# Chapter 9 Solutions

**1.** Using a $y = 0$ at the surface of the water, the gravitational energy at points A, B, and C are

$$U_{gA} = mgy = 30kg \cdot 9.8\tfrac{N}{kg} \cdot 2.8m = 823.2J$$
$$U_{gB} = mgy = 30kg \cdot 9.8\tfrac{N}{kg} \cdot 1.5m = 441J$$
$$U_{gC} = mgy = 30kg \cdot 9.8\tfrac{N}{kg} \cdot 0m = 0J$$

a) Going from A to B, the variation of energy is

$$\begin{aligned}\Delta U_g &= U_{gB} - U_{gA}\\ &= 441J - 823,2J\\ &= -382.2J\end{aligned}$$

b) Going from A to C, the variation of energy is

$$\begin{aligned}\Delta U_g &= U_{gC} - U_{gA}\\ &= 0J - 823.2J\\ &= -823.2J\end{aligned}$$

The work is therefore

$$\begin{aligned}W_g &= -\Delta U_g\\ &= --823.2J\\ &= 823.2J\end{aligned}$$

**2.** a) The formula for the potential energy is

$$\begin{aligned}U &= -\int F_x dx\\ &= -\int \left(2\tfrac{N}{m^3} \cdot x^3 + 2N\right) dx\\ &= -\frac{1}{2}\tfrac{N}{m^3} \cdot x^4 - 2N \cdot x + Cst\end{aligned}$$

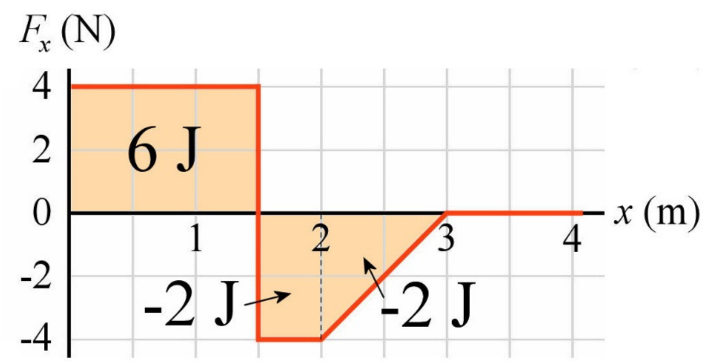
b) The potential energy difference is

$$\Delta U = \left(-\frac{1}{2}\frac{N}{m^3}\cdot(5m)^4 - 2N\cdot(5m)\right) - \left(-\frac{1}{2}\frac{N}{m^3}\cdot(-2m)^4 - 2N\cdot(-2m)\right)$$
$$= (-322.5J) - (-4J)$$
$$= -318,5J$$

c) The work done by the force is

$$W = -\Delta U$$
$$= -(-318.5J)$$
$$= 318.5J$$

**3.** The change of potential energy is equal to minus the area under the curve. Between $x$ = 0 m and $x$ = 3 m, the area is




The total area is 2 J. Therefore, the variation of potential energy is -2 J.

**4.** The force is conservative if

$$\frac{\partial F_y}{\partial x} = \frac{\partial F_x}{\partial y}$$

a) The derivatives are

$$\frac{\partial F_y}{\partial x} = \frac{\partial\left(3\frac{N}{m^2}\cdot y^2\right)}{\partial x} = 0$$

$$\frac{\partial F_x}{\partial y} == \frac{\partial\left(3\frac{N}{m^2}\cdot x^2\right)}{\partial y} = 0$$

As they are equal, the force is conservative.

b) The derivatives are

$$\frac{\partial F_y}{\partial x} = \frac{\partial\left(3\frac{N}{m^3}\cdot xy^2 + 1\frac{N}{m}\cdot y\right)}{\partial x} = 3\frac{N}{m^3}\cdot y^2$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial\left(3\frac{N}{m^3}\cdot x^2 y + 1\frac{N}{m}\cdot x\right)}{\partial y} = 3\frac{N}{m^3}\cdot x^2$$

As they are not equal, the force is not conservative.

c) The derivatives are

$$\frac{\partial F_y}{\partial x} = \frac{\partial\left(3\frac{N}{m^3}\cdot x^2 y + 1\frac{N}{m}\cdot x\right)}{\partial x} = 6\frac{N}{m^3}\cdot xy + 1\frac{N}{m}$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial\left(3\frac{N}{m^3}\cdot xy^2 + 1\frac{N}{m}\cdot y\right)}{\partial y} = 6\frac{N}{m^3}\cdot xy + 1\frac{N}{m}$$

As they are equal, the force is conservative.

**5.** The $x$-component of the force is

$$\begin{aligned} F_x &= -\frac{\partial U}{\partial x} \\ &= -\frac{\partial\left(4\frac{J}{m^2}\cdot x^2 + 2\frac{J}{m}\cdot(x+y) - 3\frac{J}{m^2}\cdot xy\right)}{\partial x} \\ &= -\left(8\frac{J}{m^2}\cdot x + 2\frac{J}{m} - 3\frac{J}{m^2}\cdot y\right) \end{aligned}$$

At $x = 1$ m and $y = 2$ m, this component of the force is

$$\begin{aligned}F_x &= -\left(8\tfrac{J}{m^2}\cdot 1m + 2\tfrac{J}{m} - 3\tfrac{J}{m^2}\cdot 2m\right)\\ &= -4N\end{aligned}$$

The $y$-component of the force is

$$\begin{aligned}F_y &= -\frac{\partial U}{\partial y}\\ &= -\frac{\partial\left(4\tfrac{J}{m^2}\cdot x^2 + 2\tfrac{J}{m}\cdot(x+y) - 3\tfrac{J}{m^2}\cdot xy\right)}{\partial y}\\ &= -\left(0 + 2\tfrac{J}{m} - 3\tfrac{J}{m^2}\cdot x\right)\end{aligned}$$

At $x$ = 1 m and $y$ = 2 m, this component of the force is

$$\begin{aligned}F_y &= -\left(2\tfrac{J}{m} - 3\tfrac{J}{m^2}\cdot 1m\right)\\ &= 1N\end{aligned}$$

Therefore, the force is

$$\vec{F} = \left(-4\vec{i} + 1\vec{j}\right)N$$

**6.** a) If $F_x$ is integrated with respect to $x$, the result is

$$\begin{aligned}U &= -\int\left(-4\tfrac{N}{m^3}\cdot xy^2 + 2\tfrac{N}{m}\cdot x\right)dx\\ &= 2\tfrac{N}{m^3}\cdot x^2y^2 - 1\tfrac{N}{m}\cdot x^2 + C_1\end{aligned}$$

If $F_y$ is integrated with respect to $y$, the result is

$$\begin{aligned}U &= -\int\left(-4\tfrac{N}{m^3}\cdot x^2y + 4\tfrac{N}{m}\cdot y\right)dy\\ &= 2\tfrac{N}{m^3}\cdot x^2y^2 - 2\tfrac{N}{m}\cdot y^2 + C_2\end{aligned}$$

The second term of the first integral is the part of $U$ that depends only on $x$. The second term of the second integral is the part of $U$ that depends only on $y$. The first terms of the two integrals are the part of $U$ that depends on both $x$ and $y$ at the same time. We, therefore, come to the conclusion that $U$ is equal to

$$U = 2\tfrac{N}{m^3}\cdot x^2y^2 - 1\tfrac{N}{m}\cdot x^2 - 2\tfrac{N}{m}\cdot y^2 + Cst$$

b) At the point (2,1), the potential energy is

$$\begin{aligned} U &= 2\tfrac{N}{m^3}\cdot(2m)^2\cdot(1m)^2 - 1\tfrac{N}{m}\cdot(2m)^2 - 2\tfrac{N}{m}\cdot(1m)^2 \\ &= 2J \end{aligned}$$

c) At the point (5,2), the potential energy is

$$\begin{aligned} U &= 2\tfrac{N}{m^3}\cdot(5m)^2\cdot(2m)^2 - 1\tfrac{N}{m}\cdot(5m)^2 - 2\tfrac{N}{m}\cdot(2m)^2 \\ &= 167J \end{aligned}$$

d) The work is

$$\begin{aligned} W &= -\Delta U \\ &= -(167J - 2J) \\ &= -165J \end{aligned}$$

**7.** The $x$-component of the force is

$$\begin{aligned} F_x &= -\frac{\partial U}{\partial x} \\ &= -\frac{\partial\left(4\tfrac{J}{m^2}\cdot x^2 + 2\tfrac{J}{m}\cdot(x+y+z) - 3\tfrac{J}{m^3}\cdot xyz\right)}{\partial x} \\ &= -\left(8\tfrac{J}{m^2}\cdot x + 2\tfrac{J}{m} - 3\tfrac{J}{m^3}\cdot yz\right) \end{aligned}$$

At $x$ = 1 m, $y$ = 2 m and $z$ = - 4 m, this component is

$$\begin{aligned} F_x &= -\left(8\tfrac{J}{m^2}\cdot 1m + 2\tfrac{J}{m} - 3\tfrac{J}{m^3}\cdot 2m\cdot(-4m)\right) \\ &= -34N \end{aligned}$$

The $y$-component of the force is

$$\begin{aligned} F_y &= -\frac{\partial U}{\partial y} \\ &= -\frac{\partial\left(4\frac{J}{m^2}\cdot x^2 + 2\frac{J}{m}\cdot(x+y+z) - 3\frac{J}{m^3}\cdot xyz\right)}{\partial y} \\ &= -\left(0 + 2\frac{J}{m} - 3\frac{J}{m^3}\cdot xz\right) \end{aligned}$$

At $x$ = 1 m, $y$ = 2 m and $z$ = - 4 m, this component is

$$\begin{aligned} F_y &= -\left(2\frac{J}{m} - 3\frac{J}{m^3}\cdot 1m\cdot(-4m)\right) \\ &= -14N \end{aligned}$$

The $z$-component of the force is

$$\begin{aligned} F_z &= -\frac{\partial U}{\partial z} \\ &= -\frac{\partial\left(4\frac{J}{m^2}\cdot x^2 + 2\frac{J}{m}\cdot(x+y+z) - 3\frac{J}{m^3}\cdot xyz\right)}{\partial z} \\ &= -\left(0 + 2\frac{J}{m} - 3\frac{J}{m^3}\cdot xy\right) \end{aligned}$$

At $x$ = 1 m, $y$ = 2 m and $z$ = - 4 m, this component is

$$\begin{aligned} F_z &= -\left(2\frac{J}{m} - 3\frac{J}{m^3}\cdot 1m\cdot 2m\right) \\ &= 4N \end{aligned}$$

The force is therefore

$$\vec{F} = \left(-34\vec{i} - 14\vec{j} + 4\vec{k}\right)N$$

**8.** a) At equilibrium, the force exerted by the spring is equal to the force of gravity.

$$kx = mg$$

The stretching of the spring is therefore

$$
\begin{aligned}
x &= \frac{mg}{k} \\
&= \frac{0.05kg \cdot 9.8\frac{N}{kg}}{50\frac{N}{m}} \\
&= 0.0098m
\end{aligned}
$$

The spring energy is therefore

$$
\begin{aligned}
U_{sp} &= \frac{1}{2}kx^2 \\
&= \frac{1}{2}\cdot 50\tfrac{N}{m}\cdot\left(0.0098m\right)^2 \\
&= 0.002401J
\end{aligned}
$$

b) At equilibrium, the force exerted by the spring is equal to the force of gravity.

$$kx' = mg$$

The stretching of the spring is therefore

$$
\begin{aligned}
x' &= \frac{mg}{k} \\
&= \frac{0.55kg \cdot 9.8\frac{N}{kg}}{50\frac{N}{m}} \\
&= 0.1078m
\end{aligned}
$$

The spring energy is therefore

$$
\begin{aligned}
U_{sp}' &= \frac{1}{2}kx'^2 \\
&= \frac{1}{2}\cdot 50\tfrac{N}{m}\cdot\left(0.1078m\right)^2 \\
&= 0.290521J
\end{aligned}
$$

c) The work done by the spring is

$$
\begin{aligned}
W_{sp} &= -\Delta U_{sp} \\
&= -\left(0.290521J - 0.002401J\right) \\
&= -0.28812J
\end{aligned}
$$

a) *Mechanical Energy Formula*

**9.** a) *Mechanical Energy Formula*

As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

*Mechanical Energy at Instant 1 (car at point A)*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy \\ &= \frac{1}{2}\cdot 2000kg \cdot v^2 + 2000kg \cdot 9.8\tfrac{N}{kg} \cdot 25m \\ &= 1000kg \cdot v^2 + 490,000J \end{aligned}$$

We have chosen to set the $y = 0$ at point B.

*Mechanical Energy at Instant 2 (car at point B)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' \\ &= \frac{1}{2}\cdot 2000kg \cdot \left(25\tfrac{m}{s}\right)^2 + 0J \\ &= 625,000J \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ 1000kg \cdot v^2 + 490,000J &= 625,000J \\ v &= 11.62\tfrac{m}{s} \end{aligned}$$

b) *Mechanical Energy at Instant 2 (car at point C)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' \\ &= \frac{1}{2}\cdot 2000kg \cdot v'^2 + 2000kg \cdot 9.8\tfrac{N}{kg} \cdot 5m \\ &= 1000kg \cdot v'^2 + 98,000J \end{aligned}$$

*Mechanical Energy Conservation*

$$E' = E'$$
$$625{,}000J = 1000kg \cdot v'^2 + 98{,}000J$$
$$v = 22.96 \tfrac{m}{s}$$

## **10.** *Mechanical Energy Formula*

As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

*Mechanical Energy at Instant 1 (spring compressed)*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0 + 0 + \frac{1}{2} \cdot 2000 \tfrac{N}{m} \cdot (0.2m)^2 \\ &= 40J \end{aligned}$$

We have chosen to set the $y = 0$ at the ground level.

a) *Mechanical Energy at Instant 2 (when the spring is compressed 5 cm)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2} \cdot 10kg \cdot v'^2 + 0 + \frac{1}{2} \cdot 2000 \tfrac{N}{m} \cdot (0.05m)^2 \\ &= 5kg \cdot v'^2 + 2.5J \end{aligned}$$

*Mechanical Energy Conservation*

$$E = E'$$
$$40J = 5kg \cdot v'^2 + 2.5J$$
$$v' = 2.739 \tfrac{m}{s}$$

b) *Mechanical Energy at Instant 2 (when the spring is no longer compressed)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}\cdot 10kg \cdot v'^2 + 0 + 0 \\ &= 5kg \cdot v'^2 \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ 40J &= 5kg \cdot v'^2 \\ v' &= 2.828\tfrac{m}{s} \end{aligned}$$

**11.** *Mechanical Energy Formula*

As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

*Mechanical Energy at Instant 1 (spring compressed)*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0 + 0 + \frac{1}{2}\cdot 500\tfrac{N}{m}\cdot (0.2m)^2 \\ &= 10J \end{aligned}$$

We have chosen to set the $y = 0$ at the ground level.

*Mechanical Energy at Instant 2 (when the mass is at its maximum height on the slope)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= 0 + 2kg \cdot 9.8\tfrac{N}{kg}\cdot y' + 0 \\ &= 19.6\tfrac{N}{kg}\cdot y' \end{aligned}$$

*Mechanical Energy Conservation*

$$E = E'$$
$$10J = 19.6N \cdot y'$$
$$y' = 0.5102m$$

The displacement is therefore

$$\sin 45° = \frac{y'}{D}$$
$$D = \frac{y'}{\sin 45°}$$
$$D = \frac{0.5102m}{\sin 45°}$$
$$D = 0.7215m = 72.15cm$$

**12.** *Mechanical Energy Formula*

As the system is composed of an object and two springs, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2$$

*Mechanical Energy at Instant 1*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2 \\ &= 0 + 0 + \frac{1}{2}\cdot 100\tfrac{N}{m}\cdot(0.5m)^2 + \frac{1}{2}\cdot 200\tfrac{N}{m}\cdot(0.1m)^2 \\ &= 12.5J + 1J \\ &= 13.5J \end{aligned}$$

We have chosen to set the $y = 0$ at the ground level.

*Mechanical Energy at Instant 2*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}k_1 x_1'^2 + \frac{1}{2}k_2 x_2'^2 \\ &= \frac{1}{2}\cdot 5kg\cdot v'^2 + 0 + \frac{1}{2}\cdot 100\tfrac{N}{m}\cdot\left(0.25m\right)^2 + \frac{1}{2}\cdot 200\tfrac{N}{m}\cdot\left(0.15m\right)^2 \\ &= 2{,}5kg\cdot v'^2 + 3.125J + 2.25J \\ &= 2.5kg\cdot v^2 + 5.375J \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ 13.5J &= 2.5kg\cdot v'^2 + 5.375J \\ v' &= 1.803\tfrac{m}{s} \end{aligned}$$

**13.** *Mechanical Energy Formula*

As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

*Mechanical Energy at Instant 1*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0 + 2kg\cdot 9.8\tfrac{N}{kg}\cdot 0.5m + 0 \\ &= 9.8J \end{aligned}$$

We have chosen to set the $y = 0$ at the end of the tube.

*Mechanical Energy at Instant 2*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= 0 + 2kg\cdot 9.8\tfrac{N}{kg}\cdot\left(-d\right) + \frac{1}{2}\cdot 100\tfrac{N}{m}\cdot d^2 \\ &= -19.6N\cdot d + 50\tfrac{N}{m}\cdot d^2 \end{aligned}$$

*Mechanical Energy Conservation*

$$E = E'$$
$$9.8J = -19.6N \cdot d + 50\tfrac{N}{m} \cdot d^2$$
$$50\tfrac{N}{m} \cdot d^2 - 19.6N \cdot d - 9.8J = 0$$

The solution of this equation is $d$ = 0.6801 m.

(The other solution $d$ = -0.2881 m must be rejected since it corresponds to a stretching of the spring, which does not make sense here.)

## 14. *Mechanical Energy Formula*

As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

*Mechanical Energy at Instant 1 (spring compressed)*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0 + 0 + \frac{1}{2} \cdot 2000\tfrac{N}{m} \cdot (0.2m)^2 \\ &= 40J \end{aligned}$$

We have chosen to set the $y = 0$ at the ground level.

*Mechanical Energy at Instant 2 (spring partially compressed)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}mv'^2 + 0 + \frac{1}{2}kx'^2 \end{aligned}$$

*Mechanical Energy Conservation*

$$E = E'$$
$$40J = \frac{1}{2}mv'^2 + \frac{1}{2}kx'^2$$

Since the kinetic energy is equal to the spring energy

$$\frac{1}{2}mv'^2 = \frac{1}{2}kx'^2$$

Our equation becomes

$$\begin{aligned}
40J &= \frac{1}{2}mv'^2 + \frac{1}{2}kx'^2 \\
40J &= \frac{1}{2}kx'^2 + \frac{1}{2}kx'^2 \\
40J &= kx'^2 \\
40J &= 2000\tfrac{N}{m} \cdot x'^2 \\
x' &= 0.1414m
\end{aligned}$$

## 15. *Mechanical Energy Formula*

As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

*Mechanical Energy at Instant 1 (car at point 1)*

$$\begin{aligned}
E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\
&= \frac{1}{2} \cdot 2000kg \cdot \left(10\tfrac{m}{s}\right)^2 + 2000kg \cdot 9.8\tfrac{N}{kg} \cdot 30m + 0 \\
&= 100,000J + 588,000J \\
&= 688,000J
\end{aligned}$$

We have chosen to set the $y = 0$ at the point 2.

a) The maximum speed will be reached at the lowest point, so at point 2.

*Mechanical Energy at Instant 2 (car at point 2)*

$$\begin{aligned}
E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\
&= \frac{1}{2} \cdot 2000kg \cdot v'^2 + 0 + 0 \\
&= 1000kg \cdot v'^2
\end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ 688{,}000J &= 1000kg \cdot v'^2 \\ v' &= 26.23 \tfrac{m}{s} \end{aligned}$$

b) *Mechanical Energy at Instant 2 (when the spring compression is maximum)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= 0 + 2000kg \cdot 9.8 \tfrac{N}{m} \cdot 15m + \frac{1}{2} \cdot 800 \tfrac{N}{m} \cdot x'^2 \\ &= 294{,}000J + 400 \tfrac{N}{m} \cdot x'^2 \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ 688{,}000J &= 294{,}000J + 400 \tfrac{N}{m} \cdot x'^2 \\ x' &= 31.38m \end{aligned}$$

c) *Mechanical Energy at Instant 2*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}mv'^2 + 2000kg \cdot 9.8 \tfrac{N}{m} \cdot 15m + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}mv'^2 + 294{,}000J + \frac{1}{2}kx'^2 \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ 688{,}000J &= \frac{1}{2}mv'^2 + 294{,}000J + \frac{1}{2}kx'^2 \\ 394{,}000J &= \frac{1}{2}mv'^2 + \frac{1}{2}kx'^2 \end{aligned}$$

Since the kinetic energy is equal to twice the spring energy, we have

$$\frac{1}{2}mv'^2 = 2 \cdot \frac{1}{2}kx'^2$$
$$\frac{1}{4}mv'^2 = \frac{1}{2}kx'^2$$

Our equation becomes

$$394,000J = \frac{1}{2}mv'^2 + \frac{1}{2}kx'^2$$
$$394,000J = \frac{1}{2}mv'^2 + \frac{1}{4}mv'^2$$
$$394,000J = \frac{3}{4}mv'^2$$
$$394,000J = \frac{3}{4} \cdot 2000kg \cdot v'^2$$
$$v' = 16.21\tfrac{m}{s}$$

**16.** *<u>Mechanical Energy Formula</u>*

As the system is composed of two objects and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}m_1v_1^2 + m_1gy_1 + \frac{1}{2}m_2v_2^2 + m_2gy_2 + \frac{1}{2}kx^2$$

*<u>Mechanical Energy at Instant 1</u>*

$$\begin{aligned} E &= \frac{1}{2}m_1v_1^2 + m_1gy_1 + \frac{1}{2}m_2v_2^2 + m_2gy_2 + \frac{1}{2}kx^2 \\ &= 0+0+0+0+\frac{1}{2} \cdot 195\tfrac{N}{m} \cdot (0.15m)^2 \\ &= 2.19375J \end{aligned}$$

We have chosen to set the $y = 0$ of each block at its position at instant 1.

*<u>Mechanical Energy at Instant 2 (when the 30 kg block is 10 cm lower while the 25 kg block has moved 10 cm towards the top of the slope)</u>*

The variation in height of the 25 kg block is given by

$$\sin 40° = \frac{y}{10cm}$$
$$y = 6.4279cm$$

The mechanical energy at instant 2 is then

$$\begin{aligned}
E' &= \frac{1}{2}m_1v'^2 + m_1gy_1' + \frac{1}{2}m_2v'^2 + m_2gy_2' + \frac{1}{2}kx'^2 \\
&= \frac{1}{2}\cdot 30kg\cdot v'^2 + 30kg\cdot 9.8\tfrac{N}{kg}\cdot(-0.1m) + \frac{1}{2}\cdot 25kg\cdot v'^2 \\
&\qquad + 25kg\cdot 9.8\tfrac{N}{kg}\cdot 0.064279m + \frac{1}{2}\cdot 195\tfrac{N}{m}\cdot(0.05m)^2 \\
&= 15kg\cdot v'^2 - 29.4J + 12.5kg\cdot v'^2 + 15.748J + 0.24375J \\
&= 27.5kg\cdot v'^2 - 13.408J
\end{aligned}$$

*Mechanical Energy Conservation*

$$E = E'$$
$$2.19375J = 27.5kg\cdot v'^2 - 13.408J$$
$$v' = 0.7532\tfrac{m}{s}$$

**17.** *Mechanical Energy Formula*

As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

*Mechanical Energy at Instant 1 (pendulum at point A)*

$$\begin{aligned}
E &= \frac{1}{2}mv^2 + mgy \\
&= 0 + mgy
\end{aligned}$$

We have chosen to set the $y = 0$ at the lowest point of the path of the pendulum.

The height of the pendulum at point A is

$$\begin{aligned} y &= L(1-\cos\theta) \\ &= 1.2m \cdot (1-\cos 35°) \\ &= 0.217m \end{aligned}$$

The mechanical energy at point A is therefore

$$\begin{aligned} E &= mgy \\ &= 4kg \cdot 9.8\tfrac{N}{m} \cdot 0.217m \\ &= 8.507J \end{aligned}$$

*Mechanical Energy at Instant 2, (pendulum at point B)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' \\ &= \frac{1}{2} \cdot 4kg \cdot v'^2 + 0 \\ &= 2kg \cdot v'^2 \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ 8.507J &= 2kg \cdot v'^2 \\ v' &= 2.062\tfrac{m}{s} \end{aligned}$$

**18.** *Mechanical Energy Formula*

As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

*Mechanical Energy at Instant 1 (Radu end of a vertical rope with a speed v)*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy \\ &= \frac{1}{2}mv^2 \end{aligned}$$

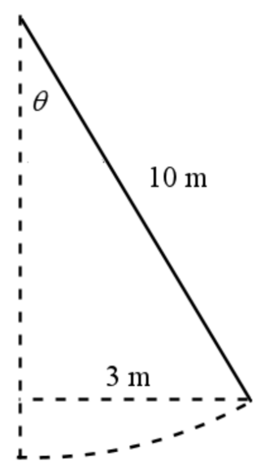
We have chosen to set the $y = 0$ at the lowest point on the path of the pendulum.

*Mechanical Energy at Instant 2 (Radu at the highest point of the pendulum's motion)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' \\ &= mgy' \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ \frac{1}{2}mv^2 &= mgy' \\ \frac{1}{2}v^2 &= gy' \end{aligned}$$




The value of $y_{min}$ when Radu crosses the ravine must be found. First, the minimum angle is found with

$$\begin{aligned} \sin\theta &= \frac{3m}{10m} \\ \theta &= 17.46° \end{aligned}$$

This corresponds to a height of

$$\begin{aligned} y_{\min} &= L(1-\cos\theta_{\min}) \\ &= 10m\cdot(1-\cos 17.46°) \\ &= 0.4606m \end{aligned}$$

The energy equation then becomes

$$\begin{aligned} \frac{1}{2}v_{\min}^2 &= gy_{\min} \\ \frac{1}{2}v_{\min}^2 &= 9.8\tfrac{N}{kg}\cdot 0.4606m \\ v_{\min} &= 3.005\tfrac{m}{s} \end{aligned}$$

## 19. *Mechanical Energy Formula*

As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

*Mechanical Energy at Instant 1 (configuration shown in the figure)*

The height of the pendulum is

$$\begin{aligned} y &= L(1-\cos\theta) \\ &= 4m \cdot (1-\cos 25°) \\ &= 0.3748m \end{aligned}$$

The mechanical energy is then

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy \\ &= \frac{1}{2} \cdot 2kg \cdot \left(2\tfrac{m}{s}\right)^2 + 2kg \cdot 9.8\tfrac{N}{kg} \cdot 0.3748m \\ &= 11.345J \end{aligned}$$

We have chosen to set the $y = 0$ at the lowest point on the path of the pendulum.

a) *Mechanical Energy at Instant 2 (vertical pendulum)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' \\ &= \frac{1}{2} \cdot 2kg \cdot v'^2 + 0 \\ &= 1kg \cdot v'^2 \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ 11.345J &= 1kg \cdot v'^2 \\ v' &= 3.368\tfrac{m}{s} \end{aligned}$$

b) *Mechanical Energy at Instant 3 (pendulum at its highest point)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' \\ &= 0 + 2kg \cdot 9.8\tfrac{N}{kg} \cdot y' \\ &= 19.6N \cdot y' \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ 11.345J &= 19.6N \cdot y' \\ y' &= 0.57885m \end{aligned}$$

This corresponds to the angle

$$\begin{aligned} \cos\theta &= \frac{L - y'}{L} \\ \cos\theta &= \frac{4m - 0.57885m}{4m} \\ \theta &= 31.2° \end{aligned}$$

**20.** *Mechanical Energy Formula*

As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

*Mechanical Energy at Instant 1*

The height of the pendulum is

$$y = L(1 - \cos\theta)$$

The mechanical energy is then

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy \\ &= 0 + mgy \\ &= mgL(1 - \cos 50°) \end{aligned}$$

We have chosen to set the $y = 0$ at the lowest point on the path of the pendulum.

*Mechanical Energy at Instant 2 (the length of the rope is now L')*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' \\ &= 0 + mgy' \\ &= mgL'(1-\cos\theta) \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ mgL(1-\cos 50°) &= mgL'(1-\cos\theta) \\ L(1-\cos 50°) &= L'(1-\cos\theta) \\ 1.5m\cdot(1-\cos 50°) &= 0.5m\cdot(1-\cos\theta) \\ \cos\theta &= -0.0716 \\ \theta &= 94.1° \end{aligned}$$

**21.** To obtain the apparent weigth, the acceleration of the car is needed. This acceleration is the centripetal acceleration

$$a_c = \frac{v^2}{r}$$

To obtain the acceleration, the speed of the car is needed. This speed can be calculated with energy conservation.

*Mechanical Energy Formula*

As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}Mv^2 + Mgy$$

(*M* is used instead of *m* to avoid the confusion between the mass and the units (metre) later.)

*Mechanical Energy at Instant 1 (car at position A)*

The height of the car is

$$\begin{aligned} y &= L(1-\cos\theta) \\ &= 5m\cdot(1-\cos 80°) \\ &= 4.132m \end{aligned}$$

The mechanical energy of the car is then

$$\begin{aligned} E &= \frac{1}{2}Mv^2 + Mgy \\ &= 0 + Mgy \\ &= Mg\cdot 4.132m \end{aligned}$$

We have chosen to set the $y = 0$ at the lowest point on the path of the car.

*Mechanical Energy at Instant 2 (car at position B)*

$$\begin{aligned} E' &= \frac{1}{2}Mv'^2 + Mgy' \\ &= \frac{1}{2}Mv'^2 + 0 \\ &= \frac{1}{2}Mv'^2 \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ Mg\cdot 4.132m &= \frac{1}{2}Mv'^2 \\ g\cdot 4.132m &= \frac{1}{2}v'^2 \\ v' &= 8.999\tfrac{m}{s} \end{aligned}$$

The centripetal acceleration of a person in the car is then

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{\left(8.999\tfrac{m}{s}\right)^2}{5m} \\ &= 16.1965\tfrac{m}{s^2} \end{aligned}$$

upwards.

The components of the apparent weight of a person in the car are therefore

$$w_{app\ x} = -Ma_x$$
$$\rightarrow w_{app\ x} = 0$$
$$w_{app\ y} = -Mg - Ma_y$$
$$\rightarrow w_{app\ y} = -M \cdot 9.8\tfrac{N}{kg} - M \cdot 16.1965\tfrac{m}{s^2}$$
$$\rightarrow w_{app\ y} = -M \cdot 25.9965\tfrac{N}{kg}$$

The number of *g* is therefore

$$n_g = \frac{\left|w_{app}\right|}{w_{\text{actual on Earth}}} = \frac{M \cdot 25.9965\tfrac{N}{kg}}{M \cdot 9.8\tfrac{N}{kg}} = 2.653$$

**22.** *Mechanical Energy Formula*

The system being formed of one object (the ball) and two springs, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2$$

*Mechanical Energy at Instant 1 (stretched springs)*

At that instant, the ball has no speed. The $y = 0$ is set between the two springs (where the ball is) so that $mgy = 0$. However, the springs are stretched. The length of the spring is equal to the hypotenuse of a triangle whose sides adjacent to the right angle are 50 cm and 30 cm long. The length of the springs is, therefore,

$$L = \sqrt{(50cm)^2 + (30cm)^2} = 58.31cm$$

As the springs were neither stretched nor compressed when their length was 50 cm, then they are stretched 8.31 cm. Therefore, the energy at the instant 1 is

$$\begin{aligned} E_{mec} &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2 \\ &= 0 + 0 + \frac{1}{2}\cdot 500\tfrac{N}{m}\cdot(0.0831m)^2 + \frac{1}{2}\cdot 500\tfrac{N}{m}\cdot(0.0831m)^2 \\ &= 3.452J \end{aligned}$$

*Mechanical Energy at Instant 2*

At that instant, the ball has some speed, the springs are no more stretched and the ball is still at $y = 0$. The energy at instant 2 is, therefore,

$$\begin{aligned} E'_{mec} &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}k_1 x_1'^2 + \frac{1}{2}k_2 x_2'^2 \\ &= \frac{1}{2}\cdot 0{,}1kg \cdot v'^2 + 0 + 0 + 0 \\ &= 0{,}05kg \cdot v'^2 \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ 3{,}452J &= 0{,}05kg \cdot v'^2 \\ v' &= 8{,}31\tfrac{m}{s} \end{aligned}$$

**23.** *Mechanical Energy Formula*

As the system is composed of two objects, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}m_1 v_1^2 + m_1 g y_1 + \frac{1}{2}m_2 v_2^2 + m_2 g y_2$$

*Mechanical Energy at Instant 1 (configuration shown in the figure)*

$$\begin{aligned} E &= \frac{1}{2}m_1 v_1^2 + m_1 g y_1 + \frac{1}{2}m_2 v_2^2 + m_2 g y_2 \\ &= 0 + 0 + 0 + 0 \\ &= 0 \end{aligned}$$

We have chosen to set the $y = 0$ each block at its initial position.

We know that the blocks moved 2 m, but we do not know in which direction. Therefore, there must be two solutions.

**1st solution: the 6 kg block moves 2 m upwards**

*Mechanical Energy at Instant 2 (after a 2 m upwards displacement)*

$$\begin{aligned}E' &= \frac{1}{2}m_1 v_1'^2 + m_1 g y_1' + \frac{1}{2}m_2 v_2'^2 + m_2 g y_2' \\ &= \frac{1}{2}\cdot 6kg\cdot v'^2 + 6kg\cdot 9.8\tfrac{N}{kg}\cdot 2m + \frac{1}{2}\cdot 10kg\cdot v'^2 + 10kg\cdot 9.8\tfrac{N}{kg}\cdot 0m \\ &= 3kg\cdot v'^2 + 117.6J + 5kg\cdot v'^2 \\ &= 8kg\cdot v'^2 + 117.6J\end{aligned}$$

*Work Done by the External Force and by the Non-Conservative Force*

As there are external forces, the work done by these forces must be calculated.

$$\begin{aligned}W_{10N} &= 10N\cdot 2m\cdot\cos(0°) = 20J \\ W_{20N} &= 20N\cdot 2m\cdot\cos(180°) = -40J\end{aligned}$$

The net work done by the external forces is thus -20 J.

*Application of $E_{mec} + W_{ext} + W_{n.cons.} = E'_{mec}$*

$$\begin{aligned}E + W_{ext} &= E' \\ 0 + -20J &= 8kg\cdot v'^2 + 117.6J \\ v'^2 &= -17.2\tfrac{m^2}{s^2}\end{aligned}$$

This has no solution. It is, therefore, impossible for the 6 kg block to move 2 m upwards.

**2nd solution: the 6 kg block moves 2 m downwards**

*Mechanical Energy at Instant 2 (after a 2 m downwards displacement)*

$$\begin{aligned}E' &= \frac{1}{2}m_1 v_1'^2 + m_1 g y_1' + \frac{1}{2}m_2 v_2'^2 + m_2 g y_2' \\ &= \frac{1}{2}\cdot 6kg\cdot v'^2 + 6kg\cdot 9.8\tfrac{N}{kg}\cdot(-2m) + \frac{1}{2}\cdot 10kg\cdot v'^2 + 10kg\cdot 9.8\tfrac{N}{kg}\cdot 0m \\ &= 3kg\cdot v'^2 - 117.6J + 5kg\cdot v'^2 \\ &= 8kg\cdot v'^2 - 117.6J\end{aligned}$$

*Work Done by the External Force and by the Non-Conservative Force*

As there are external forces, the work done by these forces must be calculated.

$$W_{10N} = 10N \cdot 2m \cdot \cos(180°) = -20J$$
$$W_{20N} = 20N \cdot 2m \cdot \cos(0°) = 40J$$

The net work done by the external forces is thus 20 J.

*Application of $E_{mec} + W_{ext} + W_{n.cons.} = E'_{mec}$*

$$E + W_{ext} = E'$$
$$0 + 20J = 8kg \cdot v'^2 - 117.6J$$
$$v'^2 = 17.2 \tfrac{m^2}{s^2}$$
$$v' = 4.147 \tfrac{m}{s}$$

**24.** *Mechanical Energy Formula*

As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

*Mechanical Energy at Instant 1 (configuration shown in the figure)*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0 + 0 + \frac{1}{2} \cdot 20{,}000 \tfrac{N}{m} \cdot (3m)^2 \\ &= 90{,}000J \end{aligned}$$

We have chosen to set the $y = 0$ at ground level.

*Mechanical Energy at Instant 2 (after a 50 m displacement)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2} \cdot 500kg \cdot v'^2 + 0 + 0 \\ &= 250kg \cdot v'^2 \end{aligned}$$

*Work Done by the External Force and by the Non-Conservative Force*

As there is a non-conservative force, the work done by this force must be calculated.

$$\begin{aligned} W_{non-cons} &= 2000N \cdot 50m \cdot \cos(0°) \\ &= 100,000J \end{aligned}$$

*Application of $E_{mec}$ + $W_{ext}$ + $W_{n.cons.}$ = $E'_{mec}$*

$$\begin{aligned} E + W_{nc} &= E' \\ 90,000J + 100,000J &= 250kg \cdot v'^2 \\ v' &= 27.568 \tfrac{m}{s} \end{aligned}$$

**25.** *Mechanical Energy Formula*

As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

*Mechanical Energy at Instant 1 (configuration shown in the figure)*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy \\ &= 0 + mg \cdot (62m) \\ &= mg \cdot (62m) \end{aligned}$$

We have chosen to set the $y = 0$ at the base of the hill.

*Mechanical Energy at Instant 2 (after the descent and the motion on the flat surface)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' \\ &= 0 + 0 \\ &= 0 \end{aligned}$$

*Work Done by the External Force and by the Non-Conservative Force*

As there is a non-conservative force (the friction force), the work done by this force must be calculated.

$$\begin{aligned} W_{non-cons} &= F_f \Delta s \cdot \cos(180°) \\ &= \mu_k F_N \Delta s \cdot \cos(180°) \\ &= \mu_k mg \Delta s \cdot \cos(180°) \\ &= -\mu_k mg \Delta s \end{aligned}$$

*Application of $E_{mec} + W_{ext} + W_{n.cons.} = E'_{mec}$*

$$\begin{gathered} E + W_{nc} = E' \\ mg \cdot (62m) + -\mu_k mg \Delta s = 0 \\ (62m) - \mu_k \Delta s = 0m \\ 62m - 0.2 \cdot \Delta s = 0m \\ \Delta s = 310m \end{gathered}$$

## 26. *Mechanical Energy Formula*

As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

*Mechanical Energy at Instant 1 (configuration shown in the figure)*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0 + 0 + \frac{1}{2} \cdot 2000\tfrac{N}{m} \cdot (0.5m)^2 \\ &= 250J \end{aligned}$$

We have chosen to set the $y = 0$ at ground level.

*Mechanical Energy at Instant 2 (when the block has stopped)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= 0 + 0 + 0 \\ &= 0 \end{aligned}$$

*Work Done by the External Force and by the Non-Conservative Force*

As there is a non-conservative force (the friction force), the work done by this force must be calculated.

$$\begin{aligned} W_{non-cons} &= F_f \Delta s \cdot \cos(180°) \\ &= \mu_k F_N \Delta s \cdot \cos(180°) \\ &= \mu_k mg \Delta s \cdot \cos(180°) \\ &= \mu_k \cdot 10kg \cdot 9.8\tfrac{N}{kg} \cdot 42m \cdot \cos(180°) \\ &= -\mu_k \cdot 4116J \end{aligned}$$

*Application of $E_{mec} + W_{ext} + W_{n.cons.} = E'_{mec}$*

$$\begin{aligned} E + W_{non-cons} &= E' \\ 250J + -\mu_k \cdot 4116J &= 0 \\ \mu_k &= 0.06074 \end{aligned}$$

**27.** a) *Mechanical Energy Formula*

As the system is composed of two objects and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}m_1 v_1^2 + m_1 g y_1 + \frac{1}{2}m_2 v_2^2 + m_2 g y_2 + \frac{1}{2}kx^2$$

*Mechanical Energy at Instant 1 (configuration shown in the diagram)*

$$\begin{aligned} E &= \frac{1}{2}m_1 v_1^2 + m_1 g y_1 + \frac{1}{2}m_2 v_2^2 + m_2 g y_2 + \frac{1}{2}kx^2 \\ &= 0+0+0+0+0 \\ &= 0 \end{aligned}$$

We have chosen to set the $y = 0$ of each block at its initial position.

*Mechanical Energy at Instant 2 (after the 1 m displacement)*

We know that the 36 kg block moved downwards 1 m (as it is impossible for this block to move upwards).

$$\begin{aligned}E' &= \frac{1}{2}m_1v_1'^2 + m_1gy_1' + \frac{1}{2}m_2v_2'^2 + m_2gy_2' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}\cdot 36kg\cdot v'^2 + 36kg\cdot 9.8\tfrac{N}{kg}\cdot(-1m) + \frac{1}{2}\cdot 12kg\cdot v'^2 + 10kg\cdot 9{,}8\tfrac{N}{kg}\cdot 0m + \frac{1}{2}\cdot 200\tfrac{N}{m}\cdot(1m)^2 \\ &= 18kg\cdot v'^2 - 352.8J + 6kg\cdot v'^2 + 0J + 100J \\ &= 24kg\cdot v'^2 - 252.8J\end{aligned}$$

*Work Done by the External Force and by the Non-Conservative Force*

As there is a non-conservative force (the friction force on the 12 kg block), the work done by this force must be calculated.

$$\begin{aligned}W_{non-cons} &= F_f\Delta s\cdot\cos(180°) \\ &= \mu_k F_N\Delta s\cdot\cos(180°) \\ &= \mu_k mg\Delta s\cos(180°) \\ &= 0.4\cdot 12kg\cdot 9.8\tfrac{N}{kg}\cdot 1m\cdot\cos 180° \\ &= -47.04J\end{aligned}$$

*Application of $E_{mec} + W_{ext} + W_{n.cons.} = E'_{mec}$*

$$\begin{aligned}E + W_{non-cons} &= E' \\ 0 + -47.04J &= 24kg\cdot v'^2 - 252.8J \\ 205.76J &= 24kg\cdot v'^2 \\ v' &= 2.928\tfrac{m}{s}\end{aligned}$$

b) *Mechanical Energy at Instant 2 (after the 3.2 m displacement)*

$$\begin{aligned}E' &= \frac{1}{2}m_1v_1'^2 + m_1gy_1' + \frac{1}{2}m_2v_2'^2 + m_2gy_2' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}\cdot 36kg\cdot v'^2 + 36kg\cdot 9.8\tfrac{N}{kg}\cdot(-3.2m) + \frac{1}{2}\cdot 12kg\cdot v'^2 + 10kg\cdot 9.8\tfrac{N}{kg}\cdot 0m + \frac{1}{2}\cdot 200\tfrac{N}{m}\cdot(3.2m)^2 \\ &= 18kg\cdot v'^2 - 1128.96J + 6kg\cdot v'^2 + 0J + 1024J \\ &= 24kg\cdot v'^2 - 104.96J\end{aligned}$$

*Work Done by the External Force and by the Non-Conservative Force*

As there is a non-conservative force (the friction force on the 12 kg block), the work done by this force must be calculated.

$$\begin{aligned} W_{non-cons} &= F_f \Delta s \cdot \cos(180°) \\ &= \mu_k F_N \Delta s \cdot \cos(180°) \\ &= \mu_k mg \Delta s \cos(180°) \\ &= 0.4 \cdot 12kg \cdot 9.8\tfrac{N}{kg} \cdot 3.2m \cdot \cos 180° \\ &= -150.528J \end{aligned}$$

*Application of $E_{mec} + W_{ext} + W_{n.cons.} = E'_{mec}$*

$$\begin{aligned} E + W_{non-cons} &= E' \\ 0 + -150.528J &= 24kg \cdot v'^2 - 104.96J \\ -45.568J &= 24kg \cdot v'^2 \end{aligned}$$

This has no solution. This means that the blocks cannot have a 3.2 m displacement.

**28.** *Mechanical Energy Formula*

As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

*Mechanical Energy at Instant 1 (configuration shown in the figure)*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0 + 0 + 0 \\ &= 0 \end{aligned}$$

We have chosen to set the $y = 0$ at the initial position of the block.

*Mechanical Energy at Instant 2 (when the block is at rest)*

At that instant, the spring is compressed by a distance $d$ et the block has moved by a distance 3m + $d$. The height of the block is then

$$\sin 30° = \frac{y}{3m+d}$$
$$y = (3m+d)\sin 30°$$
$$y = \tfrac{1}{2}(3m+d)$$

The mechanical energy at instant 2 is now

$$\begin{aligned}
E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\
&= 0J + 3kg \cdot 9.8\tfrac{N}{kg} \cdot \left(-\tfrac{1}{2}(3m+d)\right) + \frac{1}{2} \cdot 400\tfrac{N}{m} \cdot d^2 \\
&= -14.7N \cdot (3m+d) + 200\tfrac{N}{m} \cdot d^2
\end{aligned}$$

*Work Done by the External Force and by the Non-Conservative Force*

As there is a non-conservative force (the friction force), the work done by this force must be calculated.

$$\begin{aligned}
W_{non-cons} &= F_f \Delta s \cdot \cos(180°) \\
&= \mu_k F_N \Delta s \cdot \cos(180°) \\
&= \mu_k mg \sin(60°) \cdot (3m+d) \cdot \cos(180°) \\
&= 0{,}2 \cdot 3kg \cdot 9.8\tfrac{N}{kg} \cdot \sin(60°) \cdot (3m+d) \cdot \cos(180°) \\
&= -5.09223N \cdot (3m+d)
\end{aligned}$$

*Application of $E_{mec} + W_{ext} + W_{n.cons.} = E'_{mec}$*

$$E + W_{non-cons} = E'$$
$$0 + -5.09223N \cdot (3m+d) = -14.7N \cdot (3m+d) + 200\tfrac{N}{m} \cdot d^2$$
$$-15.2767J - 5.09223N \cdot d = -44{,}1J - 14.7N \cdot d + 200\tfrac{N}{m} d^2$$
$$-28.8233J - 9.60777N \cdot d + 200\tfrac{N}{m} d^2 = 0$$

The solution of this equation is $d$ = 0.4044 m.

(There is another negative solution corresponding to a stretching of the spring, which makes no sense here.)

**29.** *Mechanical Energy Formula*

As the system is composed of two objects, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}m_1v_1^2 + m_1gy_1 + \frac{1}{2}m_2v_2^2 + m_2gy_2$$

*Mechanical Energy at Instant 1 (configuration shown in the figure)*

$$\begin{aligned} E &= \frac{1}{2}m_1v_1^2 + m_1gy_1 + \frac{1}{2}m_2v_2^2 + m_2gy_2 \\ &= 0+0+0+0 \\ &= 0 \end{aligned}$$

We have chosen to set the $y = 0$ each block at its initial position.

*Mechanical Energy at Instant 2 (after a 2 m downwards displacement)*

$$\begin{aligned} E' &= \frac{1}{2}m_1v_1'^2 + m_1gy_1' + \frac{1}{2}m_2v_2'^2 + m_2gy_2' \\ &= \frac{1}{2}\cdot 6kg\cdot v'^2 + 6kg\cdot 9.8\tfrac{N}{kg}\cdot(-2m) + \frac{1}{2}\cdot 10kg\cdot v'^2 + 10kg\cdot 9.8\tfrac{N}{kg}\cdot 0m \\ &= 3kg\cdot v'^2 - 117.6J + 5kg\cdot v'^2 \\ &= 8kg\cdot v'^2 - 117.6J \end{aligned}$$

*Work Done by the External Force and by the Non-Conservative Force*

As there is an external force, the work done by this force must be calculated.

$$W_{20N} = 20N\cdot 2m\cdot\cos(0°) = 40J$$

As there is a non-conservative force (the friction force), the work done by this force must be calculated.

$$\begin{aligned} W_{non-cons} &= F_f\Delta s\cdot\cos(180°) \\ &= \mu_k F_N\Delta s\cdot\cos(180°) \\ &= \mu_k F_N\Delta s\cdot\cos(180°) \\ &= 0.4\cdot 10kg\cdot 9.8\tfrac{N}{kg}\cdot 2m\cdot\cos(180°) \\ &= -78,4J \end{aligned}$$

*Application of $E_{mec} + W_{ext} + W_{n.cons.} = E'_{mec}$*

$$\begin{aligned}E + W_{ext} + W_{non-cons} &= E' \\ 0 + 40J - 78,4J &= 8kg \cdot v'^2 - 117.6J \\ v'^2 &= 9,9 \tfrac{m^2}{s^2} \\ v' &= 3.146 \tfrac{m}{s}\end{aligned}$$

**30.** a) It cannot be at the places where $U$ is larger than $E$, therefore at

$$x < 4 \text{ m} \quad \text{and} \quad x > 26 \text{ m}$$

b) At the minimum of $U$, thus at $x$ = 10 m.

c) At $x$ = 20, we have $U$ = 2.6 J. Since the mechanical energy is 4 J, the kinetic energy is

$$\begin{aligned}E_{mec} &= E_k + U \\ 4J &= E_k + 2.6J \\ E_k &= 1.4J\end{aligned}$$

Then

$$\begin{aligned}E_K &= 1.4J \\ \frac{1}{2} m v^2 &= 1.4J \\ \frac{1}{2} \cdot 2kg \cdot v^2 &= 1.4J \\ v &= 1.1832 \tfrac{m}{s}\end{aligned}$$

As this is a rough estimate, let's say 1.2 m/s.

d) Stable equilibrium at $x$ = 10 m. The mechanical energy must be 0.2 J.
Stable equilibrium at $x$ = 22 m. The mechanical energy must be 2 J.

e) Unstable equilibrium at $x$ = 17 m. The mechanical energy must be 3.4 J.

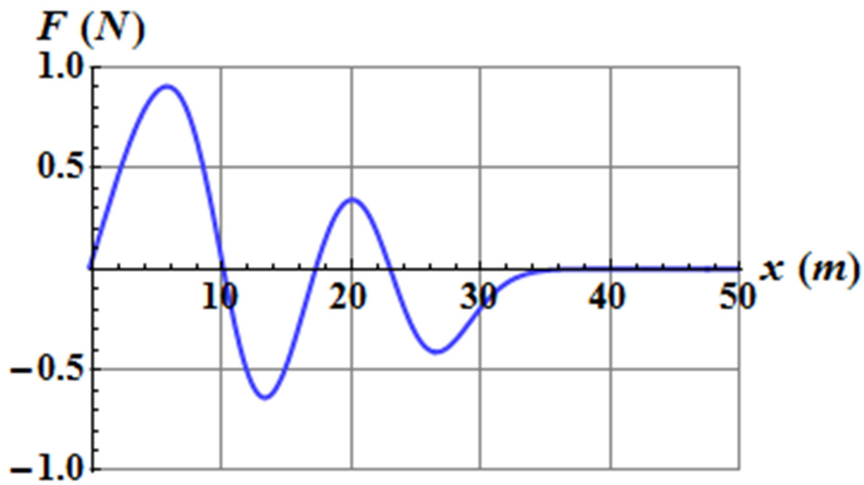
f) At $x$ = 10 m, we have $U$ = 0.2 J and at $x$ = 20 m, we have $U$ = 2.6 J. Therefore

$$\begin{aligned}\Delta U &= U_2 - U_1 \\ &= 2.6J - 0.2J \\ &= 2.4J\end{aligned}$$

The work is then

$$\begin{aligned}W &= -\Delta U \\ &= -2.4J\end{aligned}$$

g)




**31.** We know that the slope is zero at the equilibrium positions. So we need to solve the equation

$$\frac{dU}{dx} = 0$$

to find the equilibrium positions. With the derivative of the potential energy, the result is

$$\begin{aligned}6\tfrac{J}{m^3} \cdot x^2 + 6\tfrac{J}{m^2} \cdot x - 72\tfrac{J}{m} &= 0 \\ x^2 + 1m \cdot x - 12m^2 &= 0\end{aligned}$$

The solutions of this quadratic equation are $x$ = 3 m and $x$ = – 4 m. These are the 2 equilibrium positions.

Then, it should be noted that the type of equilibrium is related to the concavity of the function at the equilibrium position. With a positive concavity, the equilibrium is stable, with a negative concavity, the equilibrium is unstable and with a zero concavity, the equilibrium is neutral.

Concavity is found with the second derivative. The second derivative is

$$\frac{d^2U}{dx^2} = 12\tfrac{J}{m^3} \cdot x + 6\tfrac{J}{m^2}$$

At $x$ = 3 m, the concavity is

$$\left[\frac{d^2U}{dx^2}\right]_{x=3} = 12\tfrac{J}{m^3} \cdot 3m + 6\tfrac{J}{m^2} = 42\tfrac{J}{m^2}$$

So we have a stable equilibrium at $x$ = 3 m.

