12 NUCLEAR PHYSICS

One of the possible uranium fission reactions is

$$n^0 + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3n^0$$

What is the energy released by this reaction?

hdfons.com/atomic-bomb/atomic-bomb-wallpaper-hd/

Discover how to solve this problem in this chapter.

12.1 ATOMIC NUCLEI

Nucleus Composition

Between 1913 and 1932, physicists discovered the composition of the atomic nucleus. They came to the conclusion that it is composed of two types of particles: protons and neutrons.

Proton (p^+)	Mass: $m_p = 1.6726 \times 10^{-27}$ kg	Charge = $+1.602 \times 10^{-19}$ C
Neutron (n^0)	Mass: $m_n = 1.6750 \times 10^{-27}$ kg	Charge = 0 C

The number of protons in a nucleus is denoted Z (which comes from *zahl*, meaning *number* in German). This number is simply equal to the atomic number of the element.

The number of neutrons in a nucleus is denoted N. This number may vary for the same element. The different possibilities obtained by varying the number of neutrons for an element are the *isotopes* of the element.

Protons and neutrons are the only two members of the *nucleon* family. The number of nucleons is denoted A. Obviously, it is equal to the number of protons and neutrons.

Number of Nucleons in a Nucleus

$$A = Z + N$$

The following notation is used to represent atomic nuclei.

$$^{A}_{Z}Sy$$

Sy is the chemical symbol of the element. The value of Z can be omitted because the atomic number is a redundant information with the chemical symbol since each element has its own number. However, it is convenient to indicate it because it is easy to forget the atomic number of an element.

Here are some examples of atomic nuclei.

These are the three main hydrogen isotopes.

$^{1}_{1}H$ (also known as protium)	Composed of a single proton.
$^{2}_{1}H$ (also known as deuterium)	Composed of 1 proton and 1 neutron.
$^{3}_{1}H$ (also known as tritium)	Composed of 1 proton and 2 neutrons.

(These 3 isotopes are the only ones with special names.) These are the 16 isotopes of carbon.

$^{8}_{6}C$ Composed of 6 protons and 2 neutrons.

${}^{9}_{6}C$	Composed of 6 protons and 3 neutrons.
${}^{10}_{6}C$	Composed of 6 protons and 4 neutrons.
${}^{11}_{6}C$	Composed of 6 protons and 5 neutrons.
${}^{12}_{6}C$	**Composed of 6 protons and 6 neutrons.**
${}^{13}_{6}C$	Composed of 6 protons and 7 neutrons.
${}^{14}_{6}C$	Composed of 6 protons and 8 neutrons.
${}^{15}_{6}C$	Composed of 6 protons and 9 neutrons.
${}^{16}_{6}C$	Composed of 6 protons and 10 neutrons.
${}^{17}_{6}C$	Composed of 6 protons and 11 neutrons.
${}^{18}_{6}C$	Composed of 6 protons and 12 neutrons.
${}^{19}_{6}C$	Composed of 6 protons and 13 neutrons.
${}^{20}_{6}C$	Composed of 6 protons and 14 neutrons.
${}^{21}_{6}C$	Composed of 6 protons and 15 neutrons.
${}^{22}_{6}C$	Composed of 6 protons and 16 neutrons.

Carbon-12 (in bold), composed of 6 protons and 6 neutrons, is by far the most common carbon isotope in nature (98.9% of the mass of natural carbon). It is followed by carbon-13 (1.1%) and carbon-14 (0.000 000 000 13%). The other carbon isotopes do not exist in nature; they were created in laboratories. Some elements, such as technetium (element no. 43), promethium (element no. 61) and all the elements with an atomic number greater than 93, do not have any naturally occurring isotopes.

With this notation, the neutron can be denoted as ${}^{1}_{0}n$.

Atomic Mass Unit

To measure the mass of atomic nuclei, the atomic mass unit is used. Its value is

Atomic Mass Unit

$$1\ u = 1.660\ 539 \times 10^{-27}\ kg$$

More precisely, the atomic mass corresponds to a mass of 1 g divided by Avogadro's number (which, since 2019, is worth exactly 6.022 140 76 x 10^{-23}, all the decimals after the last 6 are 0).

In terms of u, the following masses are obtained:

Electron	Mass = 0.000 549 u
Proton	Mass = 1.007 276 u
Neutron	Mass = 1.008 665 u

Hydrogen-1 atom Mass = 1.007 825 *u*

Hydrogen-1 is slightly heavier than one proton since it is composed of one proton and one electron (in the orbitals). The mass of all known isotopes was measured. These masses can be found on the following document.
http://physique.merici.ca/ondes/atomicmasses.pdf
Note that the mass of an isotope includes the mass of the nucleus and the mass of electrons in the orbitals. Note also that the mass in *u* is equal to the molar mass of the isotope.

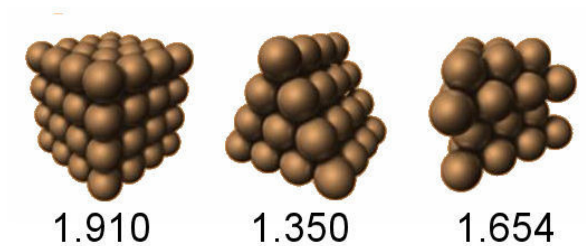
Nucleus Radii

A nucleon has a radius of 0.87 fm. The symbol fm means femtometer, or fermi, and represents 10^{-15} m. (There is a small problem in physics at the moment since one way to measure the radius of the proton gives values of about 0.84 fm and the other gives values around 0.88 fm. Even taking into account the uncertainties, the two values disagree!)

Now let's find the volume of a nucleus (V_{noyau}). If there are A nucleons in the nucleus, then the volume of the nucleus is

$$V_{nucleus} = V_{nucleon} \cdot A$$

However, there is a correction to be made. In stacking spheres, the volume obtained is a little larger than the simple sum of the volume of the spheres since there is also a small volume between the spheres. The diagram shows the links between the total volume and the volume of the spheres. This link varies depending on how the spheres are packed. The value displayed is the ratio between the total volume and the sum of the volume of the spheres. For example, for the first stack, the volume is 1.910 times larger than the sum of the volumes of the spheres. A value of 1.35 will be used here to obtain the smallest size of the nucleus.



villemin.gerard.free.fr/Wwwgvmm/Geometri/SpheEmpi.htm

Therefore,

$$V_{nucleus} = 1.35 \cdot V_{nucleon} \cdot A$$
$$\frac{4}{3}\pi\left(r_{nucleus}\right)^3 = 1.35 \cdot \frac{4}{3}\pi\left(r_{nucleon}\right)^3 \cdot A$$
$$\left(r_{nucleus}\right)^3 = 1.35 \cdot \left(r_{nucleon}\right)^3 \cdot A$$
$$\left(r_{nucleus}\right)^3 = 1.35 \cdot \left(0.87\,fm\right)^3 \cdot A$$
$$r_{nucleus} = 0.96\,fm \cdot \sqrt[3]{A}$$

This is the radius the nucleus would have if all the nucleons were closely packed on top of each other. It is a somewhat simplistic model as we will see later. The fact remains that the radius obtained is the minimum radius that the nucleus can have. The nucleus is actually a little bigger than that. The real radius is

Radius of the Nucleus

$$r_{nucleus} = 1.2\,fm \cdot \sqrt[3]{A}$$

Note that this formula is a bit approximate. The value in front of the root actually varies between 1 fm and 1.4 fm depending on the nucleus studied.

This formula was first obtained by Rutherford.

12.2 NUCLEAR ENERGY

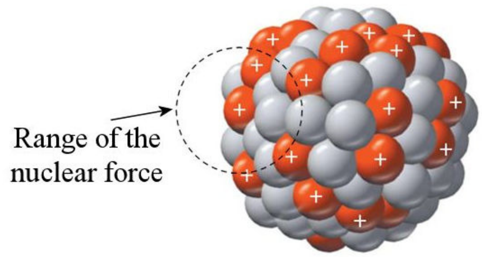
Nuclear Force

When it was realized that there are many protons in the nucleus, they wondered how these protons stay together. As they all have a positive electric charge, there is a strong electric repulsion between them, and the gravitational attraction is far too weak to cancel this repulsion. As these were the only two fundamental forces known at that time, they reached the conclusion that another force must be acting on the nucleons. This force is the nuclear force. (Actually, it is called the *strong nuclear force* to distinguish it from the weak nuclear force which is the fourth fundamental force of nature.)

The nuclear force is a very strong force of attraction between the nucleons, but it has a very short range. Two nucleons must be closer than 2 fm from each other for the nuclear force to act. If they are too far from each other, there is no force nuclear and only the repulsive electric force acts. If the nucleons are close enough, the nuclear force acts and it is larger than the electric repulsion. In this case, the two nucleons remain together and form an atomic nucleus. It is quite fortunate that the nuclear force has such a short range. Otherwise, all the nucleons of the universe would attract each other and would end up in a big ball of nucleons.

Large nuclei have a larger proportion of neutrons. For example, a small nucleus such as carbon-12 has 6 protons and 6 neutrons while a large nucleus such as lead-207 has 82 protons and 125 neutrons. The proportion shifts slowly from 1 neutron per proton for small nuclei to almost 1.5 neutrons per proton for large nuclei. This is because neutrons add nuclear attraction without adding any electrical repulsion. In large nuclei with a lot of protons which repel each other, the addition of neutrons increases the attraction between the nucleons to compensate for the large electrical repulsion. Based on this argument, it is legitimate to wonder why nuclei with only a few protons and a large number of neutrons do not exist. We will see later why there are no such nuclei with a large excess of neutrons.

If the nucleus becomes too big, the limited range of the nuclear force means that there will be a serious problem of stability. In a nucleus, a nucleon is only attracted by the neighbouring nucleons because the other nucleons are too far for the nuclear force of attraction to act. For example, the leftmost proton in the following image is only submitted to the force of nuclear attraction of nearby nucleons (those inside the dotted circle).



slides.com/tofergregg/atoms-and-the-periodic-table#/

On the other hand, this proton is subjected to the force of electric repulsion of all the other protons in the nucleus. Above a certain size, the nuclear force made by a few neighbouring nucleons is no longer sufficient to cancel the force of repulsion made by all the other protons in the nucleus and the nucleus becomes unstable. Part of the nucleus will then be able to fly away (we'll see how later). Many isotopes are unstable and no isotope is stable for elements with atomic numbers larger than 83.

The Energy Required to Remove a Proton or a Neutron From a Nucleus

To calculate the work required to remove a nucleon from the nucleus, the force or the variation of mechanical energy can be used.

$$W = \int F dr \cos\theta \qquad \text{or} \qquad W = \Delta E_{mec} = \Delta E_k + \Delta U$$

In theory, these formulae can be used since F and U are known for the nuclear force. However, the result can be obtained much more easily.

It was said in a previous chapter that energy has mass. If energy is provided to a system, its mass increases according to $E = mc^2$. Generally, it is rather futile to apply this formula because the change in mass is not very large compared to the initial mass. If a brick is lifted 1,000 m, its mass increases by only 10^{-11}%. It's rather difficult to measure such a small mass variation. On the other hand, the change (in %) is more important with the nuclear force. As energy is required to remove a nucleon from a nucleus, the mass of the system should be larger after the removal of the nucleon. Let's see if this actually happens if a neutron is removed from a carbon-12 nucleus. In this case, the transformation is

$$^{12}_{6}C \rightarrow {}^{11}_{6}C + {}^{1}_{0}n$$

Initially, the mass is 12.000 000 000 *u*. (The masses come from the atomic mass table. The mass of carbon 12 is really very close to 12 since the atomic mass unit was formerly defined with the carbon 12 isotope.)

After the transformation, there is a carbon-11 atom and a neutron. The total mass is

$$m_{^{11}C} = 11.011\ 433\ 6\ u \quad \searrow$$
$$m_{tot} = 12.020\ 098\ 5\ u$$
$$m_n = 1.008\ 664\ 916\ u \quad \nearrow$$

As predicted, the mass is indeed greater after the removal of the neutron. The mass rose by 0.020 098 5 *u*. This mass difference is called the *mass defect*. The increase in mass can be used to calculate the increase in energy in the following way.

$$\begin{aligned} E &= mc^2 \\ &= 0.020\ 098\ 5u \cdot c^2 \\ &= 0.020\ 098\ 5u \cdot 1.660\ 539 \times 10^{-27} \tfrac{kg}{u} \cdot \left(3\times 10^8 \tfrac{m}{s}\right)^2 \\ &= 3.0037\times 10^{-12} J \\ &= 18.748 MeV \end{aligned}$$

Therefore, 18.748 MeV was required to remove a neutron from the carbon-12 nucleus. It is about a million times larger than the energy required to remove an electron from the orbitals of carbon. Typically, nuclear energies are a million times larger than the chemical energy (which involves only the electrons in the orbitals).

Let's summarize this calculation, the conservation of energy gives

$$E_{before} = E_{after} + Q$$

where Q is the energy released (if Q is positive) or the energy that must be provided (if Q is negative). This equation becomes

$$\begin{aligned} Q &= E_{before} - E_{after} \\ &= m_{before}c^2 - m_{after}c^2 \\ &= \left(m_{before} - m_{after}\right)c^2 \end{aligned}$$

To go faster, a formula using the atomic mass unit and giving the energy in MeV will be devised. As the energy of 1 *u* is

$$\begin{aligned} E &= mc^2 \\ &= 1.660\ 539\times 10^{-27} kg \cdot \left(3\times 10^8 \tfrac{m}{s}\right)^2 \\ &= 1.4924\times 10^{-10} J \\ &= 931.49 MeV \end{aligned}$$

the following shortcut can be used to calculate the energy in MeV.

Nuclear Energy

$$Q = \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{MeV}{u}$$

The energy is in MeV, and the masses are in atomic mass units.

Example 12.2.1

How much energy is required to remove a neutron from a uranium-238 nucleus?

The transformation is

$$^{238}_{92}U \rightarrow {}^{237}_{92}U + {}^{1}_{0}n$$

Using the masses in the atomic mass table, the energy is

$$\begin{aligned} Q &= \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= \left(238.050\,788\,2u - \left(237.048\,730\,2u + 1.008\,664\,9u\right)\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= -0.006\,606\,9u \cdot 931.5 \tfrac{MeV}{u} \\ &= -6.154 MeV \end{aligned}$$

So, 6.154 MeV is required to remove the neutron.

Common Mistake: Rounding the Atomic Masses

When the atomic masses are subtracted, the mass difference is very small. If the masses were rounded, this small value would be lost. Therefore, keep all the digits of the atomic masses when calculating the energy.

There is a little subtlety if a proton is removed. As no electrons in the orbitals are removed, their number remains the same after the transformation. But removing a proton decreases the atomic number by 1. The previous element in the periodic table is then obtained and, normally, this element has one electron less in its orbital. Thus, the element obtained after the transformation has one electron more in its orbital compared to this element in its natural state.

The problem is that the masses in the table are the mass of the neutral atoms, i.e. the mass of the element with the right number of electrons in the orbital. As the atom here has one electron more, the mass of one electron must be added to the mass of the atom after the transformation to take into account this extra electron in the orbital. The final mass is, therefore, $m_{after} = m_{atom} + m_e + m_p$. However, this can be simplified by using the fact that hydrogen 1 is made up of one proton and one electron. The sum of the last two terms is thus equal to the mass of hydrogen-1: $m_{after} = m_{atom} + m_{H1}$.

All this to say, in conclusion, that the mass of hydrogen-1 rather than the mass of one proton must be used to calculate the final mass when a proton is removed from the nucleus.

Example 12.2.2

How much energy is required to remove a proton from a uranium-238 nucleus?

The transformation is

$$^{238}_{92}U \rightarrow {}^{237}_{91}Pa + {}^{1}_{1}H$$

Using the masses in the atomic mass table, the energy is

$$\begin{aligned} Q &= \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= \left(238.050\ 788u - \left(237.051\ 15u + 1.007\ 825u\right)\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= -0.008\ 19u \cdot 931.5 \tfrac{MeV}{u} \\ &= -7.63 MeV \end{aligned}$$

So, 7.63 MeV is required to remove the proton.

The Binding Energy of the Nucleus

The binding energy is the energy that would be required to completely destroy the nucleus. For example, it is equal to the energy required to separate a helium-4 nucleus into its constituent nucleons: 2 protons and 2 neutrons. This calculation is made exactly as in the previous section.

$$Q = \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{MeV}{u}$$

Once the atom is completely destroyed, Z protons, Z electrons, and N neutrons are obtained. The final mass is, therefore,

$$\begin{aligned} m_{after} &= Zm_p + Zm_e + Nm_n \\ &= Z\left(m_p + m_e\right) + Nm_n \\ &= Zm_{H1} + Nm_n \end{aligned}$$

For the last line, the fact that hydrogen-1 consists of one proton and one electron was used.

As the binding energy is equal the energy that must be provided (-Q), it is equal to

Binding Energy of an Element X Nucleus

$$E_{binding} = \left(Zm_{H1} + Nm_n - m_X\right) \cdot 931.5 \tfrac{MeV}{u}$$

Example 12.2.3

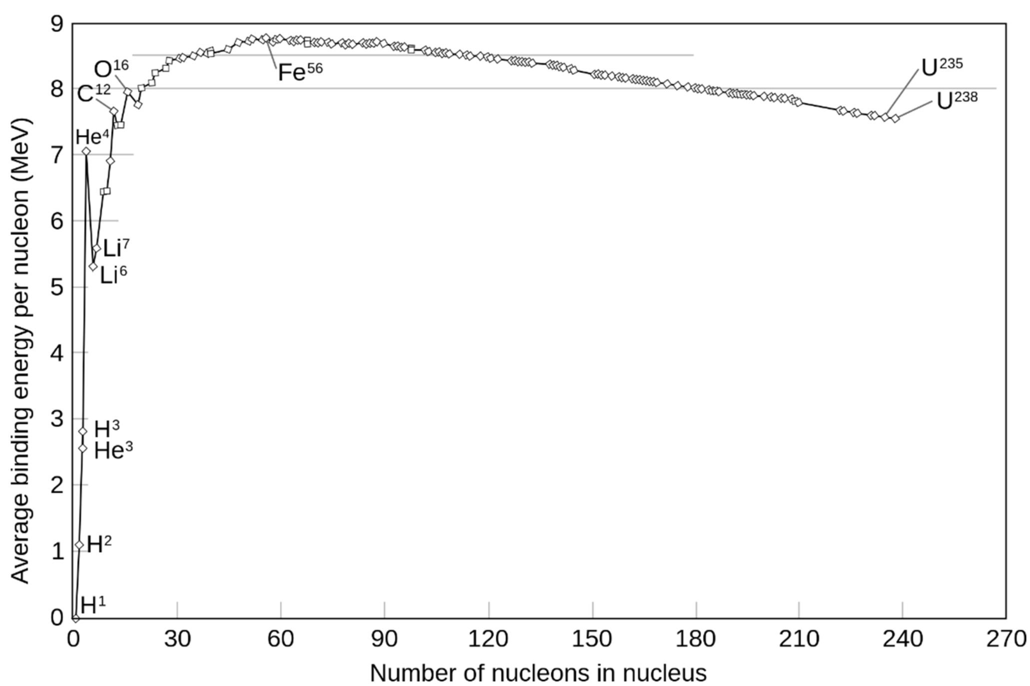
What binding energy of a helium-4 nucleus?

Using the masses in the atomic mass table, the energy is

$$\begin{aligned} E_{binding} &= \left(Zm_{H1} + Nm_n - m_X\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(2\cdot 1.007\ 825\ 032u + 2\cdot 1.008\ 664\ 916u - 4.002\ 603\ 254u\right)\cdot 931.5\tfrac{MeV}{u} \\ &= 0.030\ 377u\cdot 931.5\tfrac{MeV}{u} \\ &= 28.3MeV \end{aligned}$$

Note that if 28.3 MeV are required to destroy a helium-4 nucleus, then the creation of such a nucleus from 2 protons and 2 neutrons would release 28.3 MeV. We will see later that this nucleus formation is the basis of the energy generation in the Sun.

The binding energy can be used to find the most stable nucleus. It is not so much the total value of the binding energy that matters. Uranium has a phenomenal binding energy because there are a lot of nucleons to remove to destroy the nucleus. The average of the energy that must be provided to each nucleon to destroy the nucleus is much more interesting. It is obtained by dividing the binding energy by the number of nucleons. Then, the following graph is obtained.



en.wikipedia.org/wiki/Nuclear_binding_energy

The binding energy increases rapidly and then levels off at about 8.75 MeV per nucleon. The curve flattens because the nucleons are attracted only by other neighbouring nucleons. Even if a nucleus gets bigger, the number of neighbouring nucleons remains about the same and it takes about 8.75 MeV to snatch a nucleon from these neighbours. The slight drop comes from the electrical repulsion which increases with the size of the nucleus and makes it easier to remove the protons. The nucleus that has the largest binding energy per nucleon is iron-56 at 8.75 MeV/nucleon. Those of you who took the astrophysics course know the importance of this element for nuclear reactions in stars.

12.3 RADIOACTIVITY

Some isotopes are stable and will be able to exist forever (these are the isotopes whose mass is bolded in the mass table). On the other hand, many other isotopes are unstable and will eventually transform into another isotope. All elements that have an atomic number less than or equal to 83 (bismuth) have at least one stable isotope (with the exception of technetium, element 43, and promethium, element 61). All elements that have an atomic number greater than 83 do not have any stable isotopes. During the transformation process, the unstable isotopes emit particles. This emission of particles is called *radioactivity*. The isotopes that are transformed are called *radioactive isotopes*.

The Discovery of Radioactivity

In 1896, Henri Becquerel discovered that uranium continuously emits something. Becquerel made the discovery by chance while studying phosphorescence. When he heard of X-rays, discovered in 1895, he wanted to verify if phosphorescent substances also emit X-rays. He then discovered that phosphorescent uranium salts could mark photographic plates even if he covered them with black paper. This suggested that uranium salts emit X-rays along with visible light in phosphorescence. But then came a few days without sunshine, and Becquerel was unable to continue his experiments as he was using the Sun to excite the phosphorescence of the uranium salts. But even if the uranium salts were not emitting light anymore after these few days, Becquerel discovered they were still marking the photographic plates. This showed that the uranium salts were emitting something that has no connection whatsoever with phosphorescence. This new phenomenon was named *radioactivity* by Pierre Curie and Marie Sklodowska Curie

Becquerel was able to confirm that the radioactivity of the salts specifically comes from the uranium atoms. Soon after the publication of Becquerel's results, it was discovered, in 1898, that thorium is also radioactive. Also in 1898, Pierre and Marie Curie discovered that certain types of ore emit more radiation than if they were composed of pure uranium, the most radioactive substance known at this time. This implied that these minerals contained a substance even more radioactive. They then work hard to isolate two new radioactive elements: radium, and polonium (1898). These were the first new elements discovered of a long list of radioactive elements known today.

Nature of the Particles Emitted by a Radioactive Element

In 1899, several researchers had noticed that the emissions made by radium were deflected by magnetic fields, which suggested that radioactive elements were emitting some kind of electrically charged particles. Pierre Curie showed that there were actually two types of particles: one that was apparently not deflected (he called them alpha particles), and another that was deflected (he called them beta particles) and had more penetrating power in matter.

Becquerel, Pierre Curie and Marie Curie experiments in 1901 showed that the beta particles are actually electrons. As for alpha particles, Ernest Rutherford was able to show that a magnetic field could also deflect them. From Rutherford's experiments, it was clear that alpha particles are much more massive than electrons and have a positive electric charge. Finally, Rutherford identified them, with Hans Geiger in 1907, as doubly ionized helium atoms.

A third type of particle (known as gamma particles) was discovered in 1900 by Paul Villard. These particles are photons with a lot of energy.

Finally, in 1934, Frédéric Joliot and Irène Joliot-Curie discovered that some nucleus can also emit an antielectron (also called positron), the antiparticle of the electron.

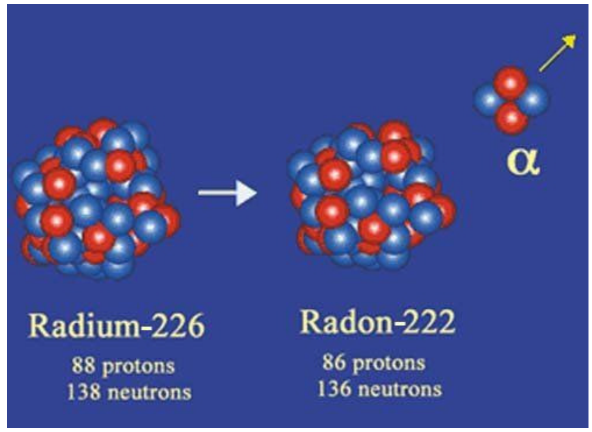
So, the particles emitted by an atomic nucleus for each type of radioactivity are:

Alpha: a helium nucleus
Beta: an electron (e^-) or an anti-electron (e^+)
Gamma: a photon

What Happens to the Atomic Nucleus When It Emits a Particle?

In 1903, Ernest Rutherford and Frederick Soddy, both working at McGill University in Montreal at that time, showed that thorium atoms undergo internal changes when they emit an alpha, a beta or a gamma particle. In 1906, they came to the conclusion that uranium is transformed into thorium when it emits an alpha particle, and then into radium and then into other elements to finally become lead, a conclusion that was strongly contested by Lord Kelvin. It is then said that the nucleus decays into another element. When the nature of the three types of emitted particle was determined, Soddy was able to give the exact rules of transformation (1911-1913). Let's examine these rules for each type of decay.

Alpha Decay



www.laradioactivite.com/fr/site/pages/laradioactivitealpha.htm

In an alpha decay, a nucleus emits a helium nucleus. As there are 2 protons and 2 neutrons in a helium nucleus, the decaying nucleus loses 2 protons and 2 neutrons. For example, here is what happens when a radium nucleus emits an alpha particle (a synonym of helium-4 nucleus).

$$ {}^{226}_{88}Ra \to {}^{222}_{86}Rn + {}^{4}_{2}He $$

The emission of a helium nucleus did remove 2 protons and 4 nucleons to the initial nucleus. So, the element changes during the decay. The initial radioactive element is referred to as the *parent atom*; the resulting atom after the decay is called the *daughter product*.

If the parent element is X and the daughter element is Y, the reaction is always

Alpha Decay

$$ {}^{A}_{Z}X \to {}^{A-4}_{Z-2}Y + {}^{4}_{2}He $$

The decay happens spontaneously if it releases energy. The energy released is calculated as before: with the difference of mass before and after the decay.

Energy Released in an Alpha Decay

$$ Q = \left(m_X - m_Y - m_{He4}\right)\cdot 931.5\tfrac{MeV}{u} $$

If Q is positive, the decay releases energy and is possible. If Q is negative, energy must be provided for this reaction to happen and it is, therefore, impossible.

Notice that the ejection of a single proton or a single neutron is almost always impossible because Q is negative for most nuclei. (This is possible for a few rare nuclei.)

In 1928, George Gamow showed that the alpha particle gets out of the nucleus by quantum tunnelling.

Example 12.3.1

Give the reaction of the alpha decay of uranium-238 and calculate the energy released by this decay.

The reaction is

$$ {}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He $$

The energy released is

$$ \begin{aligned} Q &= \left(m_X - m_Y - m_{He4}\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(238.050\,788u - 234.043\,601u - 4.002\,603u\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(0.004\,584u\right)\cdot 931.5\tfrac{MeV}{u} \\ &= 4.27MeV \end{aligned} $$

This released energy goes into the kinetic energies of the two nuclei after the decay. To find the speed of each particle, the laws of conservation of energy and momentum must be applied.

$$ Q = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 $$
$$ 0 = m_1v_1 + m_2v_2 $$

The first equation simply says that the sum of the kinetic energies of the two nuclei is equal to the energy released. The second equation says that the momentum is 0 because the parent nucleus is motionless. So, the energy of the particle alpha is found with

$$ Q = \frac{1}{2}m_{He4}v_{He4}^2 + \frac{1}{2}m_Y v_Y^2 $$
$$ Q = \frac{1}{2}m_{He4}v_{He4}^2 + \frac{1}{2m_Y}\left(m_Y v_Y\right)^2 $$
$$ Q = \frac{1}{2}m_{He4}v_{He4}^2 + \frac{1}{2m_Y}\left(m_{He4} v_{He4}\right)^2 $$
$$ Q = \frac{1}{2}m_{He4}v_{He4}^2 + \frac{m_{He4}}{m_Y}\frac{1}{2}m_{He4}v_{He4}^2 $$
$$ Q = \left(1 + \frac{m_{He4}}{m_Y}\right)\frac{1}{2}m_{He4}v_{He4}^2 $$
$$ Q = \left(1 + \frac{m_{He4}}{m_Y}\right)E_{k\ He4} $$

The energy of the alpha particle is thus

Energy of the Alpha Particle After an Alpha Decay

$$ E_{k\ He4} = \frac{m_Y}{m_{He4} + m_Y}Q $$

In the uranium decay example, it can be calculated that the kinetic energy of the alpha particle is 4.20 MeV. Therefore, only 0.07 MeV remains for the thorium nucleus. Most of the time, the alpha particle receives almost all the energy released.

Note that it is very easy to block alpha particles. These large particles interact a lot with matter and a single sheet of paper is enough to stop them.

Beta Decay

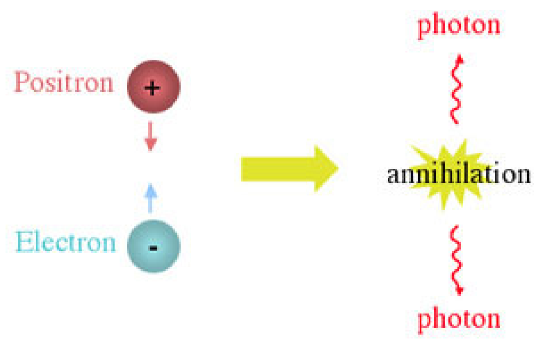
In a beta decay, a nucleus emits an electron or an antielectron (also known as a positron). The emitted electron has absolutely nothing to do with the electrons in the orbitals. This naturally leads to the question: but where does this electron or this antielectron come from if there are no electrons nor positrons in the nucleus initially?

To understand whence these particles come from, you must know that it is possible to convert a proton into a neutron or a neutron into a proton. The transformation reactions, discovered by Fermi in 1934, are

$$n^0 \to p^+ + e^- + \overline{\nu}$$
$$p^+ \to n^0 + e^+ + \nu$$

In these formulae, e^- is an electron, e^+ is an antielectron, ν is a neutrino, and $\overline{\nu}$ is an antineutrino.

The antielectrons are antimatter. These are the antiparticles of electrons. When a particle and its antiparticle meet, they disappear completely (they annihilate) by releasing a lot of energy as photons.



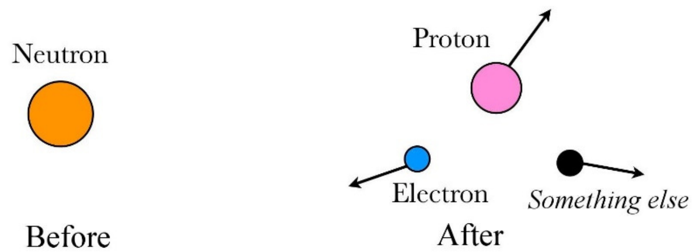
astronomy.swin.edu.au/cosmos/P/Positron

This process can also be inverted to create matter from photons. In this case, as much matter is created as antimatter. This creation of matter and antimatter is routinely achieved in particle accelerators.

Neutrinos are particles having a very small mass and no electric charge. They interact very weakly with matter. Every second, billion of neutrinos pass through your body without any effect. Perhaps, one day in your life, one of these neutrinos will interact with an atom in your body. In 1930, Wolfgang Pauli had assumed that these particles existed because energy and momentum did not seem to be conserved when the energy and the momentum of the electron (or the antielectron) and the nucleus was measured in beta decays. A part of the energy and of the momentum seemed to be missing.

This image shows what is observed. The momentum is not conserved since the two particles do not travel in opposite direction after the decay.

This new image shows what Pauli assumed. He assumed that a third undetectable particle is also present, and that this particle carries away the missing energy and momentum.



andthejoyofdiscovery.wordpress.com/tag/neutrino

Pauli called this hypothetical particle *neutrino*. The existence of the neutrino was experimentally confirmed in 1956.

The reactions

$$n^0 \rightarrow p^+ + e^- + \overline{\nu}$$
$$p^+ \rightarrow n^0 + e^+ + \nu$$

do not mean that a neutron is made up of a proton, an electron and an antineutrino (if the first transformation is taken as an example). The electron and the antineutrino appear when the neutron becomes a proton. They did not exist before the transformation.

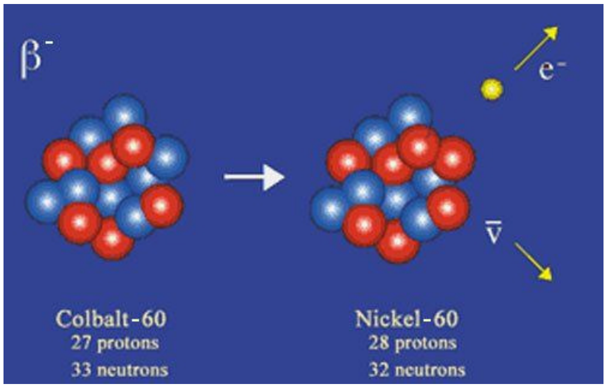
In a beta – (β^-) decay, a neutron turns into a proton and an electron is ejected (along with an antineutrino). In a beta + (β^+) decay, a proton turns into a neutron and a positron is ejected (along with a neutrino).

β^- Decay

If a neutron becomes a proton, then the nucleus gains a proton and loses a neutron. Here is an example of a β^-decay.

$$^{60}_{27}Co \rightarrow {}^{60}_{28}Ni + e^- + \overline{\nu}$$

The element then switches to the next element in the periodic table because there is an extra proton. The number of nucleons, however, remains the same.



www.laradioactivite.com/fr/site/pages/laradioactivitebeta.htm

If the parent element is X and the daughter element is Y, the reaction is always

β^- Decay

$${}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + e^- + \bar{\nu}$$

As usual, the energy released by this reaction is found with the mass variation.

$$\begin{aligned} Q &= \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{Mev}{u} \\ &= \left(m_X - \left(m_{Y \text{ with one electron missing}} + m_e + m_{\bar{\nu}}\right)\right) \cdot 931.5 \tfrac{Mev}{u} \end{aligned}$$

Notice that one electron is missing in the orbital of the daughter element because the transformation of the nucleus does not change the number of electrons in the orbitals. The daughter element being the next element in the periodic table element, there should be an extra electron in the orbital, but there isn't. So, the mass of this missing electron must be subtracted from the mass of the daughter element. In addition, the mass of the neutrino will be neglected since it is at least 250 000 times less massive than the electron. Thus, the energy released becomes

$$\begin{aligned} Q &= \left(m_X - \left(m_{Y \text{ with one electron missing}} + m_e + m_{\bar{\nu}}\right)\right) \cdot 931.5 \tfrac{Mev}{u} \\ &= \left(m_X - \left(m_Y - m_e + m_e\right)\right) \cdot 931.5 \tfrac{Mev}{u} \end{aligned}$$

Therefore,

Energy Released in a β^- Decay

$$Q = \left(m_X - m_Y\right) \cdot 931.5 \tfrac{MeV}{u}$$

Example 12.3.2

Give the reaction of the β^- decay of carbon-14 and calculate the energy released by this decay.

The reaction is

$${}_{6}^{14}C \rightarrow {}_{7}^{14}N + e^- + \bar{\nu}$$

The energy released is

$$\begin{aligned} Q &= \left(m_{{}_{6}^{14}C} - m_{{}_{7}^{14}N}\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= \left(14.003\ 241\ 989\ u - 14.003\ 074\ 005\ u\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= \left(0.000\ 167\ 984\ u\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= 0.1564 MeV = 156.4 keV \end{aligned}$$

This energy can be split into any possible ways between the electron and the antineutrino. The daughter nucleus receives virtually no kinetic energy.

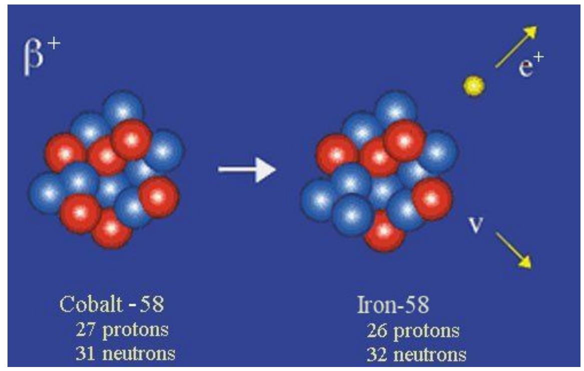
The emitted electron is relatively easy to stop, but it is a little more difficult to stop than an alpha particle. Generally, a lead plate 1 millimeter thick or a plexiglass plate 1 cm thick will do.

β^+ *Decay*

If a proton becomes a neutron, then the nucleus gains a neutron and loses a proton. Here is an example of a β^+ decay.

$$^{58}_{27}Co \rightarrow {}^{58}_{26}Fe + e^+ + \nu$$

The element then switches to the preceding element in the periodic table because there is one proton less. The number of nucleons, however, remains the same.



www.laradioactivite.com/fr/site/pages/laradioactivitebeta.htm

If the parent element is X and the daughter element is Y, the reaction is always

β^+ Decay

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ + \nu$$

The energy released by this reaction is found with the mass variation.

$$\begin{aligned} Q &= \left(m_{before} - m_{after}\right)\cdot 931.5\tfrac{Mev}{u} \\ &= \left(m_X - \left(m_{Y\text{ with one extra electron}} + m_e + m_\nu\right)\right)\cdot 931.5\tfrac{Mev}{u} \end{aligned}$$

Notice that there is one extra electron in the orbitals of the daughter element because the transformation of the nucleus does not change the number of electrons in the orbitals. The daughter element being the previous element in the periodic table element, there should be one electron less in the orbital, but it has not disappeared. The mass of one electron must, therefore, be added to the mass of the daughter element. In addition, the mass of the neutrino will be neglected. Thus, the energy released becomes

$$\begin{aligned} Q &= \left(m_X - \left(m_{Y\text{ with one extra electron}} + m_e + m_\nu\right)\right)\cdot 931.5\tfrac{Mev}{u} \\ &= \left(m_X - \left(m_Y + m_e + m_e\right)\right)\cdot 931.5\tfrac{Mev}{u} \end{aligned}$$

Therefore,

Energy Released in a β^+ Decay

$$Q = \left(m_X - m_Y - 2m_e\right)\cdot 931.5\tfrac{MeV}{u}$$

Example 12.3.3

Give the reaction of the β^+ decay of fluorine-18 and calculate the energy released by this decay.

The reaction is

$$^{18}_{9}F \rightarrow {}^{18}_{8}O + e^+ + \nu$$

The energy released is

$$\begin{aligned}Q &= \left(m_{^{18}_{9}F} - m_{^{18}_{8}O} - 2m_e\right)\cdot 931.5\tfrac{MeV}{u}\\ &= \left(18.000\,938\,0\ u - 17.999\,161\,0\ u - 2\cdot 0.000\,548\,6\ u\right)\cdot 931.5\tfrac{MeV}{u}\\ &= \left(0.000\,679\,8\ u\right)\cdot 931.5\tfrac{MeV}{u}\\ &= 0.6332 MeV = 633.2 keV\end{aligned}$$

Again, this energy can be divided in any possible way between the antielectron and the neutrino. The daughter nucleus receives virtually no kinetic energy.

The antielectron emitted will not go very far. Soon enough, he's going to encounter an electron and they're going to annihilate each other. The annihilation will generate 2 very energetic photons (in the gamma part of the electromagnetic spectrum).

Electron Capture (E.C. or ε)

Electron capture is a third variant of beta decay which was proposed in 1934 by Gian Carlo Wick and observed for the first time in 1937 by Luis Alvarez. In an electron capture, a proton is transformed into a neutron by the following reaction.

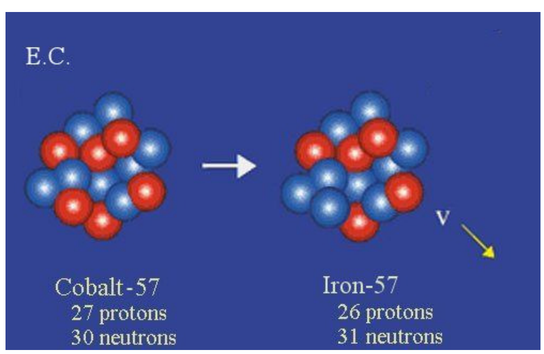
$$p^+ + e^- \rightarrow n^0 + \nu$$

In this reaction, the nucleus emits only a neutrino.

The electron was captured in the orbital and, most of the time, this electron comes from a low energy level s orbital. This creates a vacancy in the low orbitals. When an electron from a higher level takes this free spot, a photon is emitted. There may be several emitted photons if this transition is done in several steps.

Here is an example of an electron capture.

$$^{57}_{27}Co \rightarrow {}^{57}_{26}Fe + \nu$$



www.laradioactivite.com/fr/site/pages/laradioactivitebeta.htm

The daughter element corresponds to the preceding element in the periodic table because there is one proton less. The number of nucleons, however, remains the same.

If the parent element is X and the daughter element is Y, the reaction is always

Electron Capture

$$ {}^{A}_{Z}X \to {}^{A}_{Z-1}Y + \nu $$

The captured electron is not shown in this equation because the electron is already a part of the parent atom.

The energy released by this reaction is found with the mass variation.

$$ \begin{aligned} Q &= \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{Mev}{u} \\ &= \left(m_X - \left(m_Y + m_\nu\right)\right) \cdot 931.5 \tfrac{Mev}{u} \end{aligned} $$

Here, the number of electrons in the orbitals is correct. By lowering the atomic number by 1, there should be one electron less in the orbital. But one electron was actually lost in the orbital when it was captured. The number of electrons in the orbitals of the daughter element is, therefore, correct. If the mass of the neutrino is neglected, the energy released becomes

Energy released in an Electron Capture

$$ Q = \left(m_X - m_Y\right) \cdot 931.5 \tfrac{MeV}{u} $$

Example 12.3.4

Give the reaction of the electron capture in aluminum-26 and calculate the energy released by this decay.

The reaction is

$$ {}^{26}_{13}Al \to {}^{26}_{12}Mg + \nu $$

The energy released is

$$ \begin{aligned} Q &= \left(m_{{}^{26}_{13}Al} - m_{{}^{26}_{12}Mg}\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= \left(25.986\,891\,69\,u - 25.982\,592\,93\,u\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= \left(0.004\,298\,76\,u\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= 4.004 MeV \end{aligned} $$

This time, the neutrino picks up all the kinetic energy. The daughter nucleus receives virtually no kinetic energy.

Positron capture, resulting in a transformation of a neutron into a proton, is also possible theoretically but it is almost impossible for a nucleus to find an antielectron since there is no antielectron in the orbitals.

Rules for beta decay

Beta Decays follow these rules:

1) Nucleus with an excess of neutrons compared to a stable isotope of the element

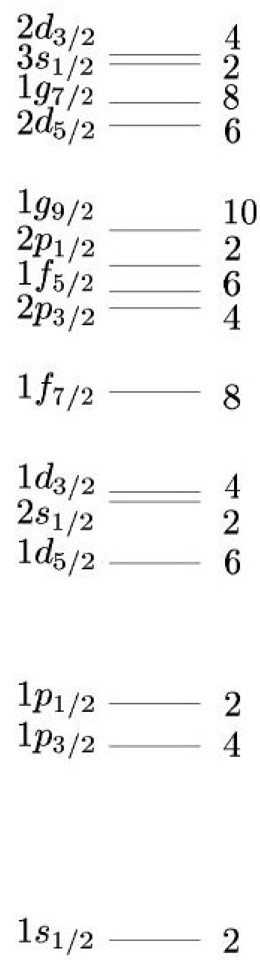
   Beta – decay to reduce the number of neutrons

2) Nucleus with an excess of protons compared to a stable isotope of the element

   Beta + decay or an electron capture to reduce the number of protons.

Obviously, only the decays that release energy, so those with a positive *Q,* are permitted.

Gamma Decay



In gamma decay, a nucleus emits a photon. This emission does not change the nature of the particles inside the nucleus. All the protons remain protons and all the neutrons remain neutrons and we are left with the same isotope that we had before the emission of the photon.

Then, what is changing when a nucleus emits a photon?

A more refined model of the nucleus predicts that protons and neutrons are also on energy levels in the nucleus, exactly as the electrons around the nucleus. The diagram on the right shows these energy levels as well as the number of neutrons or protons that can be put on these levels. If 2 is written next to a level, this level can accommodate 2 protons **and** 2 neutrons, for a total of 4 nucleons. The similitude between the notation used for these levels and the notation used for the level of the electron around the nucleus can even be noted.

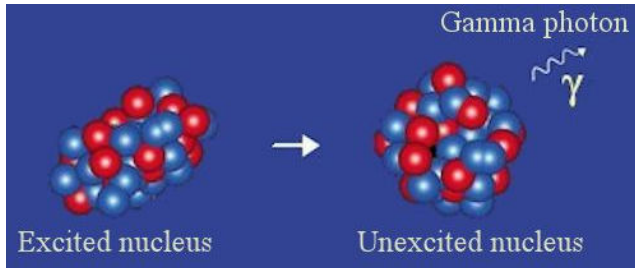
The emission of a photon by a nucleus can then be understood: If a nucleon goes down on a lower energy level, the nucleon emits a photon with the lost energy, exactly as for the electrons in the orbitals.

Obviously, for this to happen, a nucleon must be on a higher level. If this is the case, the nucleus is an excited state and this is indicated by an asterisk next to the symbol of the nucleus.

Thus, the gamma decay of an excited nucleus of cobalt-60 is written as

$$^{60}_{27}Co^{*} \rightarrow {}^{60}_{27}Co + \gamma$$

where γ is the symbol for a photon.



www.laradioactivite.com/fr/site/pages/Radioactivite_Gamma.htm

There are several ways to excite an atomic nucleus. For example, a collision with another nucleus may raise the energy level of a nucleon. Often, an excited nucleus is obtained after another decay. In this animation, the alpha decay of a plutonium-239 nucleus is followed by two gamma decays.
https://youtu.be/xZ8A7lv32q4

The emitted photon has a lot of energy and it is very difficult to block it. If it takes just one sheet of paper to block alpha rays and 1 mm of lead to block beta rays, it takes 3.8 cm of lead or 28 cm of concrete to block 90% of the gamma rays coming from the decay of cobalt-60. There will always be a certain percentage of gamma rays that will manage to pass through.

Why There Cannot Be Too Many Neutrons in a Nucleus?

These energy levels also explain why there cannot be too many neutrons in a nucleus. Suppose there is a carbon nucleus (6 protons) with 50 neutrons. The protons would be on the first 3 levels while the neutrons would occupy the first 25 levels. The last neutron would be on a level so high compared to the last proton that a significant amount of energy would be released if this neutron became a proton and goes down to the 4th level. If a process releases energy, it is possible and it will happen sooner or later. If the loss of energy is huge, as is the case when there are way more neutrons than protons, the change happens almost instantly. The neutrons will turn into protons until the ratio of the two is closer to the ratio found in the most stable isotope that can be formed with this number of nucleons.

Other Rare Decays

There are a few other possible decays but they are rather rare.

1) ***Proton Emission***: Some nuclei with a large excess of protons can eject one of these protons. This is the case for cesium-113. The decay is then

$$^{113}_{55}Cs \rightarrow {}^{112}_{54}Xe + p^{+}$$

28 nucleus that can undergo proton emission are known.

2) ***Neutron Emission***: Some nuclei (often at the beginning of the periodic table) with a very large excess of neutrons can eject one of these neutrons. This is the case of lithium-12.

   The decay is then

$${}^{12}_{3}Li \rightarrow {}^{11}_{3}Li + {}^{1}_{0}n$$

   17 nucleus that can undergo neutron emission are known.

The main mode of decay of the different isotopes can be seen on this site.
https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html

Radioactive Substances Do Not Glow

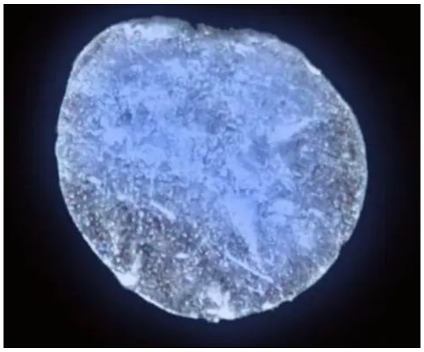
In film and television, radioactive substances often emit a kind of blueish or greenish light. However, the only light that can be emitted by a radioactive substance is gamma radiation and this radiation is not at all in the visible part of the electromagnetic spectrum. Are we shown a false representation or do radioactive substances really do emit such blue or green radiation?

klotza.blogspot.com/2016/07/do-radioactive-things-glow.html

The simple answer is that the vast majority of radioactive substances do not have this blue or green glow. They do not emit light at all. To the naked aye, nothing suggest that the substance is radioactive.

This light is associated with radioactive elements because radium was used until the 60s to make phosphorescent displays that allowed, for example, to see the hands of a watch at night. But to achieve this, radium had to be combined with zinc sulfide doped with copper. When the radium decays, the emitted particle can give some of its energy to the zinc sulfide, making an electron in the orbitals rise to a higher energy level. When the electron goes back to a lower energy level, it emits a photon in the green part of the visible spectrum. (These old watches are still radioactive, since the half-life of radium is 1600 years but their hands no longer shine in the dark because the crystal structure of zinc sulfide deteriorates quite quickly.)

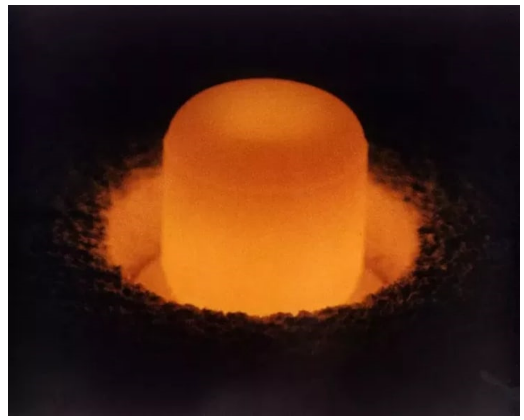
www.mentalfloss.com/article/541196/where-did-myth-radiation-glows-green-come



Note that pure radium does glow a bit but this is because the particles emitted during decay excite the nitrogen atoms in the air around the radium sample. When the electrons in the nitrogen go back to a lower energy level, photons are emitted. The light emitted in this case is rather blue.

www.mentalfloss.com/article/541196/where-did-myth-radiation-glows-green-come

Other radioactive substances shine because radioactive decays release energy that can lead to an increase in the temperature of the substance large enough for the substance to become luminous. We are not seeing the radioactivity; we are seeing the radiation emitted by a hot object. The image on the right shows a piece of plutonium-238 dioxide emitting red light because radioactive decays make it very hot.



en.wikipedia.org/wiki/Plutonium-238

Effects of Radiation on the Human Body

Radioactivity is dangerous for humans. When a high-energy particle passes through a human cell, it can cause damage. Essentially, a gamma photon will ionize an atom in a molecule, which can subsequently cause a chemical bond to rupture to form free radicals. For example, water can be split into two parts by a gamma photon.

$$H_2O \rightarrow H^+ + OH^-$$

These free radicals are extremely reactive and will generate all kinds of unwanted chemical reactions. These reactions denature the constituents of the cell and disrupt its functioning. When too many of these free radicals appear in a cell, the cell can die. Obviously, if many cells in your body die, you will not feel well…

On the other hand, radioactivity can be used to kill unwanted cell like cancer cells. Most of the time, the beta decay of cobalt-60 is used. The electron emitted during this decay could not reach deep tumors (since the electrons of the beta decay are quite easily blocked), but the two gamma decays that follow this decay generate photons with energies of 1.17 MeV and 1.33 MeV that can easily reach the tumor. By crossing several beams of gamma photons on the target cell, the latter receives a dose of radiation large enough to cause its death.

12.4 THE LAW OF RADIOACTIVE DECAY

Activity and the Number of Nuclei Remaining

At what rate will a decay happen? Will all the atoms decay immediately or will it take some time for the nuclei to decay? Here is the answer to those questions.

Let's find the number of atoms that decay in a second. This is called the *activity* and is represented by R.

Each atom has a certain probability of disintegrating during the next second. This probability is given by the *decay constant* λ. For example, if there is a probability of 1% that the atom will decay in the next second, then $\lambda = 0.01\ s^{-1}$. If there are N atoms and each has a probability λ to decay in the next second, then the number of atoms that decays during this time is obtained by multiplying these two quantities. Thus, the activity is

Activity of a Radioactive Substance

$$R = \lambda N$$

(Note that there would be nuances of interpretation to be made for substances that have a very large λ but the end result is the same.)

Common Mistake: Confusing λ With a Wavelength

The decay constant λ has nothing to do with a wavelength. It's too bad the same symbol is used for both.

Activity Units

The activity of a substance can be given in *number of decays per second*. This unit is called the *becquerel*.

The Becquerel

$$1Bq = 1 \text{ decay per second}$$

The activity can also be given in curies, which is

The Curie

$$1Ci = 3.7\times 10^{10} \text{ decays per second}$$

1 curie is very close to the activity of one gram of pure radium-226.

Number of Atoms and Activity as a Function of Time

Since there are decays, the number of atoms of the radioactive substance remaining will decrease with time. Therefore, the number of atoms will decrease at an increasingly small rate. Indeed, as the number of decays per second is given by $R = \lambda N$, we see that the number of decays per second will decrease as N decreases. Knowing that the activity decreases with N, we can find the number of atoms remaining.

The activity corresponds to the rate at which the number of nuclei decreases. If there are 1000 decays in 1 second, then the number of nuclei decreases at the rate of 1000 nuclei per second. Therefore,

$$-\frac{dN}{dt} = R$$

There is a negative sign as the number of nuclei decreases. Since $R = \lambda N$, we have

$$-\frac{dN}{dt} = \lambda N$$

This differential equation can be solved to get the number of atoms remaining as a function of time.

$$\frac{dN}{dt} = -\lambda N$$

$$\frac{dN}{N} = -\lambda dt$$

$$\ln N = -\lambda t + cst$$

The constant can be found since it is known that the number of atoms is equal to N_0 at $t = 0$. Thus,

$$\ln N_0 = -\lambda \cdot 0 + cst$$

$$cst = \ln N_0$$

Then

$$\ln N = -\lambda t + \ln N_0$$

$$\ln N - \ln N_0 = -\lambda t$$

$$\ln \frac{N}{N_0} = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

If this equation is solved for *N*, the result is

Number of Nucleus of a Radioactive Substance Remaining as a Function of Time

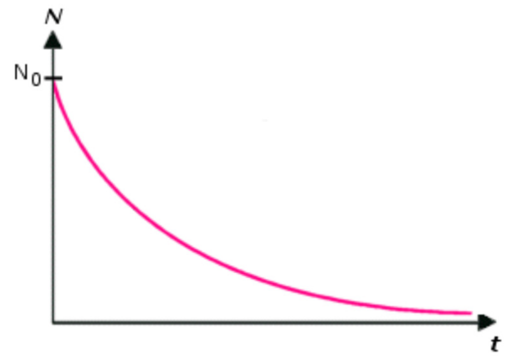
$$N = N_0 e^{-\lambda t}$$

This law was discovered by Rutherford in Montreal in 1899.

Thus, the number of remaining nuclei of a radioactive substance decreases exponentially.

Knowing this, we can find out how the activity changes over time. The activity of the substance then becomes

$$\begin{aligned} R &= \lambda N \\ &= \lambda N_0 e^{-\lambda t} \end{aligned}$$



tpe-rayons-ionisants.webnode.fr/introduction/la-radioactivite/

As the initial activity is $R_0 = \lambda N_0$, this equation can also be written as

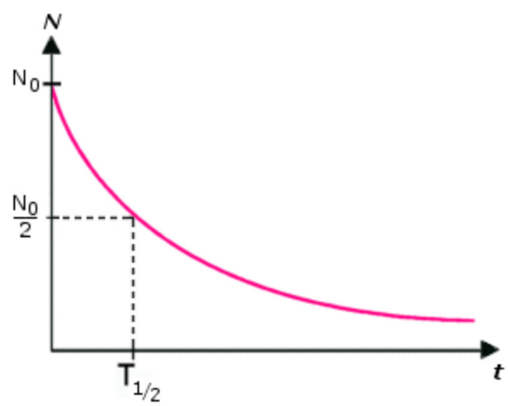
Activity as a Function of Time

$$R = R_0 e^{-\lambda t}$$

Thus, the activity of a radioactive isotope also decreases exponentially.

Half-Life

It makes no sense to ask how long will it take for all the nuclei to decay since the decline is exponential. Since an exponential function is never zero, there's always some nuclei left (in theory). To better appreciate the rate at which the nuclei decay, the half-life is used. It indicates how long it will take for half of the nuclei to decay.



Using the formula for the number of nuclei remaining as a function of time, the half-life is

$$N = N_0 e^{-\lambda t}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{½}}$$

$$\ln\frac{1}{2} = -\lambda T_{½}$$

$$T_{½} = \frac{-\ln\frac{1}{2}}{\lambda}$$

$$T_{½} = \frac{\ln\left(\frac{1}{2}\right)^{-1}}{\lambda}$$

Therefore,

Half-life

$$T_{½} = \frac{\ln 2}{\lambda}$$

The values of half-life can vary considerably from one isotope to another. Certain isotopes have half-lives as small as 10^{-22} seconds while other radioactive isotopes have half-lives as long as 10^{28} seconds (3×10^{20} years).

During each half-life, half of the remaining nuclei decay. If the half-life is 10 seconds and there 800 nuclei initially, there are 400 nuclei left after 10 seconds, 200 after 20 seconds, 100 after 30 seconds, 50 after 40 seconds and so on. The number of nuclei is divided by 2 for each half-life.

Example 12.4.1

The half-life of the alpha decay of plutonium-239 is 24,100 years. Initially, there is 1 gram of pure plutonium 239.

a) What is the initial number of nuclei?

The number of nuclei is obviously equal to the number of atoms. The number of atoms is

$$\begin{aligned} N &= \frac{0,001kg}{239\frac{u}{atom}\cdot 1.6605\times 10^{-27}\frac{kg}{u}} \\ &= 2.52\times 10^{21} atoms \end{aligned}$$

Note that 239 is also molar mass (in grams) of plutonium. Thus, the number of atoms can also have been found with

$$\begin{aligned} N &= \frac{1g}{239\frac{g}{mol}}\cdot 6.022\times 10^{23}\frac{atoms}{mol} \\ &= 2.52\times 10^{21} atoms \end{aligned}$$

b) What is the value of the decay constant?

The decay constant is

$$\begin{aligned}\lambda &= \frac{\ln 2}{T_{½}}\\ &= \frac{\ln 2}{24{,}100\ y}\\ &= \frac{\ln 2}{7.605\times 10^{11} s}\\ &= 9.114\times 10^{-13} s^{-1}\end{aligned}$$

This means that the probability that a specific plutonium nucleus decays in the next second is only 9.114 x 10^{-11} %.

c) What is the initial activity?

The initial activity is

$$\begin{aligned}R_0 &= \lambda N_0\\ &= 9.114\times 10^{-13} s^{-1}\cdot 2.52\times 10^{21}\\ &= 2.297\times 10^{9} Bq = 0.0621 Ci\end{aligned}$$

d) What will the activity of this gram of plutonium be in 10,000 years?

The activity will be

$$\begin{aligned}R &= R_0 e^{-\lambda t}\\ &= 2.297\times 10^{9} Bq\cdot e^{-\frac{\ln 2}{24{,}100 y}\cdot 10{,}000 y}\\ &= 2.297\times 10^{9} Bq\cdot 0.75\\ &= 1.723\times 10^{9} Bq = 0.0466 Ci\end{aligned}$$

Radioactive Dating

The law of radioactive decay can be used to date some objects if they contain radioactive material. One of the best-known dating methods is carbon-14 dating. All living organisms incorporate carbon during their life and a part of the natural carbon is radioactive carbon-14. While the organism is alive, the continuous absorption of carbon maintains the proportion of carbon-14 at the same level, at about 1.3 x 10^{-10} %. When the organism dies, the absorption of carbon-14 stops and the quantity of carbon-14 will then decrease exponentially with a 5730 years half-life. By measuring the proportion of carbon-14 in the dead organism, it is possible to determine for how long it has been dead.

To begin, the initial activity of one gram of natural carbon must be calculated. The number of atoms in one gram of natural carbon is

$$\begin{aligned} N_0 &= \frac{1g}{12.0107 \frac{g}{mol}} \cdot 6.022\times 10^{23} \frac{atoms}{mol} \\ &= 5.0122\times 10^{22} atoms \end{aligned}$$

The atomic weight 12.0107 g/mol was used because it is the average atomic mass of the different isotopes in natural carbon (98.9% of carbon-12, 1.1% of carbon-13 and a bit of carbon 14). 14 g/mol would have been used for pure carbon-14. The number of atoms of carbon-14 is 1.3 x 10^{-10} % of this number of atoms

$$\begin{aligned} N_{^{14}C} &= 1.3\times 10^{-12} \cdot 5.0122\times 10^{22} atoms \\ &= 6.516\times 10^{10} atoms \end{aligned}$$

The initial activity in one gram of natural carbon is, therefore,

$$\begin{aligned} R_0 &= N_0 \lambda \\ &= N_0 \frac{\ln 2}{T_{½}} \\ &= 6.516\times 10^{10} \cdot \frac{\ln 2}{5730 y} \\ &= 6.516\times 10^{10} \cdot \frac{\ln 2}{1.808\times 10^{11} s} \\ &= 0.2497 Bq \end{aligned}$$

So, let's just say that

Initial Activity of One Gram of Natural Carbon

$$R_0 = 0.25 Bq$$

Here's how to proceed to find the age of an object.

Example 12.4.2

An old piece of cotton containing 10 grams of carbon has an activity of 0.5 Bq. What is the age of this fabric?

With 10 g of carbon, the initial activity is

$$R_0 = 0.25\tfrac{Bq}{g} \cdot 10g$$
$$= 2.5Bq$$

If the activity is now 0.5 Bq, then

$$R = R_0 e^{-\lambda t}$$
$$R = R_0 e^{-\frac{\ln 2}{T_{1/2}} \cdot t}$$
$$0.5Bq = 2.5Bq \cdot e^{-\frac{\ln 2}{5730y} \cdot t}$$
$$\frac{1}{5} = e^{-\frac{\ln 2}{5730y} \cdot t}$$
$$\ln\frac{1}{5} = -\frac{\ln 2}{5730y} \cdot t$$
$$t = 5730y \cdot \frac{\ln 5}{\ln 2}$$
$$t = 13,305y$$

Limits to Carbon-14 Dating

First, objects that were not alive at some point cannot be dated with carbon-14. The cotton fabric from the previous example is a good example because cotton is a plant. There is a good chance that the fabric was made shortly after the harvest of cotton, and the cotton plant has ceased to absorb carbon then. Gold cannot be dated with carbon-14 because it was never alive.

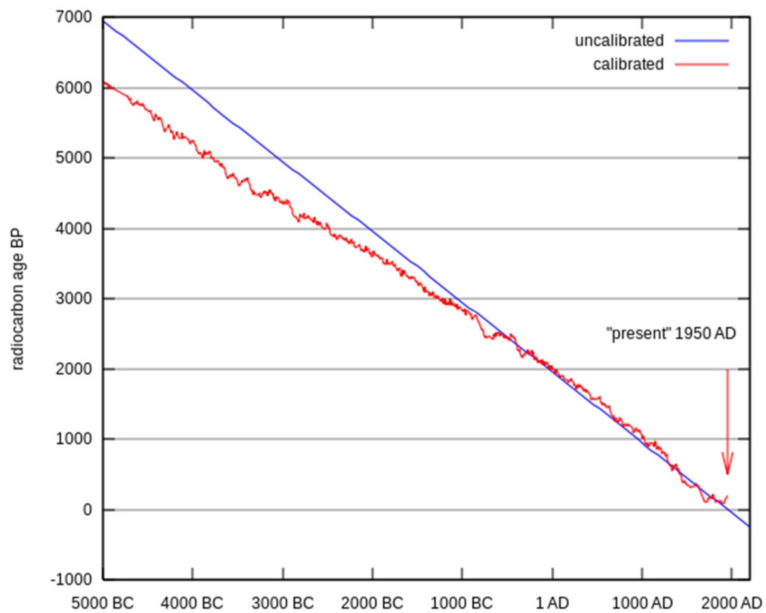
Second, very old objects (about more than 60,000 years) cannot be dated because there is not enough carbon-14 in the object after such a long time.

What Is the Origin of Natural Carbon-14?

If carbon-14 has a half-life of 5730 years, why carbon-14 atoms can still be found on Earth if it was formed 4.5 billion years ago? Perhaps this is a proof that the creationists are right after all and the Earth is only 6000-years-old! Obviously not. In fact, carbon-14 is continuously regenerated in the upper atmosphere through the following reaction.

$${}^{14}_{7}N + {}^{1}_{0}n \rightarrow {}^{14}_{6}C + {}^{1}_{1}H$$

The neutron comes from cosmic rays. Actually, the flow of neutrons in the high atmosphere is influenced by some parameters and the amount of carbon-14 produced fluctuates. Fortunately, a calibration curve can be made by comparing the ages obtained by carbon-14 dating and other possible dating methods. Here is this curve.



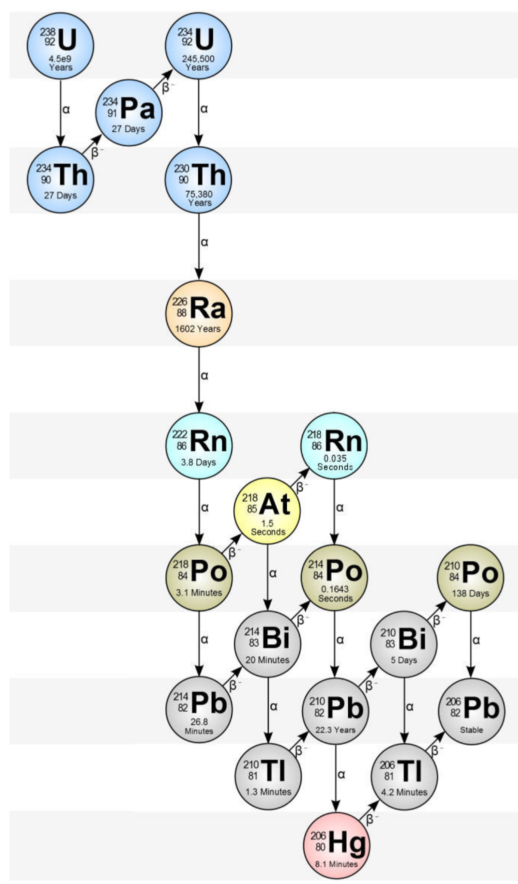
commons.wikimedia.org/wiki/File:Radiocarbon_dating_calibration.svg

If 6000 years is obtained with the calculation (age according to unmodified carbon-14 dating), this does not mean that the object dates back to 4000 B.C. (-4000) as indicated by the blue curve (uncalibrated curve), but rather that it dates back to 5000 B.C. as indicated by the red curve (calibrated curve).

Radioactive Series

Often, the result of a radioactive decay is also a radioactive nucleus, which in turn will decay into another radioactive nucleus and so on until a stable nucleus is reached. For example, here is a sequence of all the nucleus that can be obtained from uranium-238 that ends up at lead-206, including the half-lives of these decays.

Then, it is possible to understand why radium can still be found on the surface of the Earth, even if its half-life is only 1602 years. In this sequence of decays of uranium, radium is created at some point. Thus, even if radium decays pretty quickly (on a geological scale), new radium atoms are continuously created by the decay of uranium, which happens at a much slower rate (half-life of 4.51 billion years).



physicsopenlab.org/2016/01/29/uranium-gamma-spectrometry/

12.5 NUCLEAR REACTIONS

Initially, only naturally occurring isotopes were known. Most of them were stable, non-radioactive isotopes. Then, Rutherford and James Chadwick were able to change an isotope into another for the first time in 1919, with the reaction

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$$

The nuclei of helium (alpha particle) used for this reaction came from the alpha decay of radium. Thus, the alpha particle was arriving with a lot of speed so that the nucleus cannot survive unscathed to the impact.

Following this discovery, many reactions between nuclei of several isotopes were made and multiple transformations of atomic nuclei were done in this way. Using this technic, many new isotopes and elements not found in the nature were made.

The reactions are difficult to obtain by bombarding nuclei with charged particles as the nucleus also has an electrical charge that repels the incoming charged particle. A lot of speed must be given to the incoming nuclei in order to reach the target and this is why particle accelerators had to be built. But the discovery of the neutron, which has no electric charge, in 1932 changed everything. Nuclei can easily be bombarded with neutrons since there is no electric repulsion. By bombarding nuclei with neutrons, Enrico Fermi created about 40 new radioactive isotopes. New elements that do not exist naturally were also obtained. In 1940, McMillan and Abelson obtained neptunium, and in 1941, Seaborg obtained plutonium. (Fermi thought he got them in 1934-36, but it was barium and krypton.)

Once again, the energy released by the reaction is calculated with the mass defect.

Energy Released in a Nuclear Reaction

$$Q = \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{MeV}{u}$$

If Q is positive, the reaction releases energy and is possible even if the collision happens at low speed. If Q is negative, the reaction requires energy. This does not mean that the reaction is impossible but it rather means that energy must be provided for this reaction to happen. This energy is supplied in the form of kinetic energy of the nuclei before the collision. If the reaction needs 5 MeV, the colliding nuclei must have at least 5 MeV of kinetic energy so that the reaction can occur. (Often, more energy is needed since the resulting nuclei must have a certain speed after the reaction to conserve momentum so that not all the kinetic energy can be used to make the reaction because of momentum conservation.)

Example 12.5.1

One of the reactions obtained by Fermi in 1934 was

$$ {}_{0}^{1}n + {}_{92}^{238}U \rightarrow {}_{92}^{239}U $$

What is the energy released by this reaction?

The energy is

$$\begin{aligned} Q &= \left(m_{before} - m_{after}\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(1.008\ 664\ 9u + 238.050\ 788\ 2u - 239.054\ 293\ 3u\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(0.005\ 1598u\right)\cdot 931.5\tfrac{MeV}{u} \\ &= 4.81 MeV \end{aligned}$$

As the value of Q is positive, this reaction is possible even with slow neutrons.

Example 12.5.2

Consider the following nuclear reaction.

$$ p^{+} + {}_{6}^{13}C \rightarrow ? + {}_{0}^{1}n $$

a) What is the missing isotope?

In nuclear reactions, the number of protons and the number of neutrons are the same before and after the reaction. Therefore, the number of protons (Z) and the number of nucleons (A) are the same on each side of the equation.

Before the reaction, there are 7 protons. 7 protons must, therefore, be present after the reaction. They are all in the unknown isotope. The element with 7 protons is nitrogen.

Before the reaction, there are 14 nucleons. Therefore, there must be 14 nucleons after the reaction. As there is one ejected neutron, 13 nucleons are left for the unknown isotope. The unknown element is, therefore, nitrogen-13.

$$ p^{+} + {}_{6}^{13}C \rightarrow {}_{7}^{13}N + {}_{0}^{1}n $$

b) What is the energy released by this reaction?

The energy is

$$Q = \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{MeV}{u}$$
$$= \left(\left(m_p + m_C\right) - \left(m_{N \text{ with one electron less}} + m_n\right)\right) \cdot 931.5 \tfrac{MeV}{u}$$

There is one electron less for nitrogen because the electrons in the orbitals have not changed during the reaction. There were 6 electrons in the orbitals of carbon before the reaction, and there are still 6 in the orbitals of nitrogen after the collision. So one electron is missing since nitrogen normally has 7 electrons. It's the kind of thing that will happen if the incoming particle is a nucleus (such as a proton or alpha particle) instead of a complete atom. However, this problem can be solved by using the mass of the atom instead of the mass of the nucleus only. The electrons will then cancel on each side of the equation.

$$Q = \left(\left(m_p + m_C\right) - \left(m_{N \text{ with one electron less}} + m_n\right)\right) \cdot 931.5 \tfrac{MeV}{u}$$
$$= \left(\left(m_{H \text{ with one electron less}} + m_C\right) - \left(m_{N \text{ with one electron less}} + m_n\right)\right) \cdot 931.5 \tfrac{MeV}{u}$$
$$= \left(\left(m_H - m_e + m_C\right) - \left(m_N - m_e + m_n\right)\right) \cdot 931.5 \tfrac{MeV}{u}$$
$$= \left(\left(m_H + m_C\right) - \left(m_N + m_n\right)\right) \cdot 931.5 \tfrac{MeV}{u}$$

Then, we see that it is no longer necessary to consider the missing electrons.

$$Q = \left(\left(m_H + m_C\right) - \left(m_N + m_n\right)\right) \cdot 931{,}5 \tfrac{MeV}{u}$$
$$= \begin{pmatrix} \left(1.007\,825\,03u + 13.003\,354\,84u\right) - \\ \left(1.008\,664\,92u + 13.005\,738\,61u\right) \end{pmatrix} \cdot 931.5 \tfrac{MeV}{u}$$
$$= \left(\left(14.011\,179\,87u\right) - \left(14.014\,403\,53u\right)\right) \cdot 931.5 \tfrac{MeV}{u}$$
$$= \left(-0.003\,223\,66u\right) \cdot 931.5 \tfrac{MeV}{u}$$
$$= -3.003 MeV$$

The energy of the proton must, therefore, be at least 3 MeV for this reaction to occur (actually, a little more because of momentum conservation).

Sometimes a collision just excites the nucleus, which will subsequently make a gamma decay.

$${}^{1}_{1}H + {}^{13}_{6}C \rightarrow {}^{1}_{1}H + {}^{13}_{6}C^*$$
$${}^{13}_{6}C^* \rightarrow {}^{13}_{6}C + \gamma$$

Of course, the kinetic energy of the proton decreases during this collision since some of this energy is absorbed by the nucleon which goes up in the energy levels of the nucleus.

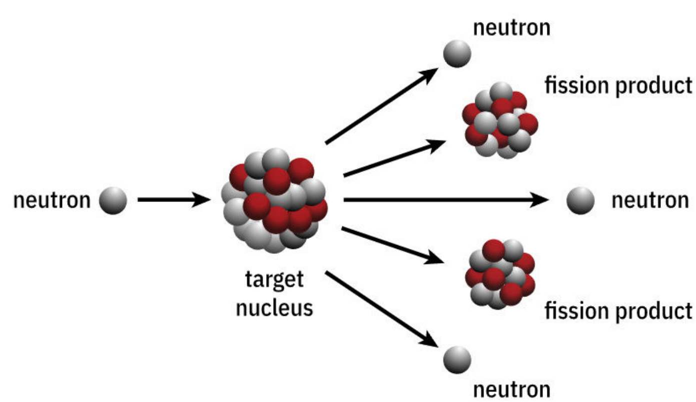
12.6 NUCLEAR FISSION

Discovery

Initially, the changes obtained by a decay or a nuclear reaction always gave elements close to the original element in the periodic. For example, Edwin McMillan and Philip Abelson obtained in 1940, by bombarding uranium (element no. 92), the element no. 93 (that they named neptunium, that does not naturally exist in the universe). It was the first of a series of elements created in the laboratory. 23 new elements were thus obtained during the 20th century.

But stranger results were also obtained with uranium. In 1938, Otto Hahn and Fritz Strassmann obtained substances whose properties were quite different from what was expected. They obtained, among other things, an element whose properties was quite similar to barium. What was this strange element obtained from uranium? They did not think it was barium (element No. 56) since it is too far from uranium (element No. 92) in the periodic table.

These results were correctly interpreted at the beginning of 1939 by Lise Meitner and Otto Frisch, former collaborators of Hahn in Germany but now refugees in Sweden (Hitler was named chancellor of Germany in 1933). They understood that these products were the result of a splitting of the nucleus of uranium into two more or less equal parts. This is called *nuclear fission*.



www.atomicarchive.com/Fission/Fission1.shtml

This time, Fermi's team in Rome had missed this discovery. In fact, several teams had enough evidence to conclude that there was nuclear fission, but many attributed the results to some apparatus defect.

Here is an animation of nuclear fission of uranium.
https://youtu.be/a5UuRjjLcLE

Example 12.6.1

One of the possible uranium fission reactions is ${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3n^{0}$

(Observe that the number of protons and the number of neutrons are the same before and after the reaction. This is always true for fission.)

What is the energy released by this reaction?

The energy is

$$\begin{aligned}Q &= \left(m_{before} - m_{after}\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(\left(m_n + m_U\right) - \left(m_{Ba} + m_{Kr} + 3\cdot m_n\right)\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(\begin{array}{l}\left(1.008\ 665u + 235.043\ 930u\right) - \\ \qquad \left(140.914\ 411u + 91.926\ 156u + 3\cdot 1.008\ 665u\right)\end{array}\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(0.186\ 033u\right)\cdot 931.5\tfrac{MeV}{u} \\ &= 173.3MeV\end{aligned}$$

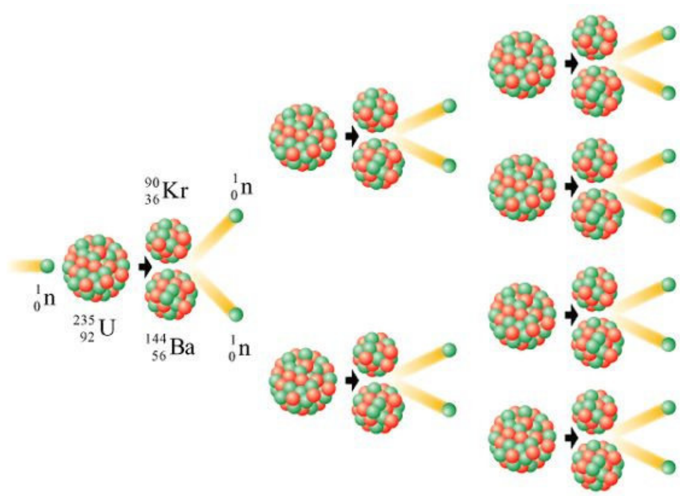
This is a lot of energy compared to the energies of other nuclear reactions that were, at best, a few tens of MeV.

Only large nuclei release energy in fission. Generally, these are the actinides (elements 89 to 102) with an odd number of neutrons. For example, uranium-235 (143 neutrons) is fissile whereas uranium-238 (146 neutrons) is not fissile (the nucleus simply absorbs the neutron). Uranium-235 is the only naturally occurring fissile isotope.

Chain Reaction

When radioactivity was discovered, ways to use it to generate energy in a controlled manner were sought. However, as the reactions occur spontaneously and randomly, this was virtually impossible. With the discovery of artificial radioactivity in 1934, scientists such as Leó Szilárd tried to achieve a chain reaction. It was hoped that the particle emitted by a decay could hit another nucleus and turn it into another radioactive nucleus. When this latter decay, the emitted particle would, in turn, cause the decay of another nucleus and so on. However, they were unable to make this idea work.

With the discovery of uranium fission in 1939, Szilárd quickly understood that a chain reaction can be obtained. The fission of the uranium nucleus is caused by the arrival of an extra neutron in the nucleus. But fission also emits neutrons, and it is possible that these neutrons subsequently cause the fission of other uranium nuclei.



resources.edb.gov.hk/~senergy/power/print/nuclear_phy_print_e.html

The reaction can be self-sustaining and can even gain quickly in intensity since each fission releases several neutrons. You can see an analogy of this chain reaction where Ping-Pong balls play the role of neutrons and mouse traps the role of uranium-235 atoms.
http://www.youtube.com/watch?v=vjqIJW_Qr3c
http://www.youtube.com/watch?v=JxzPN-vdP_0

As soon as 1939, Joliot-Curie and his collaborators showed experimentally that a chain reaction was possible, but they did not have enough uranium to make a continuous and controlled reaction. In 1942, only 4 years after the discovery of fission, Enrico Fermi's team was able to make the first controlled nuclear reaction at the University of Chicago.

There are two important characteristics of these nuclear reactors.

1) The Reaction Must Be Controlled

The number of fissions increases rapidly since each fission triggers several others fissions about 10 ns later. To prevent the reactor from getting out of hand and exploding, the reaction must be controlled. This is done with cadmium bars that are inserted into the uranium. Cadmium absorbs neutrons, which stops the reaction. When the reaction goes too fast, the cadmium bars are inserted a little more to absorb more neutrons and reduce the number of fissions. When the reaction goes too slowly, the cadmium bars are pulled out to absorb fewer neutrons and increase the number of fissions.

2) The Neutrons Must Be Slowed Down

Uranium naturally contains a lot of uranium-238 (99.3% uranium-238 and 0.7% uranium-235). However, uranium-238 absorbs neutrons emitted by the fission of uranium-235 without splitting, so natural uranium cannot do a chain reaction. On the other hand, the situation changes drastically if the neutrons emitted by fission are slowed down. This greatly increases the likelihood that neutrons interact with the rare uranium-235 atoms to the point that a slow neutron is more likely to interact with an uranium-235 atom than an uranium-238 atom, even though they are significantly fewer in number. The problem is that the neutrons emitted by fission are going too fast. The neutrons are slowed down by making them collide with the atoms of a substance called a moderator. Atoms of a good moderator do not absorb neutrons and resist to collisions. Hydrogen (^1H, ^2H and ^3H), helium and carbon nuclei are excellent moderators. Water or graphite are therefore often used as moderator (including heavy water, water in which one of the ^1H hydrogens is replaced by ^2H, which is a more efficient moderator than ordinary water). Since it is inexpensive, graphite was often used in early reactors, but it takes a lot of it because it is less efficient. Moreover, graphite can ignite and is responsible for 2 of the 4 major nuclear power plant accidents. Ordinary water absorbs some neutrons

> so that to use it as a moderator, the uranium must be enriched a little (i.e. increase the proportion of uranium-235 to 2-3%). Generally, the core of a reactor is composed of uranium dioxide (UO2) pellets separated by moderator layers which forces the neutrons emitted by a pellet to pass through the moderator before reacting with another uranium-235 atom in another pellet.

Thus, the first reactor, built in the squash court of the University of Chicago in 1942 was formed of graphite bricks each containing 2 large pellets of natural uranium. They slowly removed the cadmium bars one by one until the reaction was at a constant rate. After 28 minutes, they stopped everything by inserting the cadmium bars again.

The Atomic Bomb

If the chain reaction happens quickly, without being controlled, the result is a violent explosion and an atomic bomb is obtained.

The chain reaction can go very quickly if it is done with a large proportion of uranium-235 and without any control bar. It can also be obtained with plutonium-239, which is also a fissionable nucleus. What makes it difficult to make the bomb is to get one of these two isotopes. Uranium-235 makes up only 0.7% of natural uranium. To make a bomb, the proportion of uranium-235 must be increased (up to 85%). This task is very difficult since the 235 and 238 isotopes have the same chemical properties, making every chemical technic useless. Other technics must be used, such as centrifugation, to slowly increase the proportion of uranium-235 to obtain enriched uranium. As for plutonium, it doesn't even exist naturally. It can, however, be produced with a nuclear power plant, because the absorption of neutrons by uranium-238 eventually produce plutonium 239 after two beta decays. The design of plutonium bombs is also more complex.

On July 16, 1945, the first atomic bomb was tested in New Mexico. The atomic bomb was then used on the cities of Hiroshima (August 6, 1945) and Nagasaki (August 9, 1945). Here's a video of atomic bomb tests in the bikini atoll in 1946.
http://www.youtube.com/watch?v=nZCFrnL3W5A
The energy released in these tests was about 23 kilotons each (equivalent to 23,000 tons of TNT), about equivalent to the energy released by each of the Hiroshima's and Nagasaki's bombs.

Here are two movies showing the effects of an atomic explosion on different structures.
http://www.youtube.com/watch?v=tr76hNngqts
http://www.youtube.com/watch?v=QsB83fAtNQE
The two main effects of the explosion, heat and blast, can readily be seen.

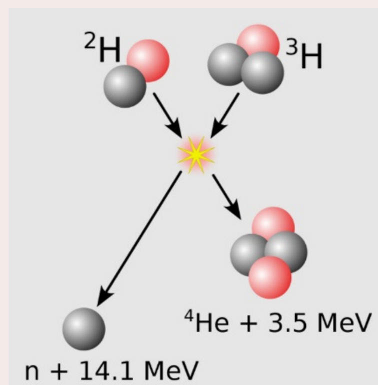
Radioactive Waste

After fission, the resulting nuclei are usually very radioactive. As the large fissile nuclei have a greater proportion of neutrons than the smaller nuclei, the mid-sized nuclei obtained after fission have an excess of neutrons. These nuclei tend to make a beta – decay to decrease the number of neutrons in the nucleus. These radioactive substances produced in a nuclear power plant must then be safely stored. These substances also contaminate the entire region where an atomic explosion or an accident in a plant occurs. As some of these products have very long half-lives, the area is contaminated for a very long time.

12.7 NUCLEAR FUSION

Energy can also be obtained by merging small nuclei to make larger ones. This is called *nuclear fusion*. Generally, this reaction releases energy if a nucleus with an atomic mass closer to 56 is formed (although there are many exceptions) since this nucleus has the highest binding energy per nucleon.

Example 12.7.1



simple.wikipedia.org/wiki/Nuclear_fusion

What is the energy released by this reaction?

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$$

(Observe that the number of protons and the number of neutrons are the same before and after the reaction. This is always true for fusion.)

The energy is

$$\begin{aligned} Q &= \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= \left(\left(m_{H2} + m_{H3}\right) - \left(m_{He} + m_{n}\right)\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= \begin{pmatrix} \left(2.014\ 101\ 778u + 3.016\ 049\ 278u\right) - \\ \left(4.002\ 603\ 254u + 1.008\ 664\ 916u\right) \end{pmatrix} \cdot 931.5 \tfrac{MeV}{u} \\ &= \left(0.018\ 882\ 886u\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= 17.6 MeV \end{aligned}$$

The H-Bomb

It was quickly realized that hydrogen fusion could increase the power of the atomic bombs. During the explosion of a fission bomb, the temperature becomes so high that hydrogen

atoms can merge and release additional energy. They thus managed to develop, in 1952, the hydrogen bomb (H-bomb) whose power is much greater than a simple fission bomb like the Hiroshima bomb. The largest bomb having been tested so far released 50 Mt of energy or about 2,500 times the energy of the Hiroshima bomb. Here are 2 videos showing H-bomb tests.
https://www.youtube.com/watch?v=yEje927dygM
https://www.youtube.com/watch?v=_D9c9kQDMis

Nuclear Fusion Power Plant

The reaction shown in the previous example is one of the reactions they are trying to produce at large scale for future nuclear fusion power plants. It's still difficult because nuclei must have sufficient speed to offset the electrical repulsion between them and get close enough to merge. As the speed of the nuclei is increased by increasing the temperature, the gas must be heated up to several tens of millions of °C! Obviously, this is a big problem: how can a gas be heated to such a high temperature and how can it be contained? There are some options, but none of them is efficient enough up to now to have a working fusion power plant.

When fusion plants will be operational (if they are one day), there are good chances that the energy crises will disappear and that the problems of CO_2 in the atmosphere will be, at least in part, resolved. To illustrate why this is so, let's calculate how long can an average Quebec home be supplied with energy with 100 grams of hydrogen (half are deuterium atoms, and half are tritium atoms). The number of atoms is

$$\begin{aligned} N &= \frac{100g}{\text{average molar mass}} \cdot N_A \\ &= \frac{100g}{2.5\frac{g}{mol}} \cdot 6.022\times 10^{23} \\ &= 2.4\times 10^{25} \end{aligned}$$

As 2 atoms are needed to make one reaction, there will be 1.2 x 10^{25} fusion reactions. The resulting energy is, therefore,

$$\begin{aligned} E &= 1.2\times 10^{25}\,fusions \cdot 17.6\frac{MeV}{fusion} \\ &= 2.1\times 10^{32}\,eV \\ &= 3.38\times 10^{13}\,J \end{aligned}$$

As an average home consumes 10^{11} J per year (30,000 kWh), energy can be provided for

$$time = \frac{3.38\times 10^{13} J}{10^{11} \frac{J}{year}} = 338\, years$$

Not bad for only 100 grams of hydrogen, isn't it? All this energy is generated without producing any $C0_2$ or radioactive substances! (In reality, neutrons generated in this reaction will hit the walls of the reactor and the absorption of these neutrons by atoms of the wall will make them radioactive…) The isotopes, such as deuterium, necessary for the fusion are simply taken from sea water. The proportion of these isotopes is not very high, but there are enough in each m³ of water to provide as much energy as 700 tons of oil.

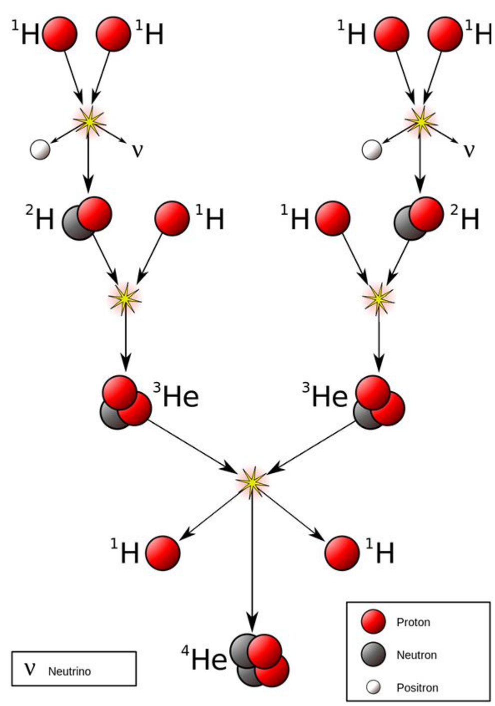
The first controlled deuterium-tritium fusion was realized at the JET in England on November 9, 1991. For the first time in 1996, at the JT-60 in Japan, more energy was obtained by a fusion reaction than the energy that had to be provided to operate the plant. Currently, an important project is under construction (ITER) in the South of France. It was designed to test the technologies needed for the first real fusion power plant (DEMO). ITER will be operational in 2020 (construction cost is $16 billion!) and DEMO should be operational by 2040.

The Sun

Following the discovery of the law of conservation of energy (1868), physicists were looking for the source of the Sun's energy. Unfortunately, no source of energy was found that would allow the Sun to shine for a few billion years. When the first nuclear reactions were made, Arthur Eddington suggested, in 1920, that these kinds of reactions were possibly the source of the Sun's energy. The details of the reaction occurring inside the Sun were discovered by Hans Bethe in 1939.

The energy of the sun comes from the fusion hydrogen nuclei, the most abundant nucleus in the Sun, into helium. The most common reaction (because there are different ways to do it) in the centre of the Sun is the one shown in this diagram.

This reaction releases 26.7 MeV. 632 million tons of hydrogen fuse in the Sun every second, thus releasing 3.83 x 10^{26} joules. 2 neutrinos are also released in each of these reactions. These neutrinos get out of the Sun almost immediately. About 10^{15} (one million billion!) of these emitted neutrinos pass through your body every second without having any effect on you.



en.wikipedia.org/wiki/Nuclear_fusion

It is then possible to say that much of the energy we use comes from nuclear fusion because almost all energy sources on Earth ultimately comes from solar energy. (The only exceptions I think of are nuclear fission and geothermal energy.)

SUMMARY OF EQUATIONS

Number of Nucleons in a Nucleus

$$A = Z + N$$

Atomic Mass Unit

$$1\ u = 1.660\ 539 \times 10^{-27}\ kg$$

Radius of the Nucleus

$$r_{nucleus} = 1.2\, fm \cdot \sqrt[3]{A}$$

Nuclear Energy

$$Q = \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{MeV}{u}$$

The energy is in MeV and the masses are in atomic mass units.

Binding Energy of an Element X Nucleus

$$E_{binding} = \left(Zm_{H1} + Nm_n - m_X\right) \cdot 931.5\ ^{MeV}/_{u}$$

Alpha Decay

$$^{A}_{Z}X \rightarrow\ ^{A-4}_{Z-2}Y +\ ^{4}_{2}He$$

Energy Released in an Alpha Decay

$$Q = \left(m_X - m_Y - m_{He4}\right) \cdot 931.5 \tfrac{MeV}{u}$$

Energy of the Alpha Particle After an Alpha Decay

$$E_{k\ He4} = \frac{m_Y}{m_{He4} + m_Y} Q$$

β^- Decay

$$^{A}_{Z}X \rightarrow\ ^{A}_{Z+1}Y + e^- + \overline{\nu}$$

Energy Released in a β^- Decay

$$Q = (m_X - m_Y) \cdot 931.5 \tfrac{MeV}{u}$$

β^+ Decay

$${}_{Z}^{A}X \to {}_{Z-1}^{A}Y + e^+ + \nu$$

Energy Released in a β^+ Decay

$$Q = (m_X - m_Y - 2m_e) \cdot 931.5 \tfrac{MeV}{u}$$

Electron Capture

$${}_{Z}^{A}X \to {}_{Z-1}^{A}Y + \nu$$

Energy released in an Electron Capture

$$Q = (m_X - m_Y) \cdot 931.5 \tfrac{MeV}{u}$$

Activity of a Radioactive Substance

$$R = \lambda N$$

Number of Nucleus Remaining as a Function of Time

$$N = N_0 e^{-\lambda t}$$

Activity as a Function of Time

$$R = R_0 e^{-\lambda t}$$

The Becquerel

$$1Bq = 1 \text{ decay per second}$$

The Curie

$$1Ci = 3.7 \times 10^{10} \text{ decays per second}$$

Half-life

$$T_{½} = \frac{\ln 2}{\lambda}$$

Initial Activity of One Gram of Natural Carbon

$$R_0 = 0.25 Bq$$

Energy Released in a Nuclear Reaction

$$Q = \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{MeV}{u}$$

EXERCISES

12.1 Atomic Nuclei

1. How many protons and neutrons are there in the following nuclei?

   a) ${}^{39}_{18}Ar$
   b) ${}^{180}_{76}Os$

2. What is the radius of the following nuclei?

   a) ${}^{18}_{8}O$
   b) ${}^{235}_{92}U$

3. What would be the radius of the Earth (m = 6 x 10^{24} kg) if it had the same density as a ${}^{12}_{6}C$ nuclei?

12.2 Nuclear Energy

Atomic Mass table
http://physique.merici.ca/ondes/atomicmasses.pdf

4. How much energy is required to remove a neutron from a ${}^{26}_{12}Mg$ nucleus?

5. How much energy is required to remove a proton from a ${}^{26}_{12}Mg$ nucleus?

6. What is the binding energy of the following nuclei?

a) ${}^{36}_{13}Al$

b) ${}^{204}_{84}Po$

7. The two nuclei ${}^{60}_{29}Cu$ and ${}^{60}_{27}Co$ have the same number of nucleons, but this does not mean that their binding energy is exactly the same. Check this by calculating the binding energy of these two nuclei.

12.3 Radioactivity

8. Let's look at the alpha decay of xenon-112.

a) Write the complete reaction of the α decay of ${}^{112}_{54}Xe$.
b) Calculate the energy released by this decay.
c) What is the kinetic energy of the alpha particle emitted in this decay?

9. Write the complete reaction of the β^- decay of ${}^{85}_{36}Kr$, and calculate the energy released by this decay.

10. Write the complete reaction of the β^+ decay of ${}^{40}_{19}K$, and calculate the energy released by this decay.

11. Write the complete reaction of an electron capture in ${}^{29}_{15}P$, and calculate the energy released by this decay.

12. Write the complete reaction of the γ decay of ${}^{49}_{24}Cr$.

13. Determine if the following decays are possible.

a) The β^- decay of ${}^{25}_{10}Ne$.
b) The β^+ decay of ${}^{28}_{13}Al$.

12.4 The Law of Radioactive Decay

14. Initially, there are 2 mg of strontium-83 whose half-life is 32.41 hours.

   a) What is the initial number of strontium-83 nuclei in this sample?
   b) How many strontium-83 nuclei will be left in this sample in 72 hours?

15. Polonium-208 decays into lead-204 with a half-life of 2.898 years. Initially, there is 1 g of pure polonium. How many grams of lead are there 10 years later?

16. Initially, there are 5 µg of polonium-201 whose half-life is 15.3 minutes.

   a) What is the initial activity of this sample (in Ci)?
   b) What will the activity of this sample be in 1 hour?

17. Initially, a radioactive substance has an activity of 20 µCi. 48 hours later, the activity of the same sample is 16.9 µCi. What is the half-life of the substance?

18. Initially, we have 10 g of radium 227 which has a 42.2 minutes half-life. How many radium atoms will decay in the next 2 hours?

19. An archaeologist discovers a piece of wood from a very old drakkar. A sample of this piece of wood contains 20 g of carbon and has an activity of 4.4 Bq. From which century comes this piece of wood? (Reminder: the half-life of carbon-14 is 5730 years.)

20. A sample of pure radium-225 has an activity of 25 Ci. What is the mass of the sample if the half-life of radium-225 is 14.9 days?

21. Uranium-235 makes a sequence of decay that eventually leads to lead-207. How many α and β^- decays occur in this sequence of decays? (Note that there are only α, β^- and γ decays in this sequence.)

22. Rocks are sometimes dated using the decay by electron capture of potassium-40.

$$^{40}_{19}K \rightarrow ^{40}_{18}Ar + \nu$$

The half-life of this decay is 1.248 billion years. In a rock, the ratio of the number of argon atoms and the number of potassium atoms is 0.15. What is the age of the rock if it is assumed that the rock contained no argon initially? (This assumption makes sense because argon is a gas that can escape from the rock as long as it is liquid.)

12.5 Nuclear Reactions

23. What is the missing nucleus in each of these nuclear reactions?

1) $^{15}_{7}N + ^{1}_{1}H \rightarrow ? + ^{4}_{2}He$
2) $^{18}_{8}O + ^{1}_{1}H \rightarrow ^{18}_{9}F + ?$
3) $^{7}_{3}Li + ^{1}_{1}H \rightarrow ? + ^{1}_{0}n$
4) $^{10}_{5}B + ? \rightarrow ^{7}_{3}Li + ^{4}_{2}He$

24. What is the energy released or what is the energy required in the following reactions?

1) $^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$
2) $^{9}_{4}Be + ^{4}_{2}He \rightarrow ^{12}_{6}C + ^{1}_{0}n$
3) $^{27}_{13}Al + ^{4}_{2}He \rightarrow ^{30}_{15}P + ^{1}_{0}n$
4) $^{14}_{7}N + ^{1}_{0}n \rightarrow ^{14}_{6}C + ^{1}_{1}H$

12.6 Nuclear Fission

25. What is the missing nucleus in following fission reaction?

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ? + ^{90}_{37}Rb + 2\,^{1}_{0}n$$

26. What is the energy released by this reaction?

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{143}_{54}Xe + ^{90}_{38}Sr + 3\,^{1}_{0}n$$

27. The fission of a uranium-235 nucleus releases 200 MeV on average. For how long can electricity be supplied to a typical Quebec home which consumes about 250 MJ per day with only 100 g of uranium-235, assuming that all the fission energy is converted to electricity?

12.7 Nuclear Fusion

28. a) What is the energy released by the following deuterium-deuterium fusion reaction?

$$ {}_1^2H + {}_1^2H \rightarrow {}_2^3He + {}_0^1n $$

b) If one atom of hydrogen in 6500 is a deuterium atom in sea water, how much energy could be obtained from 1000 kg of sea water?

Challenges

(Questions more difficult than the exam questions.)

29. Radium 232 decays into actinium 232 with a half-life of 250 s. Then, actinium decays into thorium 232 with a half-life of 119 s. Initially, the sample is made of pure radium. What is the proportion of the atoms of each element after 300 s?

Hint: The number of actinium atoms is a function of the form $N_2 = Ae^{-\lambda_1 t} + Be^{-\lambda_2 t}$

30. A star with a mass of 5 x 10^{30} kg and a luminosity of 10^{27} W gets its power through the following nuclear reaction.

$$ {}^4He + {}^4He + {}^4He \rightarrow {}^{12}C $$

Initially, 30% of the mass of the star is made of helium. How long could this star shine if helium fusion is the only source of energy (assuming that the luminosity always remains the same)?

ANSWERS

12.1 Atomic Nuclei

1. a) 18 protons, 21 neutrons b) 76 protons, 104 neutrons
2. a) 3.14 fm b) 7.41 fm

3. 184 m

12.2 Nuclear Energy

4. 11.09 MeV
5. 14.14 MeV
6. a) 274.6 MeV b) 1599 MeV
7. Copper 519.9 Mev Cobalt 524.8 MeV

12.3 Radioactivity

8. a) ${}^{112}_{54}Xe \rightarrow {}^{108}_{52}Te + {}^{4}_{2}He$ b) Energy released = 3.33 MeV
   c) Energy of the alpha particle 3.21 MeV
9. ${}^{85}_{36}Kr \rightarrow {}^{85}_{37}Rb + e^{-} + \overline{\nu}$ Energy released = 0.687 MeV
10. ${}^{40}_{19}K \rightarrow {}^{40}_{18}Ar + e^{+} + \nu$ Energy released = 0.483 MeV
11. ${}^{29}_{15}P \rightarrow {}^{29}_{14}Si + \nu$ Energy released = 4.94 MeV
12. ${}^{49}_{24}Cr^{*} \rightarrow {}^{49}_{24}Cr + \gamma$
13. a) possible b) impossible

12.4 The Law of Radioactive Decay

14. a) 1.45×10^{19} b) 3.11×10^{18}
15. 0.891 g
16. a) 305.6 Ci b) 20.17 Ci
17. 197.5 h
18. 2.283×10^{22}
19. 10^{th} century
20. 642 μg
21. 7 alpha decays, 4 beta decays
22. 252 million years

12.5 Nuclear Reactions

23.
   1) ${}^{15}_{7}N + {}^{1}_{1}H \rightarrow {}^{12}_{6}C + {}^{4}_{2}He$
   2) ${}^{18}_{8}O + {}^{1}_{1}H \rightarrow {}^{18}_{9}F + {}^{1}_{0}n$
   3) ${}^{7}_{3}Li + {}^{1}_{1}H \rightarrow {}^{7}_{4}Be + {}^{1}_{0}n$
   4) ${}^{10}_{5}B + {}^{1}_{0}n \rightarrow {}^{7}_{3}Li + {}^{4}_{2}He$
24. 1) 1.19 MeV must be provided 2) Releases 5.70 MeV
    3) 2.64 MeV must be provided 4) Releases 0.626 MeV

12.6 Nuclear Fission

25. ${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{144}_{55}Cs + {}^{90}_{37}Rb + 2\,{}^{1}_{0}n$
26. 171.2 MeV
27. 89.9 years

12.7 Nuclear Fusion

28. a) 3.27 MeV b) 2.7×10^{12} J

Challenges

29. Radium: 43.53%, actinium: 23.71%, thorium: 32.76%
30. 2.78 billion years