

# Chapter 1 Solutions

1. The average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

With the distance in metre and the time in seconds, the velocity is

$$\begin{aligned}\bar{v} &= \frac{384\,400\,000m}{262\,140s} \\ &= 1466.4 \frac{m}{s} \\ &= 5279 \frac{km}{h}\end{aligned}$$

2. The average velocity of the *Deutschland* is found with

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

The average velocity is

$$23.15kn \cdot \frac{1.853 \frac{km}{h}}{1kn} = 42.9 \frac{km}{h} = 11.92 \frac{m}{s}$$

and the crossing time is

$$\begin{aligned}\Delta t &= 5j \cdot \frac{24h}{1j} \cdot \frac{60min}{1h} \cdot \frac{60s}{1min} + 11h \cdot \frac{60min}{1h} \cdot \frac{60s}{1min} + 54min \cdot \frac{60s}{1min} \\ &= 474\,840s\end{aligned}$$

Therefore, the displacement is

$$\begin{aligned}\bar{v} &= \frac{\Delta x}{\Delta t} \\ 11.92 \frac{m}{s} &= \frac{\Delta x}{474\,840s} \\ \Delta x &= 5\,658\,100m = 5658.1km\end{aligned}$$



With this distance, the crossing time of the Lusitania can be found. The average velocity of the Lusitania was

$$23.99kn \cdot \frac{1.853 \frac{km}{h}}{1kn} = 44.45 \frac{km}{h} = 12.35 \frac{m}{s}$$

The crossing time is then

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

$$12.35 \frac{m}{s} = \frac{5\,658\,100m}{\Delta t}$$

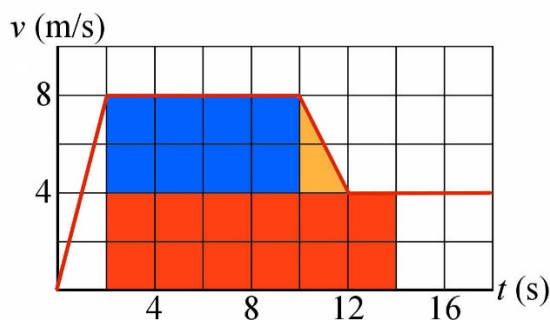
$$\Delta t = 458\,213s$$

The time difference is then

$$t = 474\,840s - 458\,213s = 16\,627s$$

This is 277 min and 7 seconds, so 4 h 37 min 7 s. Let's round up to 4 h 37 min.

**3.** The displacement is found with the area under the curve. This area is



The area of the red rectangle is  $4 \text{ m/s} \times 12 \text{ s} = 48 \text{ m}$ .

The area of the blue rectangle is  $4 \text{ m/s} \times 8 \text{ s} = 32 \text{ m}$ .

The area of the yellow triangle is  $\frac{1}{2} (4 \text{ m/s} \times 2 \text{ s}) = 4 \text{ m}$ .

As the total area is 84 m, the displacement is 84 m.

**4.** This is solved with



$$x = x_0 + vt$$

If Richard initial position was  $x = 0$ , the equation becomes

$$\begin{aligned} x &= x_0 + vt \\ 350m &= 0m + 1.389 \frac{m}{s} \cdot t \\ t &= 252s = 4 \text{ min } 12s \end{aligned}$$

**5.** The collision will occur in

$$\begin{aligned} t &= \frac{L}{v_1 - v_2} \\ t &= \frac{2000m}{13.89 \frac{m}{s} - (-19.44 \frac{m}{s})} \\ t &= 60s \end{aligned}$$

The French submarine position is then (if we set that its initial position was  $x = 0$  m)

$$\begin{aligned} x &= x_0 + vt \\ &= 0m + 13.89 \frac{m}{s} \cdot 60s \\ &= 833m \end{aligned}$$

The collision is 833 m to the right of the starting position of the French submarine.

**6.** Let's calculate first how long it takes Nicole to reach the car. Starting from  $x = 0$  m, she arrives at  $x = 100$  m with a velocity of 15 km/h. The time is therefore

$$\begin{aligned} x &= x_0 + vt \\ 100m &= 0m + 4.167 \frac{m}{s} \cdot t \\ t &= 24s \end{aligned}$$

Then, we must find out how much it will take for the bears to catch up with Nicole.

$$\begin{aligned} t &= \frac{L}{v_1 - v_2} \\ &= \frac{30m}{6.944 \frac{m}{s} - 4.167 \frac{m}{s}} \\ &= 10.8s \end{aligned}$$



As bears the catch up before she reaches the car, small Nicole must kindly give back the fish to the grizzly bears.

- 7.** Let's split this motion into two parts at a constant speed. If Dieudonné started at the position  $x = 0$  m, then its position at the end of the first part is

$$\begin{aligned}x &= x_0 + vt \\&= 0\text{km} + 110 \frac{\text{km}}{\text{h}} \cdot 4\text{h} \\&= 440\text{km}\end{aligned}$$

Using this position as the initial position of the second part, the position at the end of the second part is

$$\begin{aligned}x &= x_0 + vt \\&= 440\text{km} + 130 \frac{\text{km}}{\text{h}} \cdot 2\text{h} \\&= 700\text{km}\end{aligned}$$

His displacement is therefore 700 km.

His average velocity is then

$$\begin{aligned}\bar{v} &= \frac{\Delta x}{\Delta t} \\&= \frac{700\text{km}}{6\text{h}} \\&= 116.7 \frac{\text{km}}{\text{h}}\end{aligned}$$

- 8.** Let's split this motion into two parts at a constant speed. If Phil started at the position  $x = 0$  m, then its position at the end of the first part is

$$\begin{aligned}x &= x_0 + vt \\&= 0\text{m} + 30 \frac{\text{m}}{\text{s}} \cdot 80\text{s} \\&= 2400\text{m}\end{aligned}$$

The displacement during this part is then 2400 m.

Using this position as the initial position of the second part, the position at the end of the second part is



$$\begin{aligned}
 x &= x_0 + vt \\
 &= 2400m + \left(-20 \frac{m}{s}\right) \cdot 15s \\
 &= 2100m
 \end{aligned}$$

The displacement during this part is therefore

$$\Delta x = x_f - x_i = 2100m - 2400m = -300m$$

The total displacement is

$$\Delta x = x_f - x_i = 2100m - 0m = 2100m$$

The total distance is

$$d = 2400m + 300m = 2700m$$

The average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2100m}{95s} = 22.1 \frac{m}{s}$$

The average speed is

$$\text{Average speed} = \frac{\text{distance}}{\Delta t} = \frac{2700m}{95s} = 28.42 \frac{m}{s}$$

- 9.** Setting that the epicentre is at  $x = 0$  at the observer is at  $x = D$ , the arrival time of the primary waves is

$$\begin{aligned}
 x &= x_0 + vt \\
 D &= 0 + v_p t_p \\
 t_p &= \frac{D}{v_p}
 \end{aligned}$$

The arrival time of the secondary waves is

$$\begin{aligned}
 x &= x_0 + vt \\
 D &= 0 + v_s t_s \\
 t_s &= \frac{D}{v_s}
 \end{aligned}$$



These times are not known, but it is known that the secondary waves arrive 40 seconds after the primary waves. Therefore

$$t_s - t_p = 40s$$

This means that

$$\frac{D}{v_s} - \frac{D}{v_p} = 40s$$

$$D \left( \frac{1}{v_s} - \frac{1}{v_p} \right) = 40s$$

Using the values of the speeds, the distance can be found

$$D \left( \frac{1}{5 \frac{km}{s}} - \frac{1}{8 \frac{km}{s}} \right) = 40s$$

$$D = 533km$$

## 10. a)

At  $t = 0$  s, the position is  $x = 0$  m. At  $t = 9$  s, the position is  $x = -8$  m. The displacement is therefore

$$\Delta x = x_f - x_i = -8m - 0m = -8m$$

b) Let's calculate the displacement for each part where the speed is constant, so for each part where the slope is constant.

At  $t = 0$  s, the position is  $x = 0$  m. At  $t = 3$  s, the position is  $x = 8$  m. The displacement is therefore

$$\Delta x = x_f - x_i = 8m - 0m = 8m$$

At  $t = 3$  s, the position is  $x = 8$  m. At  $t = 3$  s, the position is  $x = 8$  m. The displacement is zero.

At  $t = 5$  s, the position is  $x = 8$  m. At  $t = 9$  s, the position is  $x = -8$  m. The displacement is therefore

$$\Delta x = x_f - x_i = -8m - 8m = -16m$$



The distance is the sum of the absolute values of the displacements.

$$\text{Distance} = 8 \text{ m} + 0 \text{ m} + 16 \text{ m} = 24 \text{ m}$$

- c) At  $t = 3 \text{ s}$ , the position is  $x = 8 \text{ m}$ . At  $t = 9 \text{ s}$ , the position is  $x = -8 \text{ m}$ . The displacement is therefore

$$\Delta x = x_f - x_i = -8\text{m} - 8\text{m} = -16\text{m}$$

The average velocity is therefore

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-16\text{m}}{9\text{s} - 3\text{s}} = -2.67 \frac{\text{m}}{\text{s}}$$

- d) The velocity at  $t = 1 \text{ s}$  is equal to the slope of the graph at  $t = 1 \text{ s}$ . As it is a straight line, any two points can be taken on this line to calculate the slope. We'll take the points  $(0 \text{ s}, 0 \text{ m})$  and  $(3 \text{ s}, 8 \text{ m})$ . The velocity is therefore

$$v = \text{slope} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{8\text{m} - 0\text{m}}{3\text{s} - 0\text{s}} = 2.67 \frac{\text{m}}{\text{s}}$$

- e) The velocity at  $t = 8 \text{ s}$  is equal to the slope of the graph at  $t = 8 \text{ s}$ . As it is a straight line, any two points can be taken on this line to calculate the slope. We'll take the points  $(7 \text{ s}, 0 \text{ m})$  and  $(9 \text{ s}, -8 \text{ m})$ . The velocity is therefore

$$v = \text{slope} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-8\text{m} - 0\text{m}}{9\text{s} - 7\text{s}} = -4 \frac{\text{m}}{\text{s}}$$

## 11. a)

To find the average velocity between  $t = 0 \text{ s}$  and  $t = 2 \text{ s}$ , we need to find the position at these two times. At  $t = 0 \text{ s}$ , the position is

$$\begin{aligned} x &= 4 \frac{\text{m}}{\text{s}^2} \cdot (0\text{s})^2 - 5 \frac{\text{m}}{\text{s}} \cdot 0\text{s} + 10\text{m} \\ &= 10\text{m} \end{aligned}$$

At  $t = 2 \text{ s}$ , the position is

$$\begin{aligned} x &= 4 \frac{\text{m}}{\text{s}^2} \cdot (2\text{s})^2 - 5 \frac{\text{m}}{\text{s}} \cdot 2\text{s} + 10\text{m} \\ &= 16\text{m} \end{aligned}$$



The average velocity is therefore

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{16m - 10m}{2s - 0s} = 3 \frac{m}{s}$$

- b) We find the velocity at  $t = 2$  s by calculating the value of the derivative at  $t = 2$  s. The derivative is

$$v = \frac{dx}{dt} = \frac{d\left(4 \frac{m}{s^2} \cdot t^2 - 5 \frac{m}{s} \cdot t + 10m\right)}{dt} = 8 \frac{m}{s^2} \cdot t - 5 \frac{m}{s}$$

At  $t = 2$  s, the velocity is therefore

$$v = 8 \frac{m}{s^2} \cdot 2s - 5 \frac{m}{s} = 11 \frac{m}{s}$$

**12.** The acceleration is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{27.78 \frac{m}{s} - 0 \frac{m}{s}}{6.1s} = 4.554 \frac{m}{s^2}$$

**13.** The acceleration is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{0 \frac{m}{s} - 33.33 \frac{m}{s}}{3.6s} = -9.26 \frac{m}{s^2}$$

**14. a)**

At  $t = 0$  s, the velocity is  $v = 0$  m/s. At  $t = 4$  s, the velocity is  $v = 8$  m/s. The average acceleration is therefore

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{8 \frac{m}{s} - 0 \frac{m}{s}}{4s - 0s} = 2 \frac{m}{s^2}$$

- b) At  $t = 10$  s, the velocity is  $v = 8$  m/s. At  $t = 12$  s, the velocity is  $v = 4$  m/s. The average acceleration is therefore

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{4 \frac{m}{s} - 8 \frac{m}{s}}{12s - 10s} = -2 \frac{m}{s^2}$$



- c) At  $t = 4$  s, the velocity is  $v = 8$  m/s. At  $t = 8$  s, the velocity is  $v = 8$  m/s. The average acceleration is therefore

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{8 \frac{m}{s} - 8 \frac{m}{s}}{8s - 4s} = 0 \frac{m}{s^2}$$

- d) The acceleration at  $t = 1$  s is equal to the slope of the graph at  $t = 1$  s. As it is a straight line, any two points can be taken on this line to calculate the slope. We'll take the points (0 s, 0 m/s) and (2 s, 8 m/s). The acceleration is therefore

$$v = slope = \frac{v_2 - v_1}{t_2 - t_1} = \frac{8 \frac{m}{s} - 0 \frac{m}{s}}{2s - 0s} = 4 \frac{m}{s^2}$$

- e) The acceleration at  $t = 14$  s is equal to the slope of the graph at  $t = 14$  s. As this is a straight line whose slope is zero, the acceleration is zero.

**15.** The answer is found with the concavity of the curve.

**16.** a)

To find the average acceleration between  $t = 0$  s and  $t = 1$  s, we need to find the velocity at these two times. Find these velocities we need, to derive the formula for the position to obtain the formula of velocity versus time. The derivative is

$$v = \frac{dx}{dt} = \frac{d\left(3 \frac{m}{s^3} \cdot t^3 - 8 \frac{m}{s^2} \cdot t^2 + 2 \frac{m}{s} \cdot t - 6m\right)}{dt} = 9 \frac{m}{s^3} \cdot t^2 - 16 \frac{m}{s^2} \cdot t + 2 \frac{m}{s}$$

At  $t = 0$  s, the velocity is

$$v = 9 \frac{m}{s^3} \cdot (0s)^2 - 16 \frac{m}{s^2} \cdot 0s + 2 \frac{m}{s} = 2 \frac{m}{s}$$

At  $t = 1$  s, the velocity is

$$v = 9 \frac{m}{s^3} \cdot (1s)^2 - 16 \frac{m}{s^2} \cdot 1s + 2 \frac{m}{s} = -5 \frac{m}{s}$$

The average acceleration is then

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-5 \frac{m}{s} - 2 \frac{m}{s}}{1s - 0s} = -7 \frac{m}{s^2}$$



- b) To find the acceleration, the formula for the velocity must be derived. The derivative is

$$a = \frac{dv}{dt} = \frac{d\left(9\frac{m}{s^3} \cdot t^2 - 16\frac{m}{s^2} \cdot t + 2\frac{m}{s}\right)}{dt} = 18\frac{m}{s^3} \cdot t - 16\frac{m}{s^2}$$

At  $t = 2$  s, the acceleration is

$$a = 18\frac{m}{s^3} \cdot 2s - 16\frac{m}{s^2} = 20\frac{m}{s^2}$$

- c) To find the jerk, the formula for the acceleration must be derived. The derivative is

$$j = \frac{da}{dt} = \frac{d\left(18\frac{m}{s^3} \cdot t - 16\frac{m}{s^2}\right)}{dt} = 18\frac{m}{s^3}$$

The jerk is always constant at  $18 \text{ m/s}^3$ .

**17.** a) The distance travelled is

$$\begin{aligned} x &= x_0 + v_0 t + \frac{1}{2} a t^2 \\ &= 0m + 0m + \frac{1}{2} \cdot 5\frac{m}{s^2} \cdot (6s)^2 \\ &= 90m \end{aligned}$$

- b) The velocity is

$$\begin{aligned} v &= v_0 + at \\ &= 0\frac{m}{s} + 5\frac{m}{s^2} \cdot 6s \\ &= 30\frac{m}{s} \end{aligned}$$

**18.** a) The time is given by this formula.

$$\begin{aligned} x &= x_0 + \frac{1}{2} (v_0 + v) t \\ 100m &= 0m + \frac{1}{2} \cdot \left(25\frac{m}{s} + 15\frac{m}{s}\right) \cdot t \\ t &= 5s \end{aligned}$$

- b) The acceleration is



$$v = v_0 + at$$

$$15 \frac{m}{s} = 25 \frac{m}{s} + a \cdot 5s$$

$$a = -2 \frac{m}{s^2}$$

c) The velocity at the third pole speed is given by

$$2a(x - x_0) = v^2 - v_0^2$$

$$2 \cdot \left(-2 \frac{m}{s^2}\right) \cdot (200m - 0m) = v^2 - \left(25 \frac{m}{s}\right)^2$$

$$v^2 = -175 \frac{m^2}{s^2}$$

As there is no solution, the car does not reach the third post.

**19.** a) The acceleration is found with

$$x = x_0 + v_0 t + \frac{1}{2} at^2$$

$$1600m = 1400m + 5 \frac{m}{s} \cdot 25s + \frac{1}{2} \cdot a \cdot (25s)^2$$

$$a = 0.24 \frac{m}{s^2}$$

b) The velocity at the end of the race is

$$v = v_0 + at$$

$$= 5 \frac{m}{s} + 0.24 \frac{m}{s^2} \cdot 25s$$

$$= 11 \frac{m}{s}$$

(Okay, this is a little fast to finish a 1600 m race...)

**20.** The distance is

$$2a(x - x_0) = v^2 - v_0^2$$

$$2 \cdot \left(-0.2 \frac{m}{s^2}\right) \cdot (x - 0m) = \left(9.65 \frac{m}{s}\right)^2 - \left(10 \frac{m}{s}\right)^2$$

$$x = 17.19m$$

**21.** The velocity is found with



$$x = x_0 + \frac{1}{2}(v_0 + v)t$$

$$30m = 0m + \frac{1}{2} \cdot (v_0 + 20 \frac{m}{s}) \cdot 1,2s$$

$$v_0 = 30 \frac{m}{s}$$

**22.** The acceleration is found with

$$2a(x - x_0) = v^2 - v_0^2$$

$$2 \cdot a \cdot (32m - 0m) = (24 \frac{m}{s})^2 - (30 \frac{m}{s})^2$$

$$a = -5.0625 \frac{m}{s^2}$$

The braking distance with an initial velocity of 42 m/s is then

$$2a(x - x_0) = v^2 - v_0^2$$

$$2 \cdot (-5.0625 \frac{m}{s^2}) \cdot (x - 0m) = (0 \frac{m}{s})^2 - (42 \frac{m}{s})^2$$

$$x = 174.2m$$

**23.** There are two parts to this movement: a first part where there's no acceleration, which lasts 0.5 s, and a second part where the car slows down.

The position at the end of the first part is

$$x = x_0 + v_0 t$$

$$= 0m + 30 \frac{m}{s} \cdot 0.5s$$

$$= 15m$$

This position is the initial position of the second part. Therefore

$$2a(x - x_0) = v^2 - v_0^2$$

$$2 \cdot (-4 \frac{m}{s^2}) \cdot (x - 15m) = (0 \frac{m}{s})^2 - (30 \frac{m}{s})^2$$

$$x = 127.5m$$

As the moose was 100 m from the car, she hits the moose.



- 24.** Let's find out how long it takes for each racer to complete the race. There are two phases to the movement of each racer: an acceleration phase and deceleration phase.

Didier's acceleration phase.

Didier's position at the end of this phase is

$$\begin{aligned}x_B &= x_{B0} + v_{B0}t + \frac{1}{2}a_B t^2 \\&= 0m + 0m + \frac{1}{2} \cdot 5 \frac{m}{s^2} \cdot (1.8s)^2 \\&= 8.1m\end{aligned}$$

Didier's velocity at the end of this phase is

$$\begin{aligned}v_B &= v_{B0} + a_B t^2 \\&= 0 \frac{m}{s} + 5 \frac{m}{s^2} \cdot 1.8s \\&= 9 \frac{m}{s}\end{aligned}$$

Didier's deceleration phase.

These values for the position and the velocity are taken as the initial values of the second phase of the movement. Therefore, the time to arrive at  $x = 100$  m is

$$\begin{aligned}x_B &= x_{B0} + v_{B0}t + \frac{1}{2}a_B t^2 \\100m &= 8.1m + 9 \frac{m}{s} \cdot t + \frac{1}{2} \cdot (-0.1 \frac{m}{s^2}) \cdot t^2 \\0m &= -91.9m + 9 \frac{m}{s} \cdot t - 0.05 \frac{m}{s^2} \cdot t^2\end{aligned}$$

The solutions to this quadratic equation are

$$\begin{aligned}t &= \frac{-9 \frac{m}{s} \pm \sqrt{\left(9 \frac{m}{s}\right)^2 - 4 \cdot (-0.05 \frac{m}{s^2}) \cdot (-91.9m)}}{2 \cdot (-0.05 \frac{m}{s^2})} \\&\nearrow t = 10.867s \\&\searrow t = 169.13s\end{aligned}$$

Only the first solution is good. The second is the time that would take Didier to return to the finish line if his negative acceleration continued forever. In this case, Didier would slow down continuously until it stops then go backwards to return to  $x = 100$  m after 169 s.



Didier therefore completed the second part of his race (the deceleration phase) in 10.867 s. The duration of his race is therefore  $1.8 \text{ s} + 10.867 \text{ s} = 12.667 \text{ s}$ .

Gilles's acceleration phase.

Gilles's position at the end of this phase is

$$\begin{aligned}x_G &= x_{G0} + v_{G0}t + \frac{1}{2}a_Gt^2 \\&= 0\text{m} + 0\text{m} + \frac{1}{2} \cdot 6 \frac{\text{m}}{\text{s}^2} \cdot (1.7\text{s})^2 \\&= 8.67\text{m}\end{aligned}$$

Gilles's velocity at the end of this phase is

$$\begin{aligned}v_G &= v_{G0} + a_Gt \\&= 0 \frac{\text{m}}{\text{s}} + 6 \frac{\text{m}}{\text{s}^2} \cdot 1.7\text{s} \\&= 10.2 \frac{\text{m}}{\text{s}}\end{aligned}$$

Gilles's deceleration phase.

These values for the position and the velocity are taken as the initial values of the second phase of the movement. Therefore, the time to arrive at  $x = 100 \text{ m}$  is

$$\begin{aligned}x_G &= x_{B0} + v_{B0}t + \frac{1}{2}a_Bt^2 \\100\text{m} &= 8.67\text{m} + 10.2 \frac{\text{m}}{\text{s}} \cdot t + \frac{1}{2} \cdot (-0.24 \frac{\text{m}}{\text{s}^2}) \cdot t^2 \\0\text{m} &= -91.33\text{m} + 10.2 \frac{\text{m}}{\text{s}} \cdot t - 0.12 \frac{\text{m}}{\text{s}^2} \cdot t^2\end{aligned}$$

The solutions to this quadratic equation are

$$t = \frac{-10.2 \frac{\text{m}}{\text{s}} \pm \sqrt{(10.2 \frac{\text{m}}{\text{s}})^2 - 4 \cdot (-0.12 \frac{\text{m}}{\text{s}^2}) \cdot (-91.33\text{m})}}{2 \cdot (-0.12 \frac{\text{m}}{\text{s}^2})}$$

$\nearrow t = 10.171\text{s}$

$\searrow t = 74.83\text{s}$

Once again, only the first solution is good.



Gilles therefore completed the second part of his race (the deceleration phase) in 10.171 s. The duration of his race is therefore  $1.7 \text{ s} + 10.171 \text{ s} = 11.871 \text{ s}$ .

Didier completed the race 12.667 s whereas Gilles completed the race 11.871 s. Gilles therefore wins by 0.796 s.

## 25. There are two phases to this movement.

First phase: only the first rocket is accelerating

$$a_1 = 5 \text{ m/s}^2 \quad a_2 = 0 \text{ m/s}^2 \quad \text{duration} = 2 \text{ s.}$$

At the end of this phase, the position and the velocity of the first rocket are

$$\begin{aligned} x_1 &= x_{10} + v_{10}t + \frac{1}{2}a_1t^2 \\ &= 0\text{m} + 0\text{m} + \frac{1}{2} \cdot 5 \frac{\text{m}}{\text{s}^2} \cdot (2\text{s})^2 \\ &= 10\text{m} \end{aligned}$$

$$\begin{aligned} v_1 &= v_{10} + a_1t \\ &= 0 \frac{\text{m}}{\text{s}} + 5 \frac{\text{m}}{\text{s}^2} \cdot 2\text{s} \\ &= 10 \frac{\text{m}}{\text{s}} \end{aligned}$$

As for rocket 2, it remains at  $x = 0 \text{ m}$  and its speed remains zero during this phase.

Second phase: both rockets are accelerating

$$a_1 = 5 \text{ m/s}^2 \quad a_2 = 6 \text{ m/s}^2$$

The positions of the rockets during this phase are

$$\begin{aligned} x_1 &= x_{10} + v_{10}t + \frac{1}{2}a_1t^2 & x_2 &= x_{20} + v_{20}t + \frac{1}{2}a_2t^2 \\ &= 10\text{m} + 10 \frac{\text{m}}{\text{s}} \cdot t + \frac{1}{2} \cdot 5 \frac{\text{m}}{\text{s}^2} \cdot t^2 & &= 0\text{m} + 0\text{m} + \frac{1}{2} \cdot 6 \frac{\text{m}}{\text{s}^2} \cdot t^2 \\ &= 10\text{m} + 10 \frac{\text{m}}{\text{s}} \cdot t + 2.5 \frac{\text{m}}{\text{s}^2} \cdot t^2 & &= 3 \frac{\text{m}}{\text{s}^2} \cdot t^2 \end{aligned}$$

When rocket 2 catches up with rocket 1, they are in the same place ( $x_1 = x_2$ ). Therefore



$$\begin{aligned}
 x_1 &= x_2 \\
 10m + 10 \frac{m}{s} \cdot t + 2.5 \frac{m}{s^2} \cdot t^2 &= 3 \frac{m}{s^2} \cdot t^2 \\
 10m + 10 \frac{m}{s} \cdot t - 0.5 \frac{m}{s^2} \cdot t^2 &= 0
 \end{aligned}$$

The solutions to the quadratic equation are

$$\begin{aligned}
 t &= \frac{-10 \frac{m}{s} \pm \sqrt{\left(10 \frac{m}{s}\right)^2 - 4 \cdot \left(-0.5 \frac{m}{s^2}\right) \cdot 10m}}{2 \cdot \left(-0.5 \frac{m}{s^2}\right)} \\
 \nearrow t &= -0.954s \\
 \searrow t &= 20.954s
 \end{aligned}$$

Rocket catches up with rocket 1 20.954 s after the departure of rocket 2 or 22.954 s after the departure of rocket 1.

The position of the rockets is then

$$\begin{aligned}
 x_2 &= 3 \frac{m}{s^2} t^2 \\
 &= 3 \frac{m}{s^2} \cdot (20.954s)^2 \\
 &= 1317m
 \end{aligned}$$

- 26.** The displacement by combining these 2 motions is 400 m. We will look for the speed reached after the acceleration phase.

For the acceleration phase, we have

$$\begin{aligned}
 2a_1 (x - x_0) &= v^2 - v_0^2 \\
 2 \cdot 2 \frac{m}{s^2} \cdot (x - 0) &= v_1^2 - 0 \\
 4 \frac{m}{s^2} \cdot x &= v_1^2
 \end{aligned}$$

For the deceleration phase, we have

$$\begin{aligned}
 2a_2 (x - x_0) &= v^2 - v_0^2 \\
 2 \cdot \left(-5 \frac{m}{s^2}\right) \cdot (400m - x) &= 0 - v_1^2 \\
 -10 \frac{m}{s^2} \cdot (400m - x) &= -v_1^2
 \end{aligned}$$

Thus, we have 2 equations and 2 unknowns.



$$4 \frac{m}{s^2} \cdot x = v_1^2$$

$$-10 \frac{m}{s^2} \cdot (400m - x) = -v_1^2$$

If we solve for  $x$  in the first equation

$$x = \frac{v_1^2}{4 \frac{m}{s^2}}$$

and replace this value in the second equation, we have

$$-10 \frac{m}{s^2} \cdot \left( 400m - \frac{v_1^2}{4 \frac{m}{s^2}} \right) = -v_1^2$$

The only thing that remains to do is to solve for the speed.

$$-10 \frac{m}{s^2} \cdot 400m + \frac{10}{4} \cdot v_1^2 = -v_1^2$$

$$-4000 \frac{m^2}{s^2} + 2.5 \cdot v_1^2 = -v_1^2$$

$$2.5 \cdot v_1^2 + v_1^2 = 4000 \frac{m^2}{s^2}$$

$$3.5 \cdot v_1^2 = 4000 \frac{m^2}{s^2}$$

$$v_1^2 = 1142.9 \frac{m^2}{s^2}$$

$$v_1 = 33.81 \frac{m}{s}$$

**27.** As the equation of motion is

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

we must have

$$5m = x_0 + 0m + 0m$$

$$5m = x_0 + (1s)v_0 + (0.5s^2)a$$

$$9m = x_0 + (2s)v_0 + (2s^2)a$$

The first equation gives us directly  $x_0 = 5$  m. The other two equations then become

$$5m = 5m + (1s)v_0 + (0.5s^2)a \qquad 9m = 5m + (2s)v_0 + (2s^2)a$$

$$0 = v_0 + (0.5s)a \qquad 4m = (2s)v_0 + (2s^2)a$$

We will solve for  $v_0$  in the equation to the left and the substitute it in the equation to the right



$$\begin{aligned}
 4m &= (2s)v_0 + (2s^2)a \\
 4m &= (2s)(-(0.5s)a) + (2s^2)a \\
 4m &= (-1s^2)a + (2s^2)a \\
 4m &= (1s^2)a \\
 a &= 4 \frac{m}{s^2}
 \end{aligned}$$

Therefore

$$\begin{aligned}
 0 &= v_0 + (0.5s)a \\
 v_0 &= -(0.5s)a \\
 v_0 &= -(0.5s) \cdot 4 \frac{m}{s^2} \\
 v_0 &= -2 \frac{m}{s}
 \end{aligned}$$

Therefore, the equation of motion is

$$\begin{aligned}
 x &= x_0 + v_0 t + \frac{1}{2}at^2 \\
 x &= 5m - 2 \frac{m}{s} \cdot t + 2 \frac{m}{s^2} \cdot t^2
 \end{aligned}$$

At  $t = 5$  s, the position is

$$\begin{aligned}
 x &= 5m - 2 \frac{m}{s} \cdot 5s + 2 \frac{m}{s^2} \cdot (5s)^2 \\
 &= 45m
 \end{aligned}$$

**28.** a) The velocity is given by (downwards positive axis,  $y_0 = 0$  at the start.)

$$\begin{aligned}
 2a(y - y_0) &= v^2 - v_0^2 \\
 2 \cdot 9.8 \frac{m}{s^2} \cdot (12m - 0m) &= v^2 - \left(0 \frac{m}{s}\right)^2 \\
 v &= 15.34 \frac{m}{s}
 \end{aligned}$$

b) The free-fall time is (downwards positive axis,  $y_0 = 0$  at the start.)



$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$12m = 0m + 0m + \frac{1}{2} \cdot 9.8 \frac{m}{s^2} \cdot t^2$$

$$t = 1.565s$$

**29.** (In this problem, an upwards positive axis is used,  $y_0 = 0$  at the start.)

a) At the maximum height, the velocity is zero. Therefore

$$2a(y - y_0) = v^2 - v_0^2$$

$$2 \cdot \left(-9.8 \frac{m}{s^2}\right) \cdot (y - 0m) = \left(0 \frac{m}{s}\right)^2 - \left(28 \frac{m}{s}\right)^2$$

$$y = 40m$$

b) The time is found with the following formula.

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$25m = 0m + 28 \frac{m}{s} \cdot t + \frac{1}{2} \cdot \left(-9.8 \frac{m}{s^2}\right) \cdot t^2$$

The solutions to this quadratic equation are

$$t = 1.108 \text{ s and } t = 4.607s$$

Both solutions are good.

c) The time is found with the following formula.

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$-25m = 0m + 28 \frac{m}{s} \cdot t + \frac{1}{2} \cdot \left(-9.8 \frac{m}{s^2}\right) \cdot t^2$$

The solutions to this quadratic equation are

$$t = -0.785s \text{ s and } t = 6.499s$$

Only the second solution is good.

d) This is solved with the following formula



$$2a(y - y_0) = v^2 - v_0^2$$

$$2 \cdot (-9.8 \frac{m}{s^2}) \cdot (20m - 0m) = v^2 - (28 \frac{m}{s})^2$$

$$v = \pm 19.8 \frac{m}{s}$$

Both solutions are good.

e) There are two possibilities here since the velocity can be positive or negative.

If the velocity is positive, we have

$$v = v_0 + at$$

$$10 \frac{m}{s} = 28 \frac{m}{s} + (-9.8 \frac{m}{s^2}) \cdot t$$

$$t = 1.837s$$

If the velocity is negative, we have

$$v = v_0 + at$$

$$-10 \frac{m}{s} = 28 \frac{m}{s} + (-9.8 \frac{m}{s^2}) \cdot t$$

$$t = 3.878s$$

f) There are two possibilities here since the velocity can be positive or negative.

If the velocity is positive, we have

$$2a(y - y_0) = v^2 - v_0^2$$

$$2 \cdot (-9.8 \frac{m}{s^2}) \cdot (y - 0m) = (12 \frac{m}{s})^2 - (28 \frac{m}{s})^2$$

$$y = 32.65m$$

If the velocity is negative, we have

$$2a(y - y_0) = v^2 - v_0^2$$

$$2 \cdot (-9.8 \frac{m}{s^2}) \cdot (y - 0m) = (-12 \frac{m}{s})^2 - (28 \frac{m}{s})^2$$

$$y = 32.65m$$

This is the same height.

g) The velocity is found with



$$\begin{aligned}
 2a(y - y_0) &= v^2 - v_0^2 \\
 2 \cdot \left(-9.8 \frac{m}{s^2}\right) \cdot (-80m - 0m) &= v^2 - \left(28 \frac{m}{s}\right)^2 \\
 v &= -48.50 \frac{m}{s}
 \end{aligned}$$

Only the negative answer is kept, because the stone is moving downwards.

h) The flight time is

$$\begin{aligned}
 v &= v_0 + at \\
 -48.50 \frac{m}{s} &= 28 \frac{m}{s} + \left(-9.8 \frac{m}{s^2}\right) \cdot t \\
 t &= 7.806s
 \end{aligned}$$

**30.** (In this problem, a downwards positive axis is used,  $y_0 = 0$  at the start.)

We have

$$\begin{aligned}
 2a(y - y_0) &= v^2 - v_0^2 \\
 2 \cdot 9.8 \frac{m}{s^2} \cdot (10m - 0m) &= \left(24 \frac{m}{s}\right)^2 - v_0^2 \\
 v &= \pm 19.49 \frac{m}{s}
 \end{aligned}$$

Both answers are good. Tryphon can have thrown the balloon upwards or downwards.

**31.** (In this problem, an upwards positive axis is used,  $y_0 = 0$  at the start.)

The velocity is found with

$$\begin{aligned}
 2a(y - y_0) &= v^2 - v_0^2 \\
 2 \cdot \left(-9.8 \frac{m}{s^2}\right) \cdot (80m - 0m) &= \left(0 \frac{m}{s}\right)^2 - v_0^2 \\
 v &= 39.6 \frac{m}{s}
 \end{aligned}$$

**32.** (In this problem, an upwards positive axis is used,  $y_0 = 0$  at the start.)

We have



$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$0m = 0m + v_0 \cdot t + \frac{1}{2} \cdot \left(-9.8 \frac{m}{s^2}\right) \cdot t^2$$

$$v_0 \cdot t = \frac{1}{2} \cdot 9.8 \frac{m}{s^2} \cdot t^2$$

$$v_0 = \frac{1}{2} \cdot 9.8 \frac{m}{s^2} \cdot t$$

$$v_0 = \frac{1}{2} \cdot 9.8 \frac{m}{s^2} \cdot 12s$$

$$v_0 = 58.8 \frac{m}{s}$$

**33.** (In this problem, an upwards positive axis is used,  $y_0 = 0$  at the start.)

This problem is solved with

$$2a(y - y_0) = v^2 - v_0^2$$

$$2 \cdot \left(-9.8 \frac{m}{s^2}\right) \cdot (5m - 0m) = (0.3v_0)^2 - v_0^2$$

$$2 \cdot \left(-9.8 \frac{m}{s^2}\right) \cdot (5m - 0m) = (0.3)^2 v_0^2 - v_0^2$$

$$2 \cdot \left(-9.8 \frac{m}{s^2}\right) \cdot (5m - 0m) = \left((0.3)^2 - 1\right) v_0^2$$

$$v_0 = 10.38 \frac{m}{s}$$

**34.** (In this problem, an upwards positive axis is used,  $y_0 = 0$  at the start.)

There are two phases to this movement: a  $4 \text{ m/s}^2$  upwards acceleration for 20 seconds, and then a  $9.8 \text{ m/s}^2$  downwards acceleration when the motors stop.

First phase:  $a = 4 \text{ m/s}^2$

At the end of this phase, we have



$$\begin{aligned}
 x &= x_0 + v_0 t + \frac{1}{2} a t^2 \\
 &= 0m + 0m + \frac{1}{2} \cdot 4 \frac{m}{s^2} \cdot (20s)^2 \\
 &= 800m
 \end{aligned}$$

$$\begin{aligned}
 v &= v_0 + at \\
 0 \frac{m}{s} + 4 \frac{m}{s^2} \cdot 20s \\
 &= 80 \frac{m}{s}
 \end{aligned}$$

Second phase:  $a = -9.8 \text{ m/s}^2$

The final values of the phase 1 are the initial values for this second phase.

At the highest point, the speed is zero. The maximum height is then found with

$$\begin{aligned}
 2a(y - y_0) &= v^2 - v_0^2 \\
 2 \cdot (-9.8 \frac{m}{s^2}) \cdot (y - 800m) &= (0 \frac{m}{s})^2 - (80 \frac{m}{s})^2 \\
 y &= 1127m
 \end{aligned}$$

The time elapsed during this phase so that the rocket is back to the ground is

$$\begin{aligned}
 y &= y_0 + v_0 t + \frac{1}{2} a t^2 \\
 0m &= 800m + 80 \frac{m}{s} \cdot t + \frac{1}{2} \cdot (-9.8 \frac{m}{s^2}) \cdot t^2
 \end{aligned}$$

The solutions to this quadratic equation are

$$t = -6.999 \text{ s and } t = 23.326 \text{ s}$$

Only the positive response is good.

The total flight time is the sum of the times of the two phases of the movement.  
Therefore, the total time is  $20 \text{ s} + 23.326 \text{ s} = 43.326 \text{ s}$ .

**35.** (In this problem, a downwards positive axis is used,  $y_0 = 0$  at the start.)

The position and the velocity of Kim's ball after 1 s are



$$\begin{aligned}
 y &= y_0 + v_0 t + \frac{1}{2} a t^2 \\
 &= 0m + 0m + \frac{1}{2} \cdot 9.8 \frac{m}{s^2} \cdot (1s)^2 \\
 &= 4.9m
 \end{aligned}$$

$$\begin{aligned}
 v &= v_0 + at \\
 &= 0 \frac{m}{s} + 9.8 \frac{m}{s^2} \cdot 1s \\
 &= 9.8 \frac{m}{s}
 \end{aligned}$$

Then began a second phase when Leo throws his ball. The position of the two balls as a function of time is then

$$\begin{aligned}
 x_K &= x_{K0} + v_{K0}t + \frac{1}{2} a_K t^2 & x_L &= x_{L0} + v_{L0}t + \frac{1}{2} a_L t^2 \\
 &= 4.9m + 9.8 \frac{m}{s} \cdot t + \frac{1}{2} \cdot 9.8 \frac{m}{s^2} \cdot t^2 & &= 0m + 12 \frac{m}{s} t + \frac{1}{2} \cdot 9.8 \frac{m}{s^2} \cdot t^2 \\
 &= 4.9m + 9.8 \frac{m}{s} \cdot t + 4.9 \frac{m}{s^2} \cdot t^2 & &= 12 \frac{m}{s} t + 4.9 \frac{m}{s^2} \cdot t^2
 \end{aligned}$$

When Leon's ball catches up with Kim's ball, the two positions are equal. We therefore have

$$\begin{aligned}
 x_K &= x_L \\
 4.9m + 9.8 \frac{m}{s} \cdot t + 4.9 \frac{m}{s^2} \cdot t^2 &= 12 \frac{m}{s} \cdot t + 4.9 \frac{m}{s^2} \cdot t^2 \\
 4.9m + 9.8 \frac{m}{s} \cdot t &= 12 \frac{m}{s} \cdot t \\
 4.9m &= 2.2 \frac{m}{s} \cdot t \\
 t &= 2.227s
 \end{aligned}$$

Leon's ball catches up Kim's ball 2.227 s following the departure of Leon's ball, therefore 3.227 s after the departure of Kim's ball.

Then, the position of Leon's ball is

$$\begin{aligned}
 x_L &= 12 \frac{m}{s} t + 4.9 \frac{m}{s^2} t^2 \\
 &= 12 \frac{m}{s} \cdot 2.227s + 4.9 \frac{m}{s^2} \cdot (2.227s)^2 \\
 &= 51.03m
 \end{aligned}$$

The ball therefore catches up with Kim's ball before hitting the ground (which is at  $y = 400$  m).



The ball is then at a height of  $400 \text{ m} - 51.03 \text{ m} = 348.97 \text{ m}$ .

**36.** (In this problem, an upwards positive axis is used,  $y_0 = 0$  at the ground.)

a) The position as function of time of the two balls is

$$\begin{aligned}x_J &= x_{J0} + v_{J0}t + \frac{1}{2}a_Jt^2 & x_F &= x_{F0} + v_{F0}t + \frac{1}{2}a_Ft^2 \\&= 51\text{m} + \left(-5\frac{\text{m}}{\text{s}}\right) \cdot t + \frac{1}{2} \cdot \left(-9.8\frac{\text{m}}{\text{s}^2}\right) \cdot t^2 & &= 1\text{m} + 15\frac{\text{m}}{\text{s}} \cdot t + \frac{1}{2} \cdot \left(-9.8\frac{\text{m}}{\text{s}^2}\right) \cdot t^2 \\&= 51\text{m} - 5\frac{\text{m}}{\text{s}} \cdot t - 4.9\frac{\text{m}}{\text{s}^2} \cdot t^2 & &= 1\text{m} + 15\frac{\text{m}}{\text{s}} \cdot t - 4.9\frac{\text{m}}{\text{s}^2} \cdot t^2\end{aligned}$$

When Johnny's ball hit Frederique's ball, the two positions are equal. We then have

$$\begin{aligned}x_J &= x_F \\51\text{m} - 5\frac{\text{m}}{\text{s}} \cdot t - 4.9\frac{\text{m}}{\text{s}^2} \cdot t^2 &= 1\text{m} + 15\frac{\text{m}}{\text{s}} \cdot t - 4.9\frac{\text{m}}{\text{s}^2} \cdot t^2 \\51\text{m} - 5\frac{\text{m}}{\text{s}} \cdot t &= 1\text{m} + 15\frac{\text{m}}{\text{s}} \cdot t \\50\text{m} &= 20\frac{\text{m}}{\text{s}} \cdot t \\t &= 2.5\text{s}\end{aligned}$$

b) Then, the position of the balls is

$$\begin{aligned}x_J &= 51\text{m} - 5\frac{\text{m}}{\text{s}} \cdot t - 4.9\frac{\text{m}}{\text{s}^2} t^2 \\&= 51\text{m} - 5\frac{\text{m}}{\text{s}} \cdot 2.5\text{s} - 4.9\frac{\text{m}}{\text{s}^2} \cdot (2.5\text{s})^2 \\&= 7.875\text{m}\end{aligned}$$

c) The velocities of the balls are

$$\begin{aligned}v_J &= v_{J0} + a_Jt & v_F &= v_{F0} + a_Ft \\&= -5\frac{\text{m}}{\text{s}} + \left(-9.8\frac{\text{m}}{\text{s}^2}\right) \cdot 2.5\text{s} & &= 15\frac{\text{m}}{\text{s}} + \left(-9.8\frac{\text{m}}{\text{s}^2}\right) \cdot 2.5\text{s} \\&= -29.5\frac{\text{m}}{\text{s}} & &= -9.5\frac{\text{m}}{\text{s}}\end{aligned}$$

It can be noted that the two balls are moving downwards. Frederique's ball had time to reach its maximum height and begin to descend before being hit by Johnny's ball.

**37.**



- a) To obtain the position from the formula of the velocity, we must integrate. We then have

$$\begin{aligned}
 x &= \int v dt \\
 &= \int \left( 2 \frac{m}{s^4} \cdot t^3 - 6 \frac{m}{s^3} \cdot t^2 + 4 \frac{m}{s^2} \cdot t + 2 \frac{m}{s} \right) dt \\
 &= 0.5 \frac{m}{s^4} \cdot t^4 - 2 \frac{m}{s^3} \cdot t^3 + 2 \frac{m}{s^2} \cdot t^2 + 2 \frac{m}{s} \cdot t + C
 \end{aligned}$$

Since the position at  $t = 0$  is  $x_0 = 5$  m. We have

$$\begin{aligned}
 5m &= 0.5 \frac{m}{s^4} \cdot (0s)^4 - 2 \frac{m}{s^3} \cdot (0s)^3 + 2 \frac{m}{s^2} \cdot (0s)^2 + 2 \frac{m}{s} \cdot (0s) + C \\
 5m &= C
 \end{aligned}$$

The formula for the position is therefore

$$v = 0.5 \frac{m}{s^4} \cdot t^4 - 2 \frac{m}{s^3} \cdot t^3 + 2 \frac{m}{s^2} \cdot t^2 + 2 \frac{m}{s} \cdot t + 5m$$

At  $t = 2$  s, the position is

$$\begin{aligned}
 v &= 0.5 \frac{m}{s^4} \cdot (2s)^4 - 2 \frac{m}{s^3} \cdot (2s)^3 + 2 \frac{m}{s^2} \cdot (2s)^2 + 2 \frac{m}{s} \cdot 2s + 5m \\
 &= 8m - 16m + 8m + 4m + 5m \\
 &= 9m
 \end{aligned}$$

- b) To obtain the acceleration, the derivative of formula of the velocity must be performed. The result is

$$\begin{aligned}
 a &= \frac{dv}{dt} \\
 &= \frac{d \left( 2 \frac{m}{s^4} \cdot t^3 - 6 \frac{m}{s^3} \cdot t^2 + 4 \frac{m}{s^2} \cdot t + 2 \frac{m}{s} \right)}{dt} \\
 &= 6 \frac{m}{s^4} \cdot t^2 - 12 \frac{m}{s^3} \cdot t + 4 \frac{m}{s^2}
 \end{aligned}$$

At  $t = 2$  s, the acceleration is

$$\begin{aligned}
 a &= 6 \frac{m}{s^4} \cdot (2s)^2 - 12 \frac{m}{s^3} \cdot 2s + 4 \frac{m}{s^2} \\
 &= 24 \frac{m}{s^2} - 24 \frac{m}{s^2} + 4 \frac{m}{s^2} \\
 &= 4 \frac{m}{s^2}
 \end{aligned}$$



**38.** a) The velocity is found by integrating the acceleration.

$$\begin{aligned} v &= \int a dt \\ &= \int \left( 36 \frac{m}{s^4} \cdot t^2 + 10 \frac{m}{s^2} \right) dt \\ &= 12 \frac{m}{s^4} \cdot t^3 + 10 \frac{m}{s^2} \cdot t + cst \end{aligned}$$

As the initial velocity is 4 m/s, we have

$$\begin{aligned} v &= 12 \frac{m}{s^4} \cdot t^3 + 10 \frac{m}{s^2} \cdot t + cst \\ 4 \frac{m}{s} &= 12 \frac{m}{s^4} \cdot (0s)^3 + 10 \frac{m}{s^2} \cdot 0s + cst \\ cst &= 4 \frac{m}{s} \end{aligned}$$

The velocity is therefore

$$v = 12 \frac{m}{s^4} \cdot t^3 + 10 \frac{m}{s^2} \cdot t + 4 \frac{m}{s}$$

The velocity at  $t = 1$  s is therefore

$$\begin{aligned} v &= 12 \frac{m}{s^4} \cdot (1s)^3 + 10 \frac{m}{s^2} \cdot 1s + 4 \frac{m}{s} \\ &= 26 \frac{m}{s} \end{aligned}$$

b) The position is found by integrating the velocity.

$$\begin{aligned} x &= \int v dt \\ &= \int \left( 12 \frac{m}{s^4} \cdot t^3 + 10 \frac{m}{s^2} \cdot t + 4 \frac{m}{s} \right) dt \\ &= 3 \frac{m}{s^4} \cdot t^4 + 5 \frac{m}{s^2} \cdot t^2 + 4 \frac{m}{s} \cdot t + cst \end{aligned}$$

If the origin is chosen so that the object is at  $x = 0$  m at the start, the equation becomes

$$\begin{aligned} 0m &= 3 \frac{m}{s^4} \cdot (0s)^4 + 5 \frac{m}{s^2} \cdot (0s)^2 + 4 \frac{m}{s} \cdot 0s + cst \\ cst &= 0m \end{aligned}$$

The position is therefore given by the formula

$$x = 3 \frac{m}{s^4} \cdot t^4 + 5 \frac{m}{s^2} \cdot t^2 + 4 \frac{m}{s} \cdot t$$



At  $t = 4$  s, the position is then

$$\begin{aligned} x &= 3 \frac{m}{s^4} \cdot (4s)^4 + 5 \frac{m}{s^2} \cdot (4s)^2 + 4 \frac{m}{s} \cdot 4s \\ &= 864m \end{aligned}$$

As we set that the position at  $t = 0$  s is 0 m, the displacement is 864 m.

**39. a)**

We will say that the position at the end of the acceleration period is  $A$ . So we must have, for the acceleration phase

$$\begin{aligned} x &= x_0 + \frac{1}{2}(v_0 + v)t_1 \\ A &= 0 + \frac{1}{2}(0 + v)t_1 \\ A &= \frac{1}{2}vt_1 \end{aligned}$$

This means that the duration of the acceleration phase is

$$t_1 = \frac{2A}{v}$$

At the end of the constant speed phase, the position is  $x = 400$  m. Thus

$$400m = A + 148.33 \frac{m}{s} \cdot t_2$$

This means that the duration of the constant speed phase is

$$t_2 = \frac{400m - A}{v}$$

Since the total time was 4.503 s, we have

$$t_1 + t_2 = 4.503s$$

It follows that

$$\frac{2A}{v} + \frac{400m - A}{v} = 4.503s$$



With this equation,  $A$  can be found

$$\begin{aligned} 2A + 400m - A &= 4.503s \cdot v \\ A + 400m &= 4.503s \cdot 148.33 \frac{m}{s} \\ A &= 4.503s \cdot 148.33 \frac{m}{s} - 400m \\ A &= 267.945m \end{aligned}$$

From this result, the duration of the acceleration phase can be found.

$$\begin{aligned} t_1 &= \frac{2A}{v} \\ &= \frac{2 \cdot 267.945m}{148.33 \frac{m}{s}} \\ &= 3.613s \end{aligned}$$

b) In the acceleration phase, we have

$$v = v_0 + at_1$$

Since the initial speed is zero, we have

$$\begin{aligned} a &= \frac{v}{t_1} \\ &= \frac{148.33 \frac{m}{s}}{3.613s} \\ &= \frac{148.33 \frac{m}{s}}{3.613s} \\ &= 41.06 \frac{m}{s^2} \end{aligned}$$

**40.** Sophie's position is

$$x_1 = vt$$

The position of the train is

$$x_2 = \frac{1}{2} \cdot 3 \frac{m}{s^2} \cdot t^2 + 10m$$

If Sophie catches the train, then



$$x_1 = x_2$$

$$vt = \frac{1}{2} \cdot 3 \frac{m}{s^2} \cdot t^2 + 10m$$

The solution of this equation is

$$vt = \frac{1}{2} \cdot 3 \frac{m}{s^2} \cdot t^2 + 10m$$

$$3 \frac{m}{s^2} \cdot t^2 - 2vt + 20m = 0$$

$$t = \frac{2v \pm \sqrt{4v^2 - 4 \cdot 3 \frac{m}{s^2} \cdot 20m}}{6 \frac{m}{s^2}}$$

If Sophie catches the train, then there must be a solution. To have a solution, the term inside the root must be positive. This means that

$$4v^2 - 4 \cdot 3 \frac{m}{s^2} \cdot 20m > 0$$

$$v^2 - 60 \frac{m^2}{s^2} \geq 0$$

$$v^2 \geq 60 \frac{m^2}{s^2}$$

$$v \geq 7.746 \frac{m}{s}$$

The minimum speed is, therefore, 7.746 m/s.