At $x$ = – 4 m, the concavity is

$$\left[\frac{d^2U}{dx^2}\right]_{x=-4m} = 12\tfrac{J}{m^3} \cdot (-4m) + 6\tfrac{J}{m^2} = -42\tfrac{J}{m^2}$$

So we have an unstable equilibrium at $x$ = – 4 m.

**32.** With the values given, the equation

$$\frac{dx}{dt} = \pm\sqrt{\frac{2}{m}\left(E_{mec} - U\right)}$$

becomes

$$\frac{dx}{dt} = \sqrt{\frac{2}{2kg}\left(144J - 9\tfrac{N}{m^3} \cdot x^4\right)}$$
$$\frac{dx}{dt} = \sqrt{144\tfrac{m^2}{s^2} - 9\tfrac{1}{m^2 s^2} \cdot x^4}$$
$$\frac{dx}{dt} = 3\tfrac{1}{s} \cdot \sqrt{16m^2 - 1\tfrac{1}{m^2} \cdot x^4}$$
$$3\tfrac{1}{s} \cdot dt = \frac{dx}{\sqrt{16m^2 - 1\tfrac{1}{m^2} \cdot x^4}}$$

(The positive sign is used because the sign in front of the root is the sign of the velocity. Since the object moves from $x$ = 0 to $x$ = 1 m, the velocity is positive.)

The equation is then integrated by taking the beginning and the end as integration limits. This means that on the left side, the limits will be the time at the position $x = 0$ (which will be called $t_1$) and the time at the position $x = 2$ m (which will be called $t_2$). On the right side, the limits will be the initial position ($x = 0$) and the final position ($x = 2$ m). Thus, the equation becomes

$$\int_{t_1}^{t_2} 3\tfrac{1}{s}\cdot dt = \int_0^{1m} \frac{dx}{\sqrt{16m^2 - 1\tfrac{1}{m^2}\cdot x^4}}$$

With Wolfram, the result is

$$3\tfrac{1}{s}\cdot\left(t_2 - t_1\right) = 0.251605$$
$$\Delta t = \frac{0.251605}{3\tfrac{1}{s}}$$
$$\Delta t = 0.08387s$$

**33.** With the values given, the equation

$$\frac{dx}{dt} = \pm\sqrt{\frac{2}{m}\left(E_{mec} - U\right)}$$

becomes

$$\frac{dx}{dt} = \pm\sqrt{\frac{2}{2kg}\left(16J - 2N\cdot x\right)}$$
$$\frac{dx}{dt} = \pm\sqrt{16\tfrac{m^2}{s^2} - 2\tfrac{m}{s^2}\cdot x}$$
$$dt = \pm\frac{dx}{\sqrt{16\tfrac{m^2}{s^2} - 2\tfrac{m}{s^2}\cdot x}}$$

Integrating on both sides, the result is

$$\int dt = \pm \int \frac{dx}{\sqrt{16\frac{m^2}{s^2} - 2\frac{m}{s^2} \cdot x}}$$

$$\int dt = \frac{\pm 1}{2\frac{m}{s^2}} \int \frac{\left(-2\frac{m}{s^2}\right) dx}{\sqrt{16\frac{m^2}{s^2} - 2\frac{m}{s^2} \cdot x}}$$

$$\int dt = \frac{\pm 1}{2\frac{m}{s^2}} \int \frac{du}{\sqrt{u}}$$

$$t + C = \frac{\pm 1}{2\frac{m}{s^2}} \frac{\sqrt{u}}{\frac{1}{2}}$$

$$t + C = \frac{\pm 1}{1\frac{m}{s^2}} \sqrt{16\frac{m^2}{s^2} - 2\frac{m}{s^2} \cdot x}$$

$$t + C = \pm \sqrt{16s^2 - 2\frac{s^2}{m} \cdot x}$$

If the object is at $x = 0$ at $t = 0$, then the integration constant is

$$0 + C = \pm \sqrt{16s^2 - 0}$$
$$C = \pm 4s$$

Therefore,

$$t \pm 4s = \pm \sqrt{16s^2 - 2\frac{s^2}{m} \cdot x}$$

Solving for $x$, the result is

$$\left(t \pm 4s\right)^2 = 16s^2 - 2\frac{s^2}{m} \cdot x$$
$$2\frac{s^2}{m} \cdot x = 16s^2 - \left(t \pm 4s\right)^2$$
$$x = 8m - \frac{1}{2}\frac{m}{s^2} \cdot \left(t \pm 4s\right)^2$$

Finally, the sign can be found since we know that the initial velocity is positive. The velocity is

$$v = \frac{dx}{dt}$$
$$= -1\frac{m}{s^2} \cdot \left(t \pm 4s\right)$$

At $t = 0$, we have

$$v_0 = -1\frac{m}{s^2} \cdot \left(\pm 4s\right)$$

Since we know that the velocity is positive, we must keep the negative sign. Thus, the position as a function of time is

$$x = 8m - \tfrac{1}{2}\tfrac{m}{s^2} \cdot (t - 4s)^2$$

Here is a graph of the position as a function of time.


**34.** *Mechanical Energy Formula*

As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + \frac{-GM_T m}{r}$$

*Mechanical Energy at Instant 1 (Neil at 400,000 km from the surface of the Earth)*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + \frac{-GM_T m}{r} \\ &= 0 + \frac{-6.674\times10^{-11}\,\tfrac{Nm^2}{kg^2}\cdot 5.972\times10^{24}\,kg\cdot 100kg}{406,371\times10^3\,m} \\ &= -9.80806\times10^7\,J \end{aligned}$$

a) *Mechanical Energy at Instant 2 (when the speed is 5000 m/s)*

$$\begin{aligned} E' &= \frac{1}{2}mv^2 + \frac{-GM_T m}{r} \\ &= \frac{1}{2}\cdot 100kg \cdot \left(5000\tfrac{m}{s}\right)^2 + \frac{-6.674\times 10^{-11}\tfrac{Nm^2}{kg^2}\cdot 5.972\times 10^{24}kg\cdot 100kg}{r'} \\ &= 1.25\times 10^9 J - \frac{3.991\times 10^{16} Jm}{r'} \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ -9.80806\times 10^7 J &= 1.25\times 10^9 J - \frac{3.991\times 10^{16} Jm}{r'} \\ -1.3481\times 10^9 J &= -\frac{3.991\times 10^{16} Jm}{r'} \\ r' &= 2.9566\times 10^7 m \\ r' &= 29{,}566 km \end{aligned}$$

Therefore, the distance from the Earth's surface is

29,566 km – 6371 km = 23,195 km

b) *Mechanical Energy at Instant 2 (the arrival at the surface of the Earth)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + \frac{-GM_T m}{r} \\ &= \frac{1}{2}\cdot 100kg \cdot v'^2 + \frac{-6.674\times 10^{-11}\tfrac{Nm^2}{kg^2}\cdot 5.972\times 10^{24}kg\cdot 100kg}{6{,}371\times 10^3 m} \\ &= 50k \cdot v'^2 - 6{,}2560\times 10^9 J \end{aligned}$$

*Mechanical Energy Conservation*

$$\begin{aligned} E &= E' \\ -9.80806\times 10^7 J &= 50kg\cdot v'^2 - 6.2556\times 10^9 J \\ 6.1579\times 10^9 J &= 50kg \cdot v'^2 \\ v' &= 11{,}098\tfrac{m}{s} = 11.098\tfrac{km}{s} \end{aligned}$$

**35.** The escape velocity is

$$\begin{aligned} v_{lib} &= \sqrt{\frac{2GM}{R}} \\ &= \sqrt{\frac{2\cdot 6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 7.34\times10^{22}\,kg}{1737\times10^{3}\,m}} \\ &= 2375\,\tfrac{m}{s} \end{aligned}$$

**36.** *Mechanical Energy Formula*

As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + \frac{-GM_M m}{r}$$

*Mechanical Energy at Instant 1 (object at the surface of the Moon)*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + \frac{-GM_M m}{r} \\ &= \frac{1}{2}mv^2 + \frac{-6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 7.34\times10^{22}\,kg\cdot m}{1737\times10^{3}\,m} \\ &= \frac{1}{2}mv^2 - 2.8202\times10^{6}\,\tfrac{J}{kg}\cdot m \end{aligned}$$

*Mechanical Energy at Instant 2 (when the object reaches its maximum height of 3000 km)*

At that instant, the velocity is zero.

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + \frac{-GM_M m}{r'} \\ &= 0 + \frac{-6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 7.34\times10^{22}\,kg\cdot m}{4737\times10^{3}\,m} \\ &= -1.0341\times10^{6}\,\tfrac{J}{kg}\cdot m \end{aligned}$$

*Mechanical Energy Conservation*

$$E = E'$$

$$\frac{1}{2}mv^2 - 2.8202\times10^6 \tfrac{J}{kg}\cdot m = -1.0341\times10^6 \tfrac{J}{kg}\cdot m$$

$$\frac{1}{2}v^2 - 2.8202\times10^6 \tfrac{J}{kg} = -1.0341\times10^6 \tfrac{J}{kg}$$

$$\frac{1}{2}v^2 = 1.786\times10^6 \tfrac{J}{kg}$$

$$v = 1890 \tfrac{m}{s}$$

**37.** *Mechanical Energy Formula*

As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + \frac{-GM_E m}{r}$$

*Mechanical Energy at Instant 1 (satellite at rest on the surface of the Earth)*

$$\begin{aligned}E &= \frac{1}{2}mv^2 + \frac{-GM_E m}{r}\\ &= 0 + \frac{-6.674\times10^{-11} \frac{Nm^2}{kg^2}\cdot 5.972\times10^{24} kg\cdot 350kg}{6371\times10^3 m}\\ &= -2.1896\times10^{10} J\end{aligned}$$

*Mechanical Energy at Instant 2 (when the satellite is in orbit)*

$$\begin{aligned}E' &= \frac{1}{2}mv'^2 + \frac{-GM_E m}{r'}\\ &= \frac{-GM_T m}{2r'}\\ &= \frac{-6.674\times10^{-11} \frac{Nm^2}{kg^2}\cdot 5.972\times10^{24} kg\cdot 350kg}{2\cdot 6871\times10^3 m}\\ &= -1.0151\times10^{10} J\end{aligned}$$

*Application of $E_{mec} + W_{ext} + W_{n.cons.} = E'_{mec}$*

$$\begin{aligned} E + W_{non-cons} &= E' \\ -2.1896\times 10^{10} J + W_{non-con} &= -1.0151\times 10^{10} J \\ W_{non-cons} &= 1.174\times 10^{10} J \end{aligned}$$

**38.** a) *Mechanical Energy Formula*

As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + \frac{-GM_E m}{r}$$

*Mechanical Energy at Instant 1 (satellite at rest on the surface of the Earth)*

$$\begin{aligned} E &= \frac{1}{2}mv^2 + \frac{-GM_T m}{R_T} \\ &= 0 + \frac{-GM_T m}{R_T} \end{aligned}$$

*Mechanical Energy at Instant 2 (when the satellite is in orbit)*

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + \frac{-GM_T m}{r'} \\ &= \frac{-GM_T m}{2r'} \end{aligned}$$

*Application of $E_{mec} + W_{ext} + W_{n.cons.} = E'_{mec}$*

$$\begin{aligned} E + W_{non-cons} &= E' \\ \frac{-GM_T m}{R_T} + W_{non-con} &= \frac{-GM_T m}{2r'} \\ W_{non-cons} &= \frac{-GM_T m}{2r'} + \frac{GM_T m}{R_T} \\ W_{non-cons} &= GM_T m\left(\frac{1}{R_T} - \frac{1}{2r'}\right) \end{aligned}$$

The kinetic energy of the satellite is ½mv². However, the speed of the orbiting satellite is

$$v^2 = \sqrt{\frac{GM_T}{r}}$$

Which means that the kinetic energy on an orbit of radius *r’* is

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{GmM_T}{r'}$$

Thus, the proportion of kinetic energy given relative to the total energy given is

$$\begin{aligned}
\frac{mv^2}{W_{non-cons}} &= \frac{\frac{1}{2}\frac{GM_T m}{r'}}{GM_T m\left(\frac{1}{R_T} - \frac{1}{2r'}\right)} \\
&= \frac{\frac{1}{2r'}}{\left(\frac{1}{R_T} - \frac{1}{2r'}\right)} \\
&= \frac{R_T}{2r' - R_T} \\
&= \frac{6371km}{2\cdot(6371km + 150km) - 6371km} \\
&= 0.9550
\end{aligned}$$

Thus, 95.5% of the energy given to this satellite in low orbit is in the form of kinetic energy.

b) If 50% of the given energy is in the form of kinetic energy, then

$$\begin{aligned}
0.5 &= \frac{R_T}{2r' - R_T} \\
0.5 &= \frac{6371km}{2r' - 6371km} \\
r' &= 9556.5km
\end{aligned}$$

**39.** a) *Mechanical Energy Formula*

As the system is composed of only one object (the Moon), the mechanical energy of the system is

$$E_{mec} = \frac{1}{2} M_M v^2 + \frac{-GM_E M_M}{r}$$

*Mechanical Energy at Instant 1 (Moon orbiting the Earth)*

$$\begin{aligned} E &= \frac{1}{2} M_M v^2 + \frac{-GM_E M_M}{r} \\ &= \frac{-GM_E M_M}{2r} \\ &= \frac{-6.674\times 10^{-11} \frac{Nm^2}{kg^2} \cdot 5.972\times 10^{24} kg \cdot 7.34\times 10^{22} kg}{2 \cdot 384,400\times 10^3 m} \\ &= -3,8053\times 10^{28} J \end{aligned}$$

*Mechanical Energy at Instant 2 (when the Moon is far from the Earth)*

$$\begin{aligned} E' &= \frac{1}{2} M_M v'^2 + \frac{-GM_E M_M}{r'} \\ &= \frac{1}{2} M_M v'^2 + 0 \end{aligned}$$

*Application of $E_{mec} + W_{ext} + W_{n.cons.} = E'_{mec}$*

$$E + W_{ext} = E'$$

$$-3.8053\times 10^{28} J + W_{ext} = \frac{1}{2} M_M v'^2$$

$$W_{ext} = 3.8053\times 10^{28} J + \frac{1}{2} M_M v'^2$$

The minimum energy is therefore $3.8053 \times 10^{28}$ J.

b) This energy corresponds to

$$\frac{3.8053\times 10^{28} J}{6.3\times 10^{13} J} = 6.04\times 10^{14}$$

times the Hiroshima atomic bomb. This is 604,000 billion times the Hiroshima bomb. Nasty explosion!

**40.** *Mechanical Energy Formula*

As the system is composed of only one object (the probe), the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + \frac{-GM_E m}{r_E} + \frac{-GM_M m}{r_M}$$

There are two gravitational energies because here we must take into account the gravitational energy made by the Earth and the Moon.

*Mechanical Energy at Instant 1 (probe at rest on the surface of the Earth)*

When the probe is on the surface of the Earth, it is 6371 km from the centre of the Earth and 378,029 km from the centre of the Moon. Thus, the energy is

$$\begin{aligned}
E &= \frac{1}{2}mv^2 + \frac{-GM_E m}{r_E} + \frac{-GM_M m}{r_M} \\
&= 0 - \frac{6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 5.972\times10^{24}\,kg\cdot 100kg}{6371\times10^3\,m} - \frac{6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 7.34\times10^{22}\,kg\cdot 100kg}{378,029\times10^3\,m} \\
&= -6.2560\times10^9\,J - 1.2959\times10^6\,J \\
&= -6.2573\times10^9\,J
\end{aligned}$$

*Mechanical Energy at Instant 2 (when the probe is at rest on the surface of the Moon)*

When the probe is on the surface of the Earth, it is 382,663 km from the centre of the Earth and 1737 km from the centre of the Moon. Thus, the energy is

$$\begin{aligned}
E' &= \frac{1}{2}mv'^2 + \frac{-GM_E m}{r_E'} + \frac{-GM_M m}{r_M'} \\
&= 0 - \frac{6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 5.972\times10^{24}\,kg\cdot 100kg}{382,663\times10^3\,m} - \frac{6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 7.34\times10^{22}\,kg\cdot 100kg}{1737\times10^3\,m} \\
&= -1.0416\times10^8\,J - 2.8202\times10^8\,J \\
&= -3.8618\times10^8\,J
\end{aligned}$$

*Application of $E_{mec} + W_{ext} + W_{n.cons.} = E'_{mec}$*

$$\begin{aligned}
E + W_{non-cons} &= E' \\
-6.2573\times10^9\,J + W_{non-cons} &= -3.8618\times10^8\,J \\
W_{non-cons} &= 5.871\times10^9\,J
\end{aligned}$$

**41.** a) The mechanical energy is

$$\begin{aligned}E_{mec} &= \frac{1}{2}mv^2 + \frac{-GM_E m}{r}\\ &= \frac{-GM_E m}{2r}\\ &= \frac{-6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 5.972\times10^{24}\,kg\cdot 500kg}{2\cdot 7371\times10^3\,m}\\ &= -1.3518\times10^{10}\,J\end{aligned}$$

b) The gravitational energy is

$$\begin{aligned}U_g &= \frac{-GM_E m}{r}\\ &= \frac{-6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 5.972\times10^{24}\,kg\cdot 500kg}{7371\times10^3\,m}\\ &= -2.7036\times10^{10}\,J\end{aligned}$$

c) The kinetic energy is

$$\begin{aligned}E_{mec} &= E_k + U_g\\ -1,3518\times10^{10}\,J &= E_k + -2,7036\times10^{10}\,J\\ E_k &= 1,3518\times10^{10}\,J\end{aligned}$$

*Alternate solution*

The speed of the satellite is

$$\begin{aligned}v &= \sqrt{\frac{GM_E}{r}}\\ &= \sqrt{\frac{6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 5.972\times10^{24}\,kg}{7371\times10^3\,m}}\\ &= 7353\,\tfrac{m}{s}\end{aligned}$$

The kinetic energy is then

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}\cdot 500kg\cdot\left(7353\tfrac{m}{s}\right)^2 \\ &= 1.3518\times10^{10}J \end{aligned}$$

d) For the satellite to escape the Earth, its energy must be positive. At a minimum, it vanishes. Therefore

$$\begin{aligned} E + W_{ext} &= E' \\ -1.3518\times10^{10}J + W_{ext} &= 0 \\ W_{ext} &= 1.3518\times10^{10}J \end{aligned}$$

(We assumed that this work is done by an external force, but we might also have assumed that this work is done by a non-conservative force.)

**42.** a) Let's check if the force respects all 3 conditions. The first condition is

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

The derivatives are

$$\frac{\partial F_x}{\partial y} = \frac{\partial\left(5\tfrac{N}{m^2}\cdot yz - 2\tfrac{N}{m}\cdot x + 2\tfrac{N}{m}\cdot y\right)}{\partial y} = 5\tfrac{N}{m^2}\cdot z + 2\tfrac{N}{m}$$

$$\frac{\partial F_y}{\partial x} = \frac{\partial\left(5\tfrac{N}{m^2}\cdot xz - 4\tfrac{N}{m}\cdot y + 2\tfrac{N}{m}\cdot x\right)}{\partial x} = 5\tfrac{N}{m^2}\cdot z + 2\tfrac{N}{m}$$

Since the derivatives are equal, the first condition is met.

The second condition is

$$\frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x}$$

The derivatives are

$$\frac{\partial F_x}{\partial z}=\frac{\partial\left(5\tfrac{N}{m^2}\cdot yz-2\tfrac{N}{m}\cdot x+2\tfrac{N}{m}\cdot y\right)}{\partial z}=5\tfrac{N}{m^2}\cdot y$$

$$\frac{\partial F_z}{\partial x}=\frac{\partial\left(5\tfrac{N}{m^2}\cdot xy+6\tfrac{N}{m}\cdot z\right)}{\partial x}=5\tfrac{N}{m^2}\cdot y$$

Since the derivatives are equal, the second condition is met.

The third condition is

$$\frac{\partial F_y}{\partial z}=\frac{\partial F_z}{\partial y}$$

The derivatives are

$$\frac{\partial F_y}{\partial z}=\frac{\partial\left(5\tfrac{N}{m^2}\cdot xz-4\tfrac{N}{m}\cdot y+2\tfrac{N}{m}\cdot x\right)}{\partial z}=5\tfrac{N}{m^2}\cdot x$$

$$\frac{\partial F_z}{\partial y}=\frac{\partial\left(5\tfrac{N}{m^2}\cdot xy+6\tfrac{N}{m}\cdot z\right)}{\partial y}=5\tfrac{N}{m^2}\cdot x$$

Since the derivatives are equal, the third condition is met.

Since all three conditions are met, the force is conservative.

b) To find $U$, the partial integrals must be calculated.

The partial integral with respect to $x$ will give us all the terms that depend on $x$ only, on $x$ and $y$ at the same time, on $x$ and $z$ at the same time and on $x$, $y$ and $z$ at the same time.

$$\begin{aligned} U &= -\int F_x dx \\ &= -\int\left(5\tfrac{N}{m^2}\cdot yx-2\tfrac{N}{m}\cdot x+2\tfrac{N}{m}\cdot y\right)dx \\ &= -5\tfrac{N}{m^2}\cdot xyx+1\tfrac{N}{m}\cdot x^2-2\tfrac{N}{m}\cdot xy+C_1 \end{aligned}$$

The partial integral with respect to $x$ will give us all the terms that depend on $y$ only, on $x$ and $y$ at the same time, on $y$ and $z$ at the same time and on $x$, $y$ and $z$ at the same time.

$$\begin{aligned} U &= -\int F_y dy \\ &= -\int \left(5\tfrac{N}{m^2}\cdot yx - 4\tfrac{N}{m}\cdot y + 2\tfrac{N}{m}\cdot x\right)dx \\ &= -5\tfrac{N}{m^2}\cdot xyx + 2\tfrac{N}{m}\cdot y^2 - 2\tfrac{N}{m}\cdot xy + C_2 \end{aligned}$$

The partial integral with respect to $z$ will give us all the terms that depend on $z$ only, on $x$ and $z$ at the same time, on $y$ and $z$ at the same time and on $x$, $y$ and $z$ at the same time.

$$\begin{aligned} U &= -\int F_z dz \\ &= -\int \left(5\tfrac{N}{m^2}\cdot xy + 6\tfrac{N}{m}\cdot z\right)dx \\ &= -5\tfrac{N}{m^2}\cdot xyx + 3\tfrac{N}{m}\cdot z^2 + C_3 \end{aligned}$$

Combining these results, it can be inferred that $U$ is

$$U = -5\tfrac{N}{m^2}\cdot xyx + 1\tfrac{N}{m}\cdot x^2 + 2\tfrac{N}{m}\cdot y^2 + 3\tfrac{N}{m}\cdot z^2 - 2\tfrac{N}{m}\cdot xy + Cst$$

Here, we'll choose $cst = 0$.

At the starting point (1 m, 1 m, 0 m), the potential energy is

$$\begin{aligned} U &= -5\tfrac{N}{m^2}\cdot 1m\cdot 1m\cdot 0m + 1\tfrac{N}{m}\cdot(1m)^2 + 2\tfrac{N}{m}\cdot(1m)^2 + 3\tfrac{N}{m}\cdot(0m)^2 - 2\tfrac{N}{m}\cdot 1m\cdot 1m \\ &= 0J + 1J + 2J + 0J - 2J \\ &= 1J \end{aligned}$$

At the ending point (4 m, -2 m, 3 m), the potential energy is

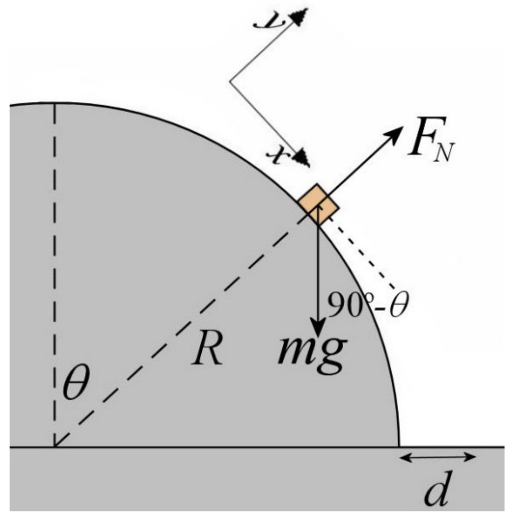
$$\begin{aligned} U &= -5\tfrac{N}{m^2}\cdot 4m\cdot(-2m)\cdot 3m + 1\tfrac{N}{m}\cdot(4m)^2 + 2\tfrac{N}{m}\cdot(-2m)^2 + 3\tfrac{N}{m}\cdot(3m)^2 - 2\tfrac{N}{m}\cdot 4m\cdot(-2m) \\ &= 120J + 16J + 8J + 27J + 16J \\ &= 187J \end{aligned}$$

Thus, the work done is

$$\begin{aligned} W &= -\Delta U \\ &= -(187J - 1J) \\ &= -186J \end{aligned}$$

**43.** a)

When the person is in contact with the sphere, there is a normal force. Therefore, the angle when the normal force becomes zero must be found.




There are two forces exerted on the person:

1) The weight
2) A normal force

Using the axes shown in the figure, the equations of forces are:

$$\sum F_x = mg\cos-(90°-\theta) = ma_t$$

$$\sum F_x = mg\sin-(90°-\theta) + N = -\frac{mv^2}{R}$$

The second equation gives

$$mg\sin-(90°-\theta) + N = -\frac{mv^2}{R}$$

$$-mg\cos\theta + N = -\frac{mv^2}{R}$$

$$N = mg\cos\theta - \frac{mv^2}{R}$$

Initially (small angle), the first term is greater than the second, and there is a normal force. As the person slides, the angle increases (and the first term decreases) and the speed increases (and the second term increases). At a certain angle, the normal force becomes zero and the contact is lost. When normal is zero, the equation becomes

$$0 = mg\cos\theta - \frac{mv^2}{r}$$

$$g\cos\theta = \frac{v^2}{r}$$

To find the angle, the speed as a function of the angle must be found. This speed can be found with the conservation of mechanical energy.

*Mechanical Energy Formula*

As there is only a single object (the person who slides), the formula of the mechanical energy is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

*Mechanical Energy at Instant 1, (the person is at the top of the dome)*

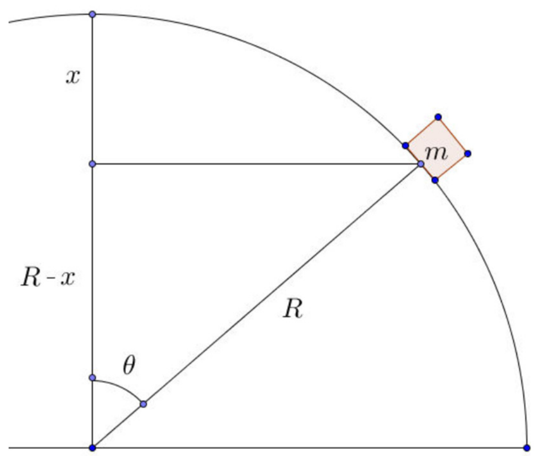
$$\begin{aligned} E_{mec} &= \frac{1}{2}mv^2 + mgy \\ &= 0 + 0 \\ &= 0 \end{aligned}$$

(The $y = 0$ was put at the top of the dome.)

*Mechanical Energy at Instant 2 (the person is at the angle $\theta$)*

$$E'_{mec} = \frac{1}{2}mv'^2 + mgy'$$

There is a link between height and angle. This link can be found with the following figure.




The height is $-x$. On the figure, it is easy to see that

$$\cos\theta = \frac{R - x}{R}$$

Therefore,

$$\begin{aligned} R\cos\theta &= R - x \\ x &= R - R\cos\theta \\ x &= R(1 - \cos\theta) \end{aligned}$$

Thus, the height is

$$y' = -R(1-\cos\theta)$$

Therefore, the mechanical energy at the instant 2 is

$$E'_{mec} = \frac{1}{2}mv'^2 - mgR(1-\cos\theta)$$

*Mechanical Energy Conservation*

$$\begin{aligned}
E_{mec} &= E'_{mec} \\
0 &= \frac{1}{2}mv'^2 - mgR(1-\cos\theta) \\
\frac{1}{2}mv'^2 &= mgR(1-\cos\theta) \\
v'^2 &= 2gR(1-\cos\theta)
\end{aligned}$$

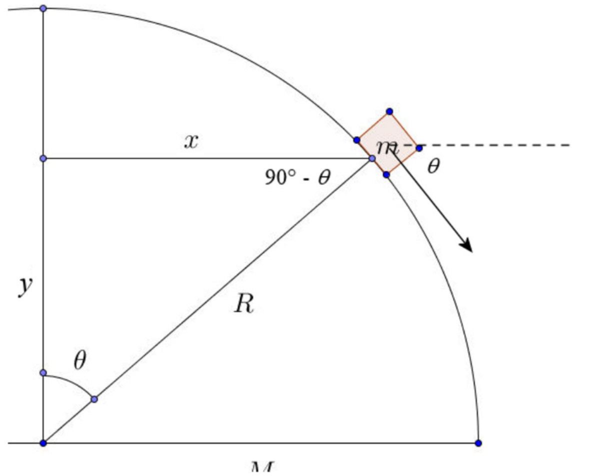
The condition for having a vanishing normal force then becomes

$$\begin{aligned}
g\cos\theta &= \frac{v^2}{R} \\
g\cos\theta &= \frac{2gR(1-\cos\theta)}{R}
\end{aligned}$$

It only remains to solve this equation for $\theta$.

$$\begin{aligned}
g\cos\theta &= \frac{2gR(1-\cos\theta)}{R} \\
g\cos\theta &= 2g(1-\cos\theta) \\
\cos\theta &= 2(1-\cos\theta) \\
\cos\theta &= 2-2\cos\theta \\
3\cos\theta &= 2 \\
\cos\theta &= 2/3 \\
\theta &= 48.19°
\end{aligned}$$

b) When the person leaves the surface, the direction of the velocity is tangent to the circle. From there on, it's a projectile that follows a parabolic trajectory.




Using an origin at the centre of the hemisphere, the initial position of the object is

$$x = R\sin(48.19°) = 6m \cdot \sin(48.19°) = 4.472m$$
$$y = R\cos(48.19°) = 6m \cdot \cos(48.19°) = 4m$$

The initial speed of the projectile motion is

$$\begin{aligned} v'^2 &= 2gR(1-\cos\theta) \\ &= 2\cdot 9.8\tfrac{m}{s^2}\cdot 6m\left(1-\tfrac{2}{3}\right) \\ &= 39.2\tfrac{m^2}{s^3} \\ v' &= 6.261\tfrac{m}{s} \end{aligned}$$

The components of this velocity (with an *x*-axis towards the right and a *y*-axis upwards) are

$$v_x = 6.261\tfrac{m}{s}\cdot\cos(-48.19°) = 4.174\tfrac{m}{s}$$
$$v_y = 6.261\tfrac{m}{s}\cdot\sin(-48.19°) = -4.667\tfrac{m}{s}$$

Therefore, the time of flight is

$$\begin{aligned} y &= y_0 + v_{0y}t - 4.9\tfrac{m}{s^2}t^2 \\ 0 &= 4m - 4.667\tfrac{m}{s}\cdot t - 4.9\tfrac{m}{s^2}\cdot t^2 \\ t &= 1.09025s \end{aligned}$$

The *x*-component of the position at the end of the free-fall is

$$\begin{aligned} x &= x_0 + v_{0x}t \\ &= 4.472m + 4.174\tfrac{m}{s} \cdot 1.09025s \\ &= 9.023m \end{aligned}$$

As the edge of the hemisphere is at $x = 6$ m, the person hits the ground 3.023 m away from the edge of the sphere.