Chapter 8 Solutions

1. The work is

$$\begin{aligned} W &= F\Delta s\cos\theta \\ &= 30N\cdot 15m\cdot\cos(40°) \\ &= 344.7J \end{aligned}$$

2. a) The work done by gravity is

$$\begin{aligned} W_g &= F_g\Delta s\cos\theta \\ &= \left(30kg\cdot 9.8\tfrac{N}{kg}\right)\cdot 25m\cdot\cos(98°) \\ &= -1022.9J \end{aligned}$$

b) The work done by the friction force is

$$\begin{aligned} W_f &= F_f\Delta s\cos\theta \\ &= 70N\cdot 25m\cdot\cos(180°) \\ &= -1750J \end{aligned}$$

c) To find the work done by Honoré, the force exerted by the latter must be found. This force can be found with the sum of the x-component of the forces on the crate.

There are 4 forces on the crate.

1) The 294 N weight directed downwards.
2) A normal force perpendicular to the slope.
3) A 70 N friction force directed downhill.
4) The force exerted by Honoré (F) directed uphill.

The equation for the x-components of the forces then gives (using an x-axis directed uphill)

$$\begin{aligned} \sum F_x &= ma_x \\ 294N\cos(-98°) - 70N + F &= 30kg\cdot 1\tfrac{m}{s^2} \\ F &= 140.9N \end{aligned}$$

Therefore, the work done by Honoré is

$$\begin{aligned} W_H &= F\Delta s\cos\theta \\ &= 140.9N \cdot 25m \cdot \cos(0°) \\ &= 3522.9J \end{aligned}$$

d) As the normal force does not work in this case, the net work is the sum of the 3 works calculated previously. It is then

$$\begin{aligned} W_{net} &= W_g + W_f + W_H \\ &= -1022.9J + -1750N + 3522.9J \\ &= 750J \end{aligned}$$

3. *First part of the motion*

The displacement is

$$\begin{aligned} \overrightarrow{\Delta s} &= \Delta x\vec{i} + \Delta y\vec{j} + \Delta z\vec{k} \\ &= \left(5\vec{i} - 3\vec{j} - 5\vec{k}\right)m \end{aligned}$$

The work done by the force 1 is

$$\begin{aligned} W_1 &= F_{1x}\Delta x + F_{1y}\Delta y + F_{1z}\Delta z \\ &= 2N \cdot 5m + 1N \cdot (-3m) + (-4N) \cdot (-5m) \\ &= 27J \end{aligned}$$

The work done by the force 2 is

$$\begin{aligned} W_2 &= F_{2x}\Delta x + F_{2y}\Delta y + F_{2z}\Delta z \\ &= (-4N) \cdot 5m + 5N \cdot (-3m) + 2N \cdot (-5m) \\ &= -45J \end{aligned}$$

The net work for the first part of the motion is therefore

$$\begin{aligned} W_{net1} &= W_1 + W_2 \\ &= 27J - 45J = -18J \end{aligned}$$

Second part of the motion

The displacement is

$$\begin{aligned}\overrightarrow{\Delta s} &= \Delta x\vec{i} + \Delta y\vec{j} + \Delta z\vec{k} \\ &= \left(3\vec{i} + 4\vec{j} - 2\vec{k}\right)m\end{aligned}$$

The work done by the force 1 is

$$\begin{aligned}W_1 &= F_{1x}\Delta x + F_{1y}\Delta y + F_{1z}\Delta z \\ &= 2N \cdot 3m + 1N \cdot 4m + (-4N)\cdot(-2m) \\ &= 18J\end{aligned}$$

The work done by the force 2 is

$$\begin{aligned}W_2 &= F_{2x}\Delta x + F_{2y}\Delta y + F_{2z}\Delta z \\ &= (-4N)\cdot 3m + 5N \cdot 4m + 2N \cdot (-2m) \\ &= 4J\end{aligned}$$

The net work for the second part of the motion is therefore

$$\begin{aligned}W_{net2} &= W_1 + W_2 \\ &= 18J + 4J = 22J\end{aligned}$$

Total Work

$$\begin{aligned}W_{net} &= W_{net1} + W_{net2} \\ &= -18J + 22J \\ &= 4J\end{aligned}$$

N.B.
The net force could also have been found by summing the forces

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

and calculating the net work of each part of the movement with

$$W_{net} = F_{net\,x}\Delta x + F_{net\,y}\Delta y + F_{net\,z}\Delta z$$

4. a) The work done by the force of gravity is

$$\begin{aligned} W_g &= F_g \Delta s \cos\theta \\ &= \left(80kg \cdot 9.8\tfrac{N}{kg}\right) \cdot 300m \cdot \cos\left(60°\right) \\ &= 117{,}600J \end{aligned}$$

b) To find the work done by the force of friction, the magnitude of the friction force must be found. To find it, the magnitude of the normal is needed. This magnitude is found with the sum of the *y*-components of the forces.

There are 3 forces exerted on Rita.

1) Rita's weight (784 N) directed downwards.
2) The normal force perpendicular to the surface.
3) The friction force directed uphill.

The equation for the *y*-components of the forces then gