg/m² s or Ns/m³. This combination of units is also called a rayl sometimes.

Thus, the intensity of sound waves is given by the following formula.

Intensity of a Sound Wave

$$I = \frac{1}{2}\rho v\omega^2 A^2 = \frac{1}{2}Z\omega^2 A^2$$

Example 3.4.2

What is the amplitude of oscillation of the air molecules when a 500 Hz sound wave having an intensity of 0.01 W/m² travels in air? (The density of air is 1.29 kg/m³ and the speed of sound is 340 m/s.)

The amplitude is found with

$$I = \frac{1}{2}\rho v\omega^2 A^2$$

$$0.01\tfrac{W}{m^2} = \frac{1}{2}\cdot 1.29\tfrac{kg}{m^3}\cdot 340\tfrac{m}{s}\cdot\left(2\pi\cdot 500Hz\right)^2\cdot A^2$$

$$A = 2.149\times 10^{-6}m$$

$$A = 2.149\mu m$$

The Decibel Scale

The human ear can pick up sounds whose intensity is at least 10^{-12} W/m². If the intensity of the sound is smaller than this value, no sound is heard. On the other hand, the sound becomes too intense if its intensity exceeds 1 W/m² (approximately). This is the threshold of pain.

The ear, like the other senses, does not have a linear response. This means that if sound A has twice the intensity of sound B, sound A will not seem to be twice as loud. An intensity scale that better represents what is perceived was therefore devised. This is the decibel scale. The intensity in decibels is calculated with the following formula.

Sound Intensity in Decibel

$$\beta = 10dB\cdot\log\frac{I}{10^{-12}\frac{W}{m^2}}$$

The perception of sound follows this rule: a sound appears to be 2 times louder if the intensity is increased by 10 dB. In the following demonstration, the intensity of a sound is decreased by 3 decibels 15 times.
http://physique.merici.ca/ondes/sons/Thedecibelscale.mp3

Here's a table showing typical sound intensities.

	Intensity (I)	**Intensity (β)**
Minimum intensity	10^{-12} W/m²	0 dB
Whisper	10^{-10} W/m²	20 dB
Conversation (at 50 cm)	10^{-6} W/m²	60 dB
Inside a car at 75 km/h	10^{-5} W/m²	70 dB
Threshold of pain	1 W/m²	120 dB
Jet airplane at 20 m	10 W/m²	130 dB

Example 3.4.3

When a 100 m distant lone person shouts, the intensity of the sound is 55 dB. What will the intensity be if 20 000 people shout together at a distance of 100 m, assuming that each person generates the same sound intensity as the lone person?

The intensity in decibels must never be directly added. The intensities in W/m² must be added to obtain the total intensity. The intensity of the sound made by one person is

$$\beta = 10dB \cdot \log \frac{I}{10^{-12} \frac{W}{m^2}}$$

$$55dB = 10dB \cdot \log \frac{I}{10^{-12} \frac{W}{m^2}}$$

$$I = 3.1623 \times 10^{-7} \frac{W}{m^2}$$

Therefore, the intensity of the sound made by 20 000 people is

$$I = 20\,000 \cdot 3.1623 \times 10^{-7} \frac{W}{m^2}$$
$$= 6.3246 \times 10^{-3} \frac{W}{m^2}$$

In decibels, this intensity is

$$\beta = 10dB \cdot \log \frac{I}{10^{-12} \frac{W}{m^2}}$$
$$= 10dB \cdot \log \frac{6.3246 \times 10^{-3} \frac{W}{m^2}}{10^{-12} \frac{W}{m^2}}$$
$$= 98.01dB$$

Therefore, the difference in intensity is 43 dB. This corresponds to adding 4.3 times 10 decibels. With 20 000 people, the sound will then seem to be $2^{4.3}$ = 19.7 times louder than the sound made by one person. It is clear that the ear does not have a linear response to sound. If it had been linear, it would have seemed 20 000 times louder.

Example 3.4.4

An isotropic source makes a sound having an intensity of 100 dB at a distance of 50 m. What is the intensity of the sound (in decibels) 1 km away from the source?

Knowing that the source generates 100 dB at 50 m, the power of the source can be found. From this power, the intensity 1 km away can then be found.
The intensity at 50 m is

$$\beta = 10dB \cdot \log \frac{I}{10^{-12} \frac{W}{m^2}}$$

$$100dB = 10dB \cdot \log \frac{I}{10^{-12} \frac{W}{m^2}}$$

$$I = 0.01 \frac{W}{m^2}$$

This means that the power of the source is

$$I = \frac{P}{4\pi r^2}$$

$$0.01\tfrac{W}{m^2} = \frac{P}{4\pi(50m)^2}$$

$$P = 314.16W$$

At a distance of 1000 m, the intensity is thus

$$\begin{aligned} I &= \frac{P}{4\pi r^2} \\ &= \frac{314.16W}{4\pi(1000m)^2} \\ &= 2.5\times10^{-5}\tfrac{W}{m^2} \end{aligned}$$

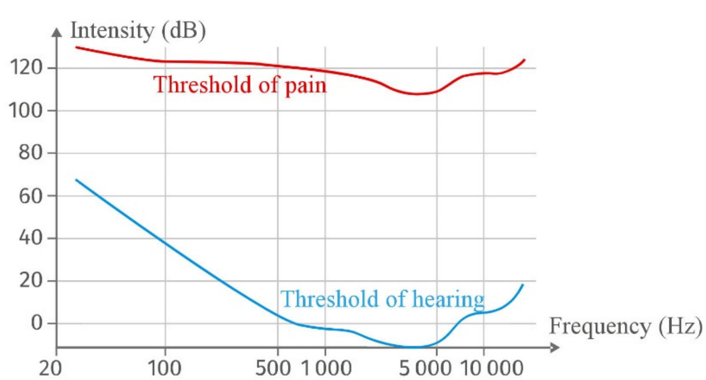
In decibels, this intensity is

$$\begin{aligned} \beta &= 10dB\cdot\log\frac{I}{10^{-12}\tfrac{W}{m^2}} \\ &= 10dB\cdot\log\frac{2.5\times10^{-5}\tfrac{W}{m^2}}{10^{-12}\tfrac{W}{m^2}} \\ &= 74dB \end{aligned}$$

Two small notes.

1- A 0 dB sound at 1000 Hz has an amplitude of 10^{-11} m. This sound can be (barely) heard even if the amplitude of motion of air molecules is smaller than the size of the hydrogen atom! Without contest, the human ear is quite sensitive.

2- Actually, the levels of intensity in decibels are much more complex than what was explained before. The following graph shows the real thresholds of audibility and of pain as functions of the frequency.



www.lelivrescolaire.fr/page/6225300

Light Intensity

We will see (last chapter in electromagnetism) that the exact relationship for light is

Light Intensity

$$I = \frac{1}{2} cn\varepsilon_0 E_0^2$$

where c is the speed of light, n is the refractive index of the medium and ε_0 is a constant (8.854 x 10^{-12} C²/Nm²).

From there, the impedance of the medium for an electromagnetic wave could have been defined. This part, usually made of the parts that depend only on the medium in the intensity formula, should be $Z = cn\varepsilon_0$. However, there are a few subtleties (that will not be given here) which come from the fact that this wave is composed of electric field and magnetic field and that the impedance is actually associated with the magnetic field and not to the electric field. This is why the impedance is instead given by $Z = 1/cn\varepsilon_0$. So, in vacuum, the impedance is 376.7 Ω.

Sensitivity of the Human Eye

The sensitivity of the human eye changes with the wavelengths of the light. Even in the visible part of the spectrum, the sensitivity is not the same for all wavelengths. The intensity perceived by the human eye is measured in lux (symbol *lx*). The lux measures the illuminance. The number of lux is calculated by multiplying the intensity of light by a factor that depends on the wavelength. Here are the values of this factor for a few wavelengths.

Wavelength	Factor	Wavelength	Factor
375 nm	0.015	575 nm	625
400 nm	0.026	600 nm	431
425 nm	4.99	625 nm	216
450 nm	26.0	650 nm	73.1
475 nm	78.9	675 nm	15.8
500 nm	220	700 nm	2.80
525 nm	542	725 nm	0.505
550 nm	680	750 nm	0.0820

(These are the values for the day vision for a field of view of 2°. There are a lot of subtleties in these conversions…)

Thus, a light wave with a wavelength of 500 nm and an intensity of 1 W/m² gives a light with an illumination of 220 lux.

Here are some values of illuminance.

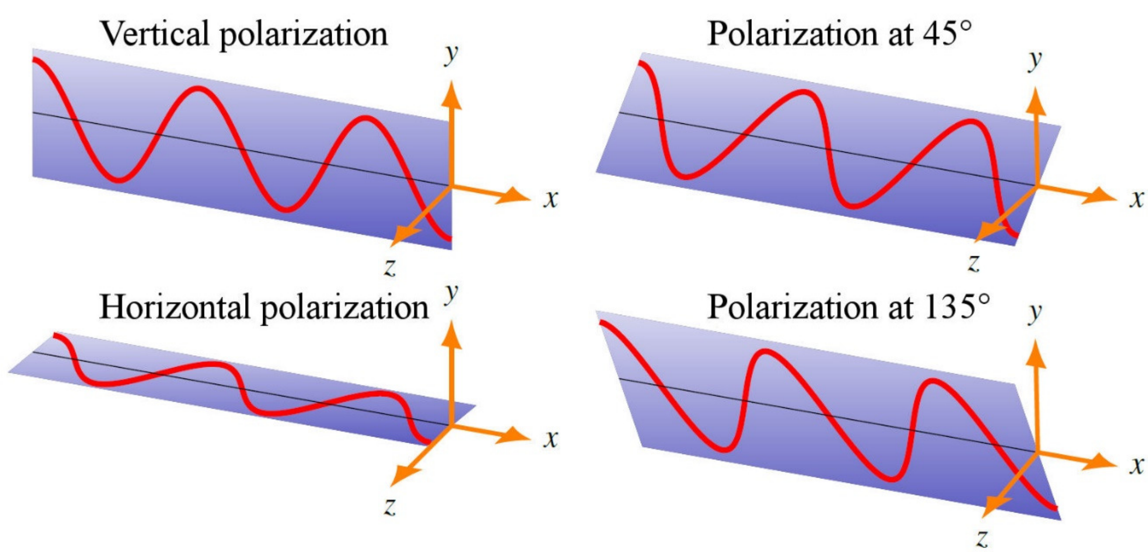
	Illuminance
Camera sensibility	0.001 lx
Full-moon night	0.5 lx
Well-lit street by night	20 to 70 lx
Well-lit apartment	200 to 400 lx
Workplace	200 to 3000 lx
Outdoor, cloudy	500 to 25,000 lx
Outdoor, Sunny	50,000 to 100,000 lx

Note that by multiplying the number of lux by the surface of the sensor, the luminous flux, which is measured in lumen (lm symbol), is obtained.

3.5 POLARIZATION

The Direction of Oscillation of the Electric Field

Since light is a transverse wave, there are several possible directions for the electric field. When the direction of oscillation of the field changes, it is said that the polarization of light changes. The following image shows different possible directions for the direction of the electric field.

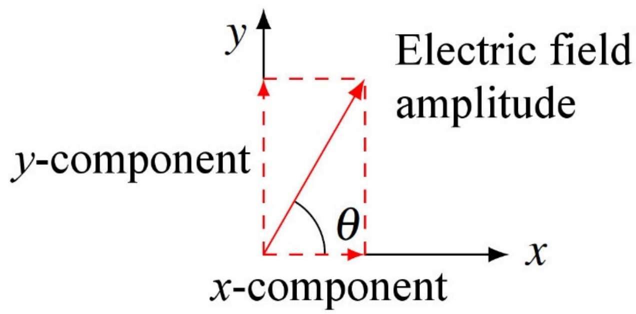


In each of these cases, the oscillation is perpendicular to the direction of propagation of the wave, as it must be for a transverse wave.

Note that these different possibilities do not exist for sound since it is a longitudinal wave. With a longitudinal wave, there is only one possible direction of oscillation.

Principal Components

There is an infinite number of possible directions of oscillation. Should all these different possibilities be examined? Of course not. It is possible to work with only two main directions of polarization (e.g. horizontal and vertical) and resolve all the other polarization with these components. For example, a polarization at 45° can be resolved into one half of horizontal polarization and one half of vertical polarization. If the behaviour of each component is known, then the behaviour of any polarization can be known since it is a combination of the behaviour of the two main components.



A wave can be easily resolved into its two components along the selected axes. The components are

$$E_{0x} = E_0 \cos\theta$$
$$E_{0y} = E_0 \sin\theta$$

where E_0 is the amplitude of the wave, E_{0x} is the amplitude of the x-component, E_{0y} is the amplitude of the y-component and θ is the angle between the direction of the polarization and the x-axis. Note that these axes can be rotated depending on the conditions. However, the two axes must always be perpendicular to each other.

Polarized and Unpolarized Light

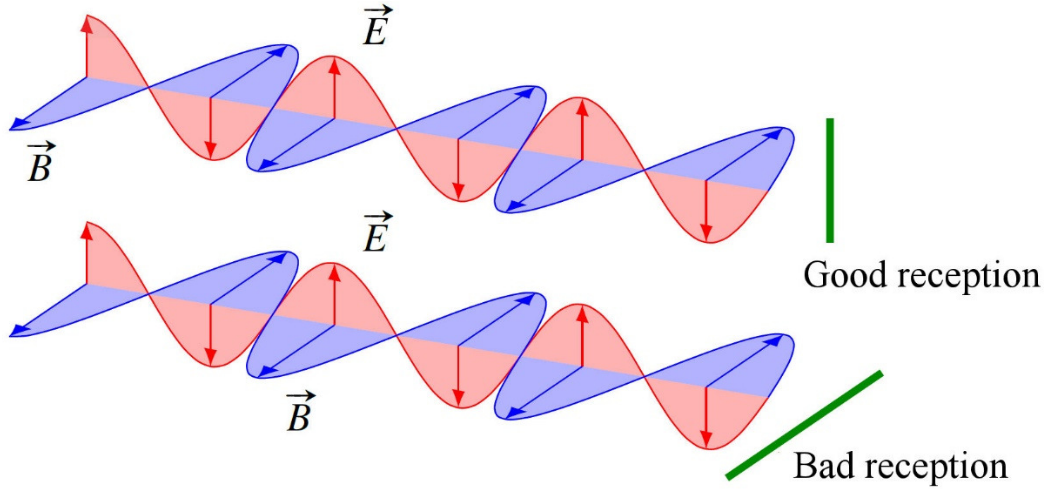
Light is polarized if the electric field oscillates uniquely along one direction. Generally, light is made up of several superimposed waves, and all these waves have the same direction of oscillation in polarized light.

In unpolarized light, the different superimposed waves have different directions of oscillation. It's actually a superposition of all possible directions of oscillation with an equal amount for each direction. Most of the time, light sources around us emit unpolarized light. For example, the light coming from the Sun and the light coming from light bulbs are not polarized.

In partially polarized light, all the directions of oscillations are present, but some polarizations are more intense than the others.

Radio Waves and Microwaves Polarization

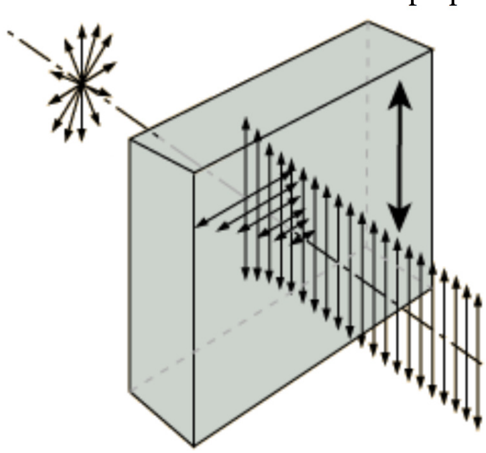
All electromagnetic waves can be polarized. The waves used in telecommunications are very often polarized. This means that the orientation of a rod-shaped antenna must be the same as the direction of the electric field oscillations to get a good reception.



With the right orientation, the electric field oscillates in the same direction as the antenna. The electric field can then move charged particles in the direction of the antenna and generate a current in the antenna.

Polarizing Filters

Light can be polarized with a filter that absorbs the light polarized in one direction and let the light polarized in another direction pass. This is a polarizing filter. For example, in the following image, unpolarized light arrives on such a filter. Unpolarized light is often represented by several arrows in directions perpendicular to the direction of propagation of the wave to show that it is a superposition of every possible direction of transverse oscillations. This polarizer lets the light polarized in the vertical direction pass. This direction is indicated by the big double arrow on the filter that shows the direction of polarization that can pass. This direction is the *polarization axis* of the polarizing filter. Then, the light polarized in a direction perpendicular to the polarization axis of the polarizing filter is absorbed. When the light comes out of this polarizing filter, only a single polarization remains, and the light is now polarized in the direction of the polarization axis of the filter



hyperphysics.phy-astr.gsu.edu/hbase/phyopt/polabs.html

Polarizing filters are made of a material composed of very long molecules aligned in the same direction. These molecules absorb light oscillating in one direction, but they cannot absorb light that oscillates in the other direction. This kind of filter was invented in 1928.

Unpolarized Light Arriving on a Filter

Light can always be resolved into two principal polarizations components whether or not it is polarized. For unpolarized light, the two components have exactly the same amplitude. An axis in the direction of the polarization axis of the filter and an axis perpendicular to the polarization axis of the filter are then used. When the light passes through the filter, the perpendicular component disappears and only the parallel component remains. Half of the light is then lost so that the light intensity is divided by two after the light has passed through the filter. Therefore,

Unpolarized Light Passing Through a Polarizing Filter

The light is now polarized in the direction of the polarization axis of the filter.

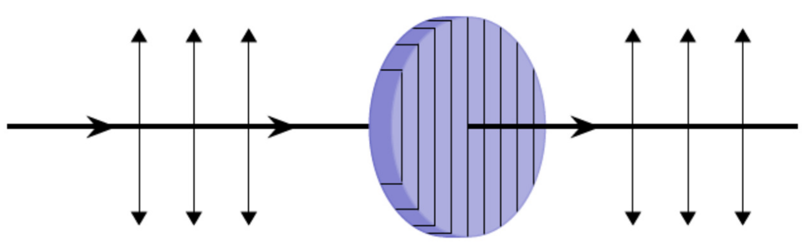
$$I = \frac{I_0}{2}$$

where I_0 is the intensity of the light before its passage through the filter.

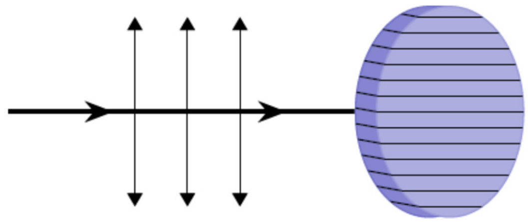
Polarized Light Arriving on a Filter

It's possible to think that nothing changes when polarized light passes through a polarizing filter because the light is already polarized. This is not necessarily true because the direction of the polarization axis of the filter can be different from the direction of polarization of the light.

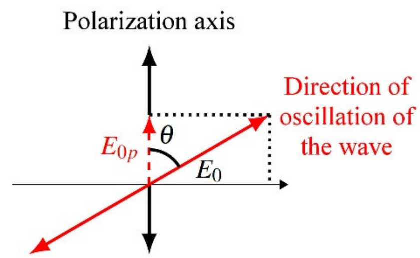
If the filter axis and the direction of the polarization are parallel, then it is true that all the light passes through the filter.



On the other hand, if the axis is perpendicular to the direction of the polarization, then no light passes through.



Actually, the axis of the polarizer can make any angle with the direction of polarization. To know the proportion of light that passes then, the light must be resolved into two components: a component parallel to the axis and a component perpendicular to the axis. Only the parallel component will pass through.



If the angle between the axis of polarization of the filter and the direction of the polarization is θ, then the parallel component is

$$E_{0p} = E_0 \cos\theta$$

Thus, the initial intensity of the light is

$$I_0 = \frac{1}{2} cn\varepsilon_0 E_0^2$$

and the light intensity of the component parallel to the axis is

$$I = \frac{1}{2} cn\varepsilon_0 \left(E_0 \cos\theta\right)^2$$

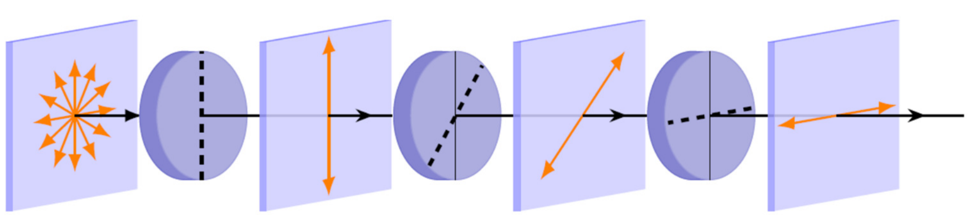
Dividing one by the other, the result is

$$\frac{I}{I_0} = \frac{\frac{1}{2} cn\varepsilon_0 \left(E_0 \cos\theta\right)^2}{\frac{1}{2} cn\varepsilon_0 \left(E_0\right)^2}$$

Therefore,

$$I = I_0 \cos^2\theta$$

Moreover, as the filter let only pass the component of the light polarized in the direction of the axis of polarization, the light that comes out of the polarizer is polarized in the direction of the axis of the polarizer. In the following diagram, the polarization of the light is always in the same direction as the axis of the last polarizer that the light has passed through.



In summary, this is what happens when light passes through a polarizing filter. (This is Malus's law.)

Polarized Light Passing Through a Polarizing Filter

The light is polarized in the direction of the polarization axis of the filter.

$$I = I_0 \cos^2 \theta$$

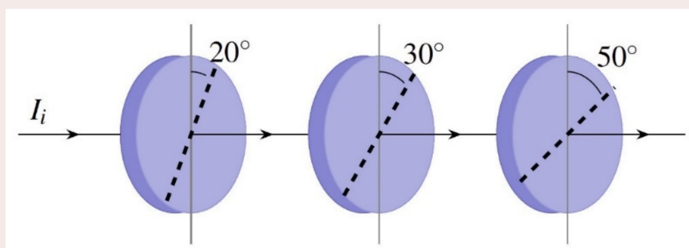
where I_0 is the intensity of the light before the passage through the filter.

Thus, if the angle between the axes is zero, all the light passes. If the angle is 90°, no light passes. That's what Grandpa John says
http://www.youtube.com/watch?v=QgA6L2n476Y
and the Department of Physics and Astronomy of the University of California.
http://www.youtube.com/watch?v=E9qpbt0v5Hw

In this video, a nice magic trick is made.
http://www.youtube.com/watch?v=9flduws7EsQ

Example 3.5.1

Unpolarized light with an initial intensity I_i passes through 3 polarizers whose axes are oriented as shown in the diagram. What percentage of light is left after the light has passed through the three polarizers?



First Polarizer

Unpolarized light arrives on a polarizer. The intensity of the light after its passage through the polarizer is thus

$$I = \frac{I_i}{2} = 0.5 I_i$$

The light is now polarized in the direction of the axis of the polarizer, so at 20° from the vertical.

Second polarizer

Polarized light arrives on a polarizer. The angle between the axis of the polarizer (30°) and the direction of polarization of the light (20°) is 30° - 20° = 10°. The intensity of the light after its passage through the polarizer is thus

$$I = I_0 \cos^2 \theta$$
$$= 0.5I_i \cdot \cos^2 10°$$
$$= 0.485I_i$$

The light is now polarized in the direction of the axis of the polarizer, so at 30° from the vertical.

Third polarizer

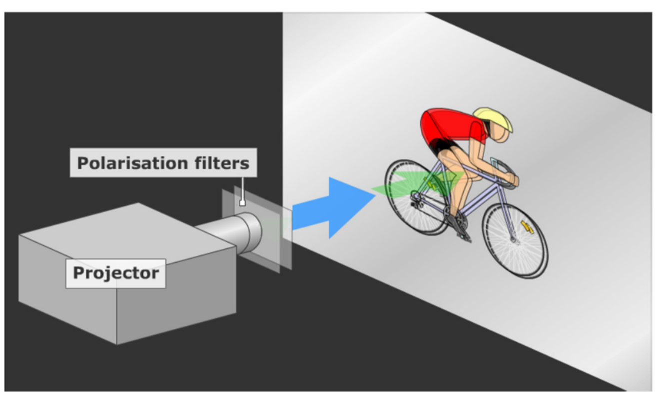
Polarized light arrives on a polarizer. The angle between the axis of the polarizer (50°) and the direction of polarization of the light (30°) is 50° - 30° = 20°. The intensity of the light after its passage through the polarizer is thus

$$I = I_0 \cos^2 \theta$$
$$I = 0.485I_i \cdot \cos^2 20°$$
$$I = 0.428I_i$$

Only 42.8% of the initial light intensity remains.

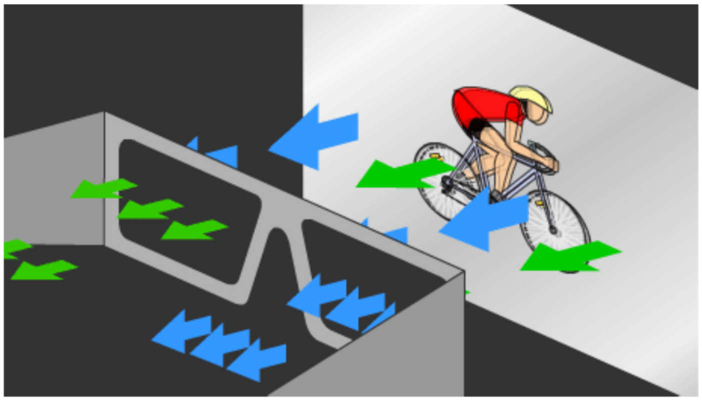
Three Dimensional Movies

To have a three-dimensional image, the image received by each eye must be slightly different. When an observer looks at an image projected onto a screen, both eyes see the same image and all the elements of the image seem to be at the same distance. For each eye to capture a different image, two images must be projected onto the screen. One way to achieve this is to use polarized images. One image is made of vertically polarized light and the other image is made of horizontally polarized light. Alternating polarizing filters (vertical and horizontal) in front of the projector polarized these two images.



news.bbc.co.uk/2/hi/entertainment/7976385.stm

To make sure that each eye sees a single image, glasses fitted with polarizing filters are used. For one eye, the axis of the polarizer is vertical, and only the vertically polarized image is seen by this eye. For the other eye, the axis of the polarizer is horizontal, and only the horizontally polarized image is seen by this eye. Each eye then receives a different image.



news.bbc.co.uk/2/hi/entertainment/7976385.stm

This way of making 3D movies explained here actually corresponds to the technology formerly used. The glasses then looked like those in the image to the right.

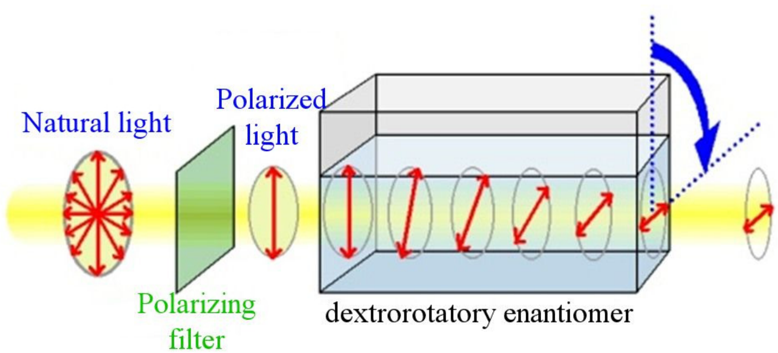
tpe3d-2013.e-monsite.com/pages/3d-polarisundefinede.html

Now, circularly polarized light is used. The glasses rather look like those in the image to the left. Circular polarization will not be explained here, but the idea is quite similar.

michaelaisms.wordpress.com/category/3-d-glasses/

Optical Activity

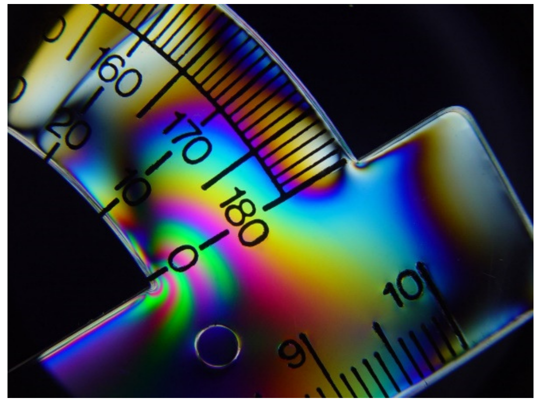
Certain kinds of molecules in solution can make the direction of polarization of polarized light rotate. This ability to rotate the polarization direction is called *optical activity* and the molecules that can rotate the direction are called *enantiomers*. In the following diagram, a substance in solution rotates the direction of polarization clockwise when looking at the beam of light heading towards us. This means that a dextrorotatory enantiomer was used. If the direction turns to the left, a levorotatory enantiomer was used.



158.64.21.3/chemistry/stuff1/EX1/notions/optique.htm

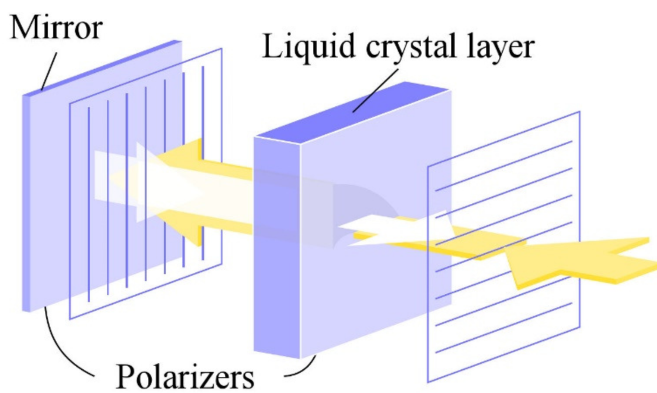
As the angle of rotation depends on the concentration of the substance, the rotation angle can be used to determine the enantiomer concentration.
Here is a demonstration with sugar molecules.
http://www.youtube.com/watch?v=GchTURvBz68

The optical activity of some transparent substances depends on the tension forces in the object and the wavelength of the light passing through the object. When polarized white light passes through these objects, the areas of tension in the object can easily be seen if the object is looked at through a polarizing filter.

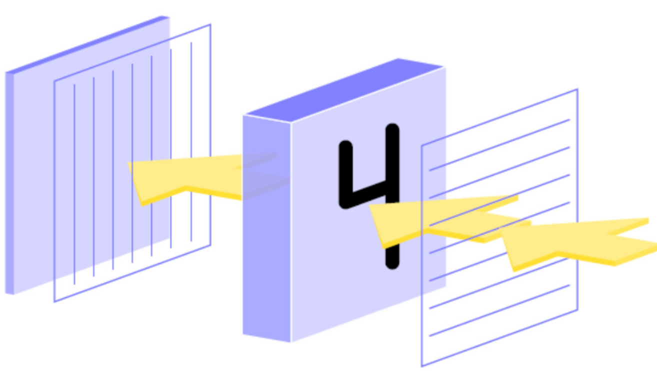


en.wikipedia.org/wiki/Photoelasticity

Optical activity is also used in liquid crystal displays. These displays consist of a layer of liquid crystal inserted between two crossed polarizers (which have perpendicular axes relative to each other). When there is no electric field, the liquid crystals are optically active. The thickness of the liquid crystal is exactly chosen so that the direction of polarization turns by 90° during its journey through the crystal. Therefore, light can pass when it arrives at the other polarizer. The light beam is then reflected on a mirror, passes through the polarizer again, in the crystal layer that changes the direction of polarization by 90° again, and through the other polarizer. Since light can get out, the display is then white.



When an electric field is applied, the liquid crystals lose their optical activity. Thus, the direction of polarization of the polarized light that passes through the liquid crystal layer does not rotate, and the light is blocked by the polarizer located on the other side of the layer. Therefore, no light gets to the mirror and there is no reflected light. The display is then black.



This also means that the image that comes out of a liquid crystal display is polarized. This can easily be seen by looking at those screens with polarized glasses. Then, the intensity of the light change depending on the orientation of the glasses.

This video shows the light coming out of LCD screens is actually polarized while the light coming out of old CRT screens was not.
http://www.youtube.com/watch?v=GwzUMEuGZHs

3.6 DOPPLER EFFECT

When a sound source moves, the frequency of the sound heard is different from the sound heard when the source is stationary. When the source is moving towards the observer, the sound becomes higher-pitched and when the source is moving away from the observer, the sound becomes more low-pitched. Here are a few demonstrations of this phenomenon.
http://www.youtube.com/watch?v=imoxDcn2Sgo
https://www.youtube.com/watch?v=iOB6-hs-tME

There is also a frequency shift if the observer is moving and the source is stationary. The sound has a higher pitch when the observer is moving towards the source and a lower pitch when the observer is moving away from the source.
http://www.youtube.com/watch?v=6-AIiJxFXFw

This change of frequency due to the motion of the observer or the source is called the Doppler Effect since the theory explaining this effect was developed by the German physicist Christian Doppler in 1842.

Formula for the Frequency Shift in One Dimension

Let's begin by getting the formula for the frequency shift if the source and the observer are limited to motions in one dimension.

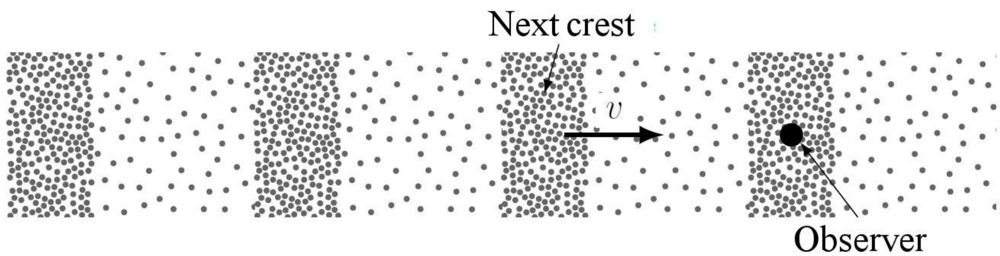
(The directions of each speed can be inverted.)

The following notation will be used here.

> f' and T' are the frequency and the period of the wave received when the source or the observer are moving.

f and T are the frequency and the period of the wave received when the source or the observer are not moving.

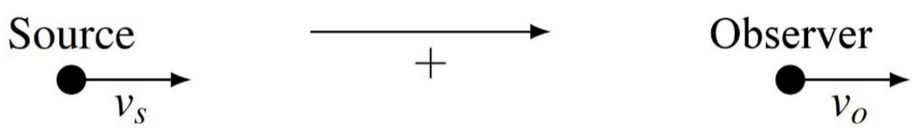
To calculate the period of the wave received by the observer when the source and the observer are moving (T'), the time it takes for the next crest of the wave to arrive at the observer when he is receiving a crest will be used.



As there are three speeds involved here, the following notation will be used:

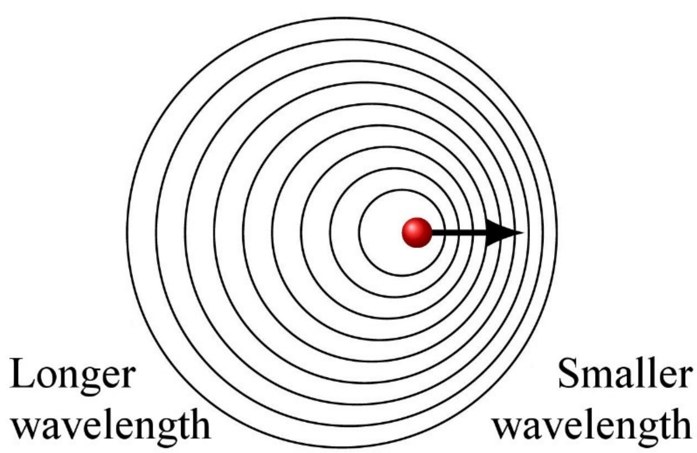
v = speed of the wave
v_s = speed of the source
v_o = speed of the observer

The sign of the speeds of the source and of the observer is determined by the following sign convention: the positive direction goes from the source to the observer.



Change of Wavelength Due to the Motion of the Source

The following animation shows the wave emitted by the source in motion.
http://www.youtube.com/watch?v=Gz8JxhosvW8
The crests of the wave spread from the source. It is clear that the motion of the source changes the wavelength of the wave. The wavelength is smaller on the side where the observer sees the source approaching whereas the wavelength is larger on the side where the observer sees the source moving away.



Suppose the source emits a crest at $t = 0$. A time T later, the source emits a crest again since it emits a crest every period. The wavelength λ' (the wavelength when the source is moving) is the distance between these two crests. It is, therefore, the distance between the crest issued at $t = 0$ and the source when it is emitting the following crest at time T.

Between $t = 0$ and $t = T$, the first crest, which moves at speed v, has travelled a distance equal to

$$D_{crest} = vT$$

Between $t = 0$ and $t = T$, the source, which is moving at speed v_s, has travelled a distance equal to

$$D_{source} = v_s T$$

The distance between the crest and the source, which is the new wavelength, is

$$\lambda' = vT - v_s T$$

$$\lambda' = vT\left(1 - \frac{v_s}{v}\right)$$

If the source were not moving, vT would have been the wavelength λ. Therefore

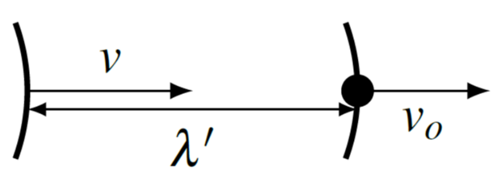
Doppler Effect: Wavelength Shift

$$\lambda' = \lambda\left(1 - \frac{v_s}{v}\right)$$

Period Shift Due to the Motion of the Observer

In this video, the period shift can be observed in a situation when the observer moves. The period decreases when he moves towards the source, and it increases when he moves away from the source.
http://www.youtube.com/watch?v=4mUjM1qMaa8

To find the period shift, consider the following situation: an observer receives a crest of the wave at $t = 0$. The new period is the time it takes for the next crest to arrive. This next crest is at a distance λ' if the motion of the source is taken into account.



The time it takes for the crest to catch up with the observer is

$$T' = \frac{\lambda'}{v - v_o}$$

Using the formula for λ' (not the one in the box, but the one just before that), the period becomes

$$T' = \frac{vT\left(1 - \frac{v_s}{v}\right)}{v - v_o}$$

$$T' = T\frac{v - v_s}{v - v_o}$$

The frequency shift is then

$$f' = \frac{1}{T'}$$

$$= \frac{1}{T\left(\frac{v - v_s}{v - v_o}\right)}$$

Simplifying, the end result is

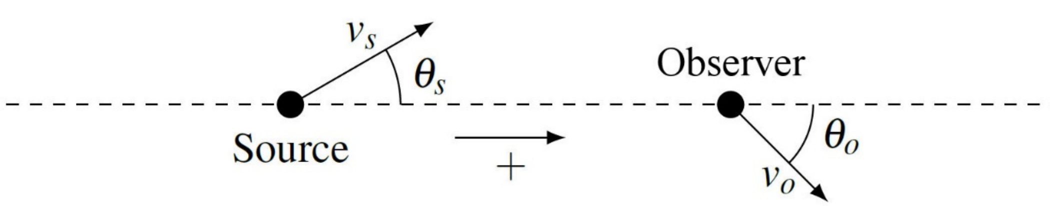
Doppler Effect: Frequency Shift

$$f' = f\frac{v - v_o}{v - v_s}$$

Sign convention for the speeds in this formula: **the positive direction goes from the source to the observer.**

In 2 or 3 Dimensions

In two or three dimensions, velocities can have any direction, as shown in this diagram.



Only the components of the velocity in the direction of the dotted line can change the frequency. As the components of these speeds are $v \cos \theta$, the frequency is

Doppler Effect in 2 or 3 dimensions: Frequency Shift

$$f' = f\frac{v - v_o\cos\theta_o}{v - v_s\cos\theta_s}$$

The angle is always the angle between the velocity and the axis, which goes from the source to the observer.

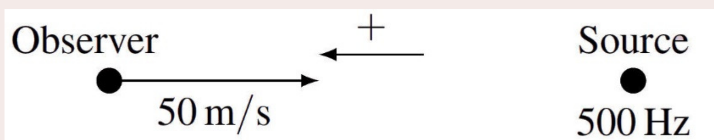
Doppler Effect With Sound

As seen in the clips at the beginning of this section, the Doppler effect can change the frequency of sound.

Example 3.6.1

An observer is moving towards a stationary sound source at 180 km/h. The frequency of the sound emitted by the source is 500 Hz. The speed of sound is 343 m/s.

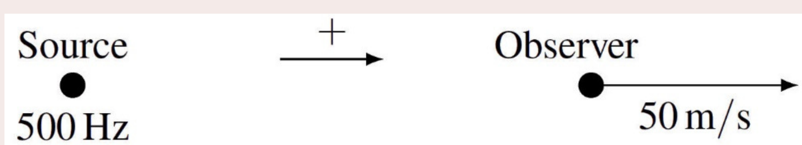
a) What is the frequency of the sound heard by the observer?



The frequency of the sound heard by the observer is

$$\begin{aligned} f' &= f\frac{v - v_o}{v - v_s} \\ &= 500Hz\cdot\frac{343\frac{m}{s} - \left(-50\frac{m}{s}\right)}{343\frac{m}{s}} \\ &= 572.3Hz \end{aligned}$$

b) What is the frequency of the sound heard by the observer once he has passed next to the source? (He is then moving away from the source.)



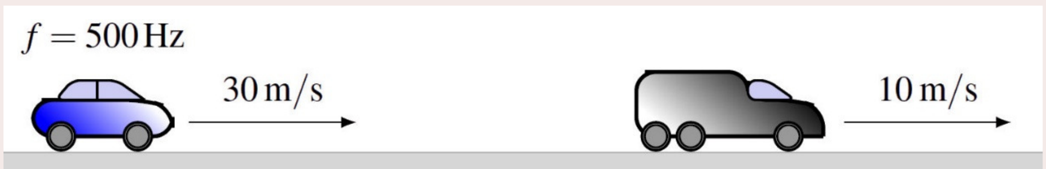
The frequency of the sound heard by the observer is

$$f' = f\frac{v - v_o}{v - v_s}$$

$$= 500Hz \cdot \frac{343\frac{m}{s} - \left(50\frac{m}{s}\right)}{343\frac{m}{s}}$$
$$= 427.1Hz$$

Example 3.6.2

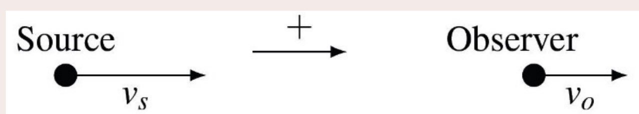
The sound emitted by the horn of the car is reflected on the back of a truck and travels back towards the person in the car. What is the frequency of the sound heard by the person in the car? The speed of sound is 340 m/s.



For this type of problem, we must know what to do when a sound is reflected. The trick is to do this problem in two parts. In the first part, the automobile is the source, and the truck is the observer. The truck then receives a sound with the frequency f' which is

$$f' = f\frac{v - v_o}{v - v_s}$$
$$= 500Hz \cdot \frac{340\frac{m}{s} - 10\frac{m}{s}}{340\frac{m}{s} - 30\frac{m}{s}}$$
$$= 532.26Hz$$

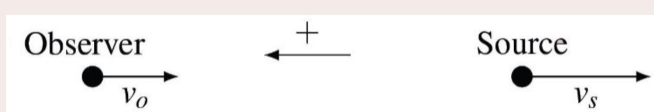
The speeds are positive in this case.



Thereafter, the sound is reflected with the same frequency as the one received by the truck. Thus, the truck becomes the source with a frequency f' and the person in the car becomes the observer. The frequency of the sound heard by the person is then

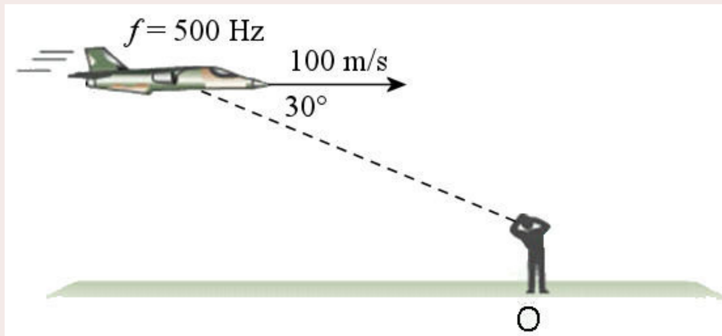
$$f'' = f'\frac{v - v_o}{v - v_s}$$
$$= 532.26Hz \cdot \frac{340\frac{m}{s} - \left(-30\frac{m}{s}\right)}{340\frac{m}{s} - \left(-10\frac{m}{s}\right)}$$
$$= 562.67Hz$$

The speeds are negative in this case.



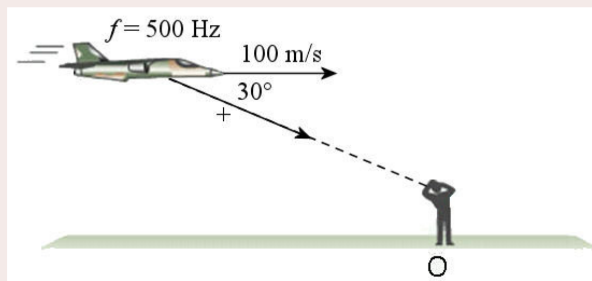
Example 3.6.3

What is the frequency heard by the observer in the following situation? The speed of sound is 343 m/s.



people.highline.edu/iglozman/classes/physnotes/doppler.htm

The angle for the velocity of the source is the angle between the velocity and the positive axis, which goes from the source to the observer. This angle is 30°.



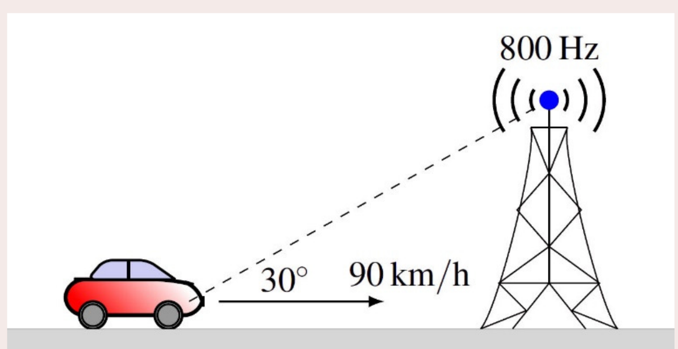
Thus, the frequency heard is

$$\begin{aligned} f' &= f\frac{v - v_o\cos\theta_o}{v - v_s\cos\theta_s} \\ &= 500Hz\cdot\frac{343\frac{m}{s} - 0}{343\frac{m}{s} - 100\frac{m}{s}\cdot\cos 30°} \\ &= 668.9Hz \end{aligned}$$

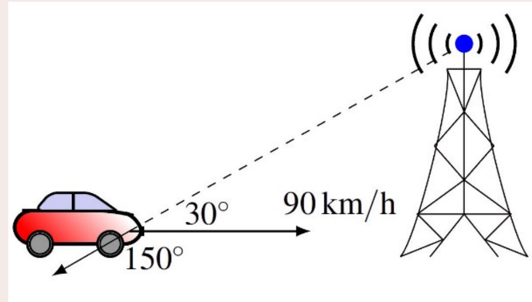
(Note that the plane will not be at this place when the observer hears the sound. During the time it takes for the sound to reach the observer, the plane will have moved.)

Example 3.6.4

What is the frequency heard by the observer in the following situation? The speed of sound is 343 m/s.



The angle for the velocity of the observer is the angle between the velocity and the positive axis, which goes from the source to the observer. This angle is 150°.



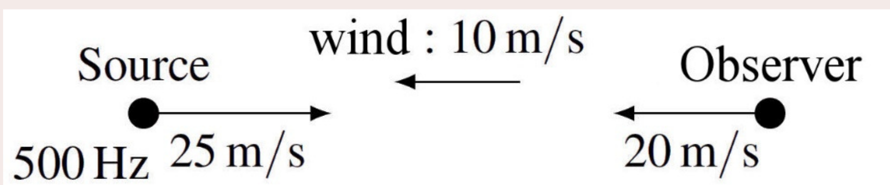
Thus, the frequency heard is

$$\begin{aligned} f' &= f\,\frac{v - v_o\cos\theta_o}{v - v_s\cos\theta_s} \\ &= 800Hz\cdot\frac{343\frac{m}{s} - 25\frac{m}{s}\cdot\cos(150°)}{343\frac{m}{s} - 0\frac{m}{s}} \\ &= 850.5Hz \end{aligned}$$

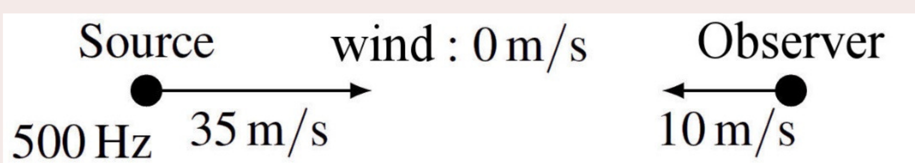
With sound, the speed of the source and the observer are always the speeds relative to air.

Example 3.6.5

What is the frequency of the sound heard by the observer in the following situation? The speed of sound is 343 m/s.



As the speeds in the Doppler shift formula are the speeds relative to air, the effect of the wind must be taken into account. To do so, the speeds according to a person in the wind frame (i.e. the speed according to a person moving with the wind) must be found. To obtain these speeds, just add a 10 m/s speed towards the right to every speed shown in the diagram. By adding a 10 m/s speed towards the right to the speed of the wind, the air is now at rest. The following situation is thus obtained.



The frequency of the sound heard by the observer is then

$$f' = f\,\frac{v - v_o}{v - v_s}$$

$$= 500Hz \cdot \frac{343\frac{m}{s} - \left(-10\frac{m}{s}\right)}{343\frac{m}{s} - 35\frac{m}{s}}$$
$$= 573Hz$$

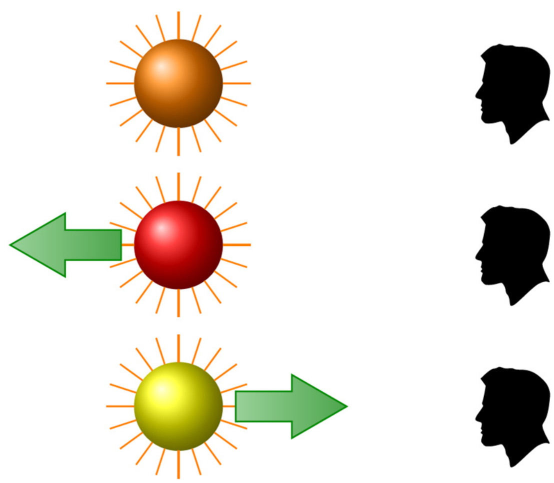
Doppler Effect With Light

The results obtained for the Doppler Effect are not only valid for sound, but they are also valid for every type of waves, including light. (In France, the effect is called the Doppler-Fizeau effect when it is applied to light.) Thus, if the source or the observer is moving, the frequency of the light perceived by the observer will be different. As colour depends on frequency, this means that the perceived colour will be different if there is a Doppler effect.

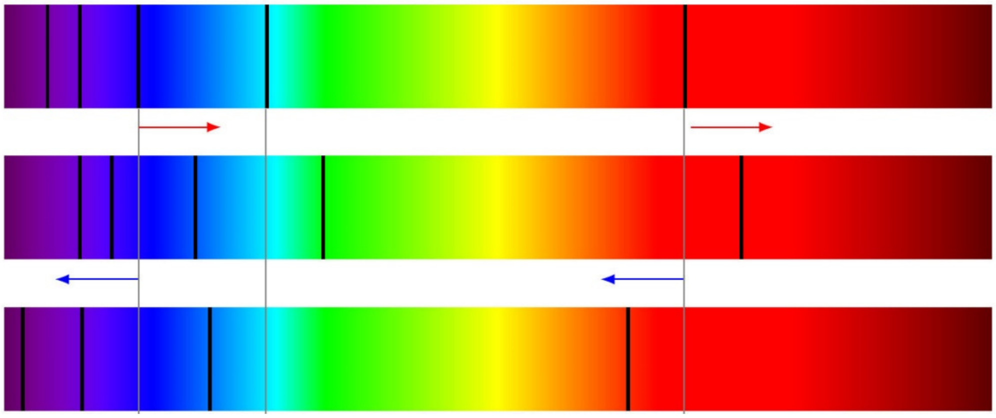
Caution: The Doppler effect formula seen in this chapter are valid only if the source and the observer have velocities much smaller than the speed of light. 10% of the speed of light can be taken as the maximum speed allowed to apply these formulas. We will see why this limit exists and how to modify the formulas so that they are always valid in the chapter on relativity.

Application in Astronomy

Suppose that there is a light source emitting light with a certain frequency or wavelength corresponding to orange light. (In the diagram, it is a star, even if it is impossible to have a star emitting a single wavelength.) If the source moves away from the observer, the frequency of the light decreases, which means that the wavelength of the light increases. The colour of the light, therefore, shifts towards the red part of the spectrum. It could then appear to be red for the observer. If the source is moving towards the observer, the frequency of the light increases, which means that the wavelength of the light decreases. The colour of the light, therefore, shifts towards the blue part of the spectrum. It could then appear yellow for the observer.



Actually, the change in colour of the star cannot be observed for a star because they do not emit a single wavelength. Instead, they emit a continuous spectrum and this whole spectrum will shift to one side or the other because of the Doppler Effect. This spectrum shift does not change the colour of the star because shifted colours are replaced by other colours that are also shifted. However, the shift of the spectrum can be detected because there are absorption lines in the spectrum of a star. These lines are caused by the absorption of specific wavelengths by the gas surrounding the star. When the star is moving away from us or is moving towards us, the Doppler Effect also shifts the position of these absorption lines in the spectrum. The following changes can then be observed.



When the source is moving away from us, the spectral lines shift towards the red part of the spectrum. This is called a *redshift.*

When the source is moving towards us, the spectral lines shift towards the blue part of the spectrum. This is called a *blueshift*.

Example 3.6.6

The spectral line of hydrogen, normally having a 656.279 nm wavelength, has a 656.263 nm wavelength in the spectrum of the star Sirius. With what speed Sirius is approaching us or receding from us?

In this case, the observer (us) is stationary. Therefore,

$$\lambda' = \lambda\left(1-\frac{v_s}{c}\right)$$

$$656.263nm = 656.279nm \cdot \left(1-\frac{v_s}{3\times10^8 \frac{m}{s}}\right)$$

$$v_s = 7314 \frac{m}{s}$$

As the positive direction is going from the source to the observer, a positive answer means that this star is approaching us at 7314 m/s.

SUMMARY OF EQUATIONS

Displacement of the Molecules of the Medium When a Sinusoidal Sound Wave is Passing

$$s = A\sin\left(kx \pm \omega t + \phi\right)$$

Speed of Sound in Air

$$v = 331.3\tfrac{m}{s}\cdot\sqrt{\frac{T}{273.15K}}$$

where T is the air temperature in kelvin

Electric Field if an Electromagnetic Sine Wave

$$E = E_0\sin\left(kx \pm \omega t + \phi\right)$$

where E_0 is the amplitude of the electric field, measured in N/C (newton per coulomb).

Speed of Light in Vacuum

$$c = 299\ 792\ 458 \text{ m/s}$$

Speed of Light in a Transparent Substance

$$v = \frac{c}{n}$$

Received Power

$$P_{received} = IA_{receiver}$$

Intensity of a Wave Emitted by an Isotropic Source

$$I = \frac{P}{4\pi r^2}$$

Impedance of the Medium for Sound Waves

$$Z = \rho v$$

Wave Intensity

$$I = \frac{1}{2}\rho v\omega^2 A^2 = \frac{1}{2}Z\omega^2 A^2$$

Sound Intensity in Decibel (β)

$$\beta = 10dB\cdot\log\frac{I}{10^{-12}\frac{W}{m^2}}$$

Light Intensity

$$I = \frac{cn\varepsilon_0 E_0^2}{2}$$

where c is the speed of light and ε_0 is a constant (8.854 x 10^{-12} C²/Nm²).

Unpolarized Light Passing Through a Polarizing Filter

The light is now polarized in the direction of the polarization axis of the filter.

$$I = \frac{I_0}{2}$$

where I_0 is the intensity of the light before the passage through the filter.

Polarized Light Passing Through a Polarizing Filter

The light is polarized in the direction of the polarization axis of the filter.

$$I = I_0 \cos^2 \theta$$

where I_0 is the intensity of the light before the passage through the filter.

Doppler Effect: Wavelength Shift (if *v* << *c* for light)

$$\lambda' = \lambda \left(1 - \frac{v_s}{v}\right)$$

Doppler Effect: Frequency Shift (if *v* << *c* for light)

$$f' = f \frac{v - v_o}{v - v_s}$$

Sign convention for the speeds in this formula: **the positive direction goes from the source to the observer.**

Doppler Effect in 2 or 3 Dimensions: Frequency Shift (if *v* << *c* for light)

$$f' = f \frac{v - v_o \cos\theta_o}{v - v_s \cos\theta_s}$$

Sign convention for the speeds in this formula: **the positive direction goes from the source to the observer.**

EXERCISES

3.2 Sound Waves

1. Dogs can hear sounds having frequencies up to 50 000 Hz. What is the wavelength of the sound at this frequency if the speed of sound is 340 m/s?

2. What is the speed of sound waves when the air temperature is 25 °C?

3. A sound wave is described by the equation

$$s = 0.01mm \cdot \sin\left(1.6\tfrac{rad}{m} \cdot x - 560\tfrac{rad}{s} \cdot t + 0.5\right)$$

a) What is the air temperature?
b) What is the maximum speed of air molecules during the passage of this wave?

3.3 Light

4. How much time does it take for light to travel from the Sun to the Earth knowing that the distance between the Sun and the Earth is 149 600 000 km?

5. A light-year is a unit of distance, which is equivalent to the distance travelled by light in vacuum during a year. What is this distance in metres?

6. What is the speed of light in diamond (n = 2.4) ?

7. An electromagnetic wave has a 10^{15} Hz frequency. In which region of the electromagnetic spectrum can this wave be found? (Radio, microwave, infrared, visible, ultraviolet, X-rays or gamma rays?)

3.4 Intensity

8. A sound source has a power of 50 W. What is the intensity (in dB) of the sound 30 m from the source?

9. A light wave with a 0.1 W/m² intensity arrives on a receiver with an area of 10 cm². What is the energy received by the receiver in 2 minutes?

10. At 10 m from a light source, the intensity is 0.001 W/m². How far away from the source should one be to measure an intensity of 0.00001 W/m²?

11. A firecracker explosion produces a sound with an intensity of 40 dB at 50 m from the firecracker. What is the intensity (in dB) of the sound produced by the explosion of 1000 firecrackers at 200 m from the explosion?

12. What is the resulting intensity (in dB) if a 90 dB sound is superimposed to a 95 dB sound?

13. A loudspeaker with a power of 50 W produces a 200 Hz sound. What is the amplitude of the sound wave 25 m from the speaker if the sound velocity is 340 m/s and if the air density is 1.3 kg/m³?

14. A loudspeaker produces a 400 Hz sound. Hz with an amplitude of 0.1 µm. What is the intensity of this sound (in dB) if the sound velocity is 340 m/s and if the air density is 1.3 kg/m³?

15. An explosion produces a sound with a power of 20 000 W. Consider that this is an isotropic source.

a) What is the intensity (in dB) 5 km from the source if the absorption of sound by the air is not taken into account?
b) What is the intensity (in dB) 5 km from the source if the air absorbs the sound at a rate of 7 dB/km?

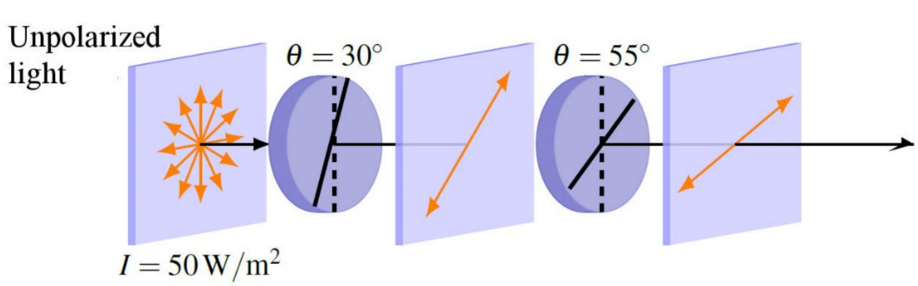
16. A light source has a power of 100 W.

a) What is the intensity of the light 30 m from the source?
b) What is the amplitude of the electric field of this wave 30 m from this source? (In air)

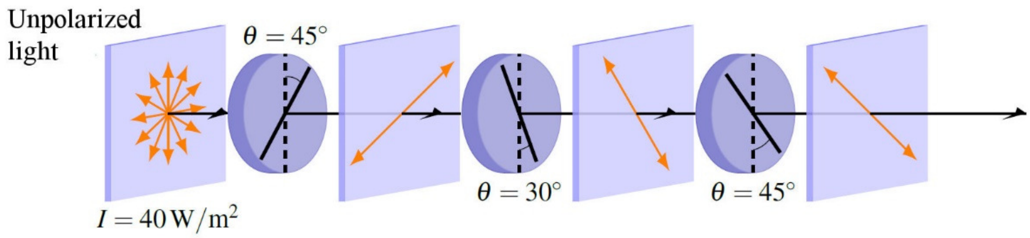
17. 20 m from an isotropic sound source, the amplitude of oscillation of the air molecules is 20 µm. What is the amplitude 100 m from the source?

3.5 Polarization

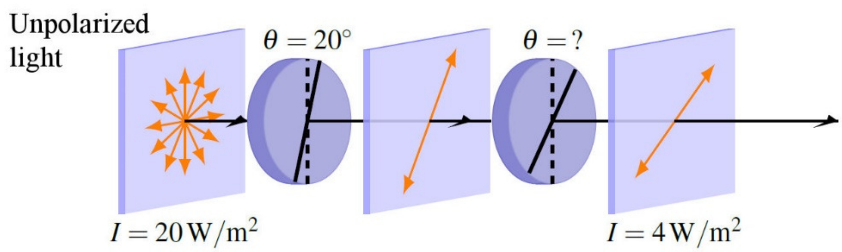
18. What is the light intensity after its passage through these two polarizers?



19. What is the light intensity after its passage through these three polarizers if it was not polarized initially?



20. What should the angle of the second polarizer be to obtain the intensity indicated in the diagram if the light is not polarized initially?

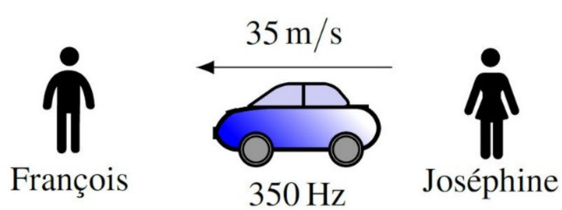


21. Polarized light passes through a polarizer. After passing through the polarizer, the intensity of the light is 5 W/m². The intensity decreases to 3 W/m² if the polarizer is rotated by 20°. What is the light intensity before its passage through the polarizer?

Hint: $\cos(a+b) = \cos a \cos b - \sin a \sin b$

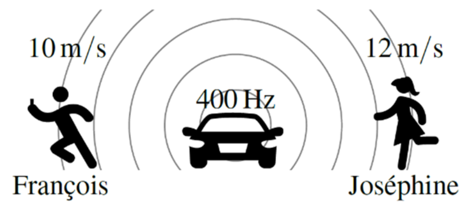
3.6 Doppler Effect

22. In this situation, the speed of sound is 340 m/s. The driver sounds the car horn, which has a frequency of 350 Hz, to warn François who is in the middle of the road.



a) What is the frequency of the sound heard by François?
b) What is the wavelength of the sound heard by François?
c) What is the frequency of the sound heard by Joséphine?
d) What is the wavelength of the sound heard by Joséphine?

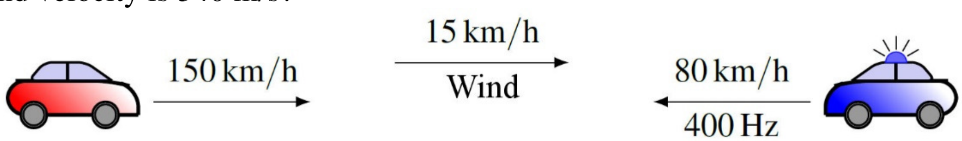
23. In this situation, the speed of sound is 340 m/s. François attempted to steal Josephine's car, and this triggered the alarm system. François flees while Joséphine is running to see what is happening.



a) What is the frequency of the sound heard by François?
b) What is the wavelength of the sound heard by François?
c) What is the frequency of the sound heard by Joséphine?
d) What is the wavelength of the sound heard by Joséphine?

24. What is the frequency of the sound heard by the person in the car to the right if the sound velocity is 340 m/s?

25. What is the frequency of the sound heard by the person in the car to the left if the sound velocity is 340 m/s?



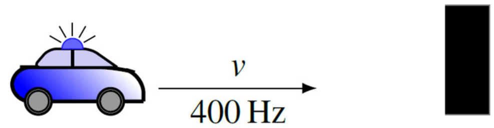
26. When a train is heading towards a person while the whistle of the train is sounding, the person hears a 150 Hz sound. When the train is moving away from the person with the same speed while the whistle of the train is still sounding, the person hears a 125 Hz sound. The speed of sound is 335 m/s.

a) What is the speed of the train?
b) What would be the frequency of the sound heard by the person if the train was not moving?

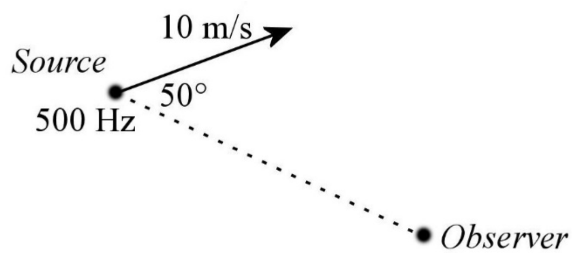
27. In the following situation, the person in the red car (the car to the right) hears the police siren with a frequency of 420 Hz. Knowing that the speed of the police car is 15 m/s higher than the speed of the red car, determine the speed of the two cars. The speed of sound is 345 m/s.

28. In the following situation, the person in the car hears the sound arriving directly from the police car and the sound of the police car reflected on the wall. What are the frequencies of the two sounds heard by the person in the car if the temperature is 25 °C?

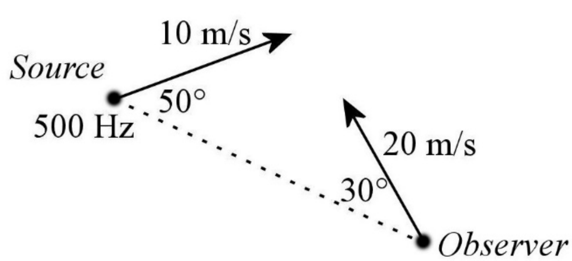
29. In the following situation, the policeman hears that the sound reflected on the wall has a 415 Hz frequency. What is the speed of the car if the temperature is 20 °C?



30. In the situation shown in the diagram, what is the frequency heard by the observer if the speed of sound is 340 m/s?



31. In the situation shown in the diagram, what is the frequency heard by the observer if the speed of sound is 340 m/s?



For the following question, use this table.

Colour	Wavelength (nm)	Colour	Wavelength
Red	700 to 625	**Green-Blue**	530 to 492
Orange	625 to 590	**Cyan**	492 to 487
Yellow	590 to 580	**Blue-Green**	487 to 482
Yellow-Green	580 to 575	**Blue**	482 to 465
Green-Yellow	575 to 560	**Indigo**	465 to 435
Green	560 to 530	**Violet**	435 to 400

32. A light source emits a 585 nm wavelength yellow light when it is at rest.

a) What will be the colour perceived by an observer at rest if this light source is moving towards the observer at 10% of the speed of light?
b) What will be the colour perceived by an observer at rest if this light source is moving away from the observer at 10% of the speed of light?

33. A stationary light source emits a 600 nm wavelength red light. How fast should an observer move towards this source so that it appears blue (λ = 470 nm)? (If we assume that the formula is still good at speeds close to the speed of light.)

Challenges

(Questions more difficult than the exam questions.)

34. An explosion produces a sound with a power of 20 000 W. Consider that this is an isotropic source.

a) How far should an observer be from the explosion to hear a 10 dB sound if the absorption of sound by the air is not taken into account?
b) How far should an observer be from the explosion to hear a 10 dB sound if the air absorbs the sound at a rate of 7 dB/km?

35. The percentage of polarization of a partially polarized beam of light is defined with the following formula.

$$p = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

where I_{max} and I_{min} are the maximum and minimum intensities obtained when light passes through a polarizer that is rotated slowly.

Show that if a beam of partially polarized light passes through a polarizer which makes an angle θ with the direction where the intensity is I_{max}, the intensity of the light passing through the polarizer is given by

$$I = \frac{1 + p\cos(2\theta)}{1 + p}$$

Hints: A partially polarized beam can be considered as a superposition of two perpendicularly polarized waves with different intensities and $\cos(a+b) = \cos a \cos b - \sin a \sin b$

ANSWERS

3.2 Sound Waves

1. 6.8 mm
2. 346.1 m/s
3. a) 31.7 °C b) 5.6 mm/s

3.3 Light

4. 8 min 19 s
5. 9.46×10^{15} m
6. 21.25×10^{8} m/s
7. The wavelength is 300 nm. Those are ultraviolet rays.

3.4 Intensity

8. 96.5 dB
9. 0.012 J
10. 100 m
11. 57.96 dB
12. 96.2 dB
13. 4.271 µm
14. 71.44 dB
15. a) 78.04 dB b) 43.04 dB
16. a) 8.842×10^{-3} W/m² b) 2.58 N/C
17. 4 µm

3.5 Polarization

18. 20.53 W/m²
19. 1.25 W/m²
20. 70.8° or 149.2°
21. 6.165 W/m²

3.6 Doppler Effect

22. a) 390.2 Hz b) 87.14 cm c) 317.3 Hz d) 107.1 cm
23. a) 388.2 Hz b) 85 cm c) 414.1 Hz d) 85 cm
24. 480.4 Hz
25. 481.4 Hz
26. a) 30.45 m/s b) 136.4 Hz
27. car: 30 m/s police: 45 m/s
28. The person in the car receives a sound at 424.91 Hz and sound at 437.37 Hz.
29. 6.317 m/s

30. 509.6 Hz
31. 535.6 Hz
32. a) Green-blue (λ = 526.5 nm) b) Red (λ = 643.5 nm)
33. 8.3 x 10^7 m/s

Challenges

34. a) 12 616 km b) 8.992 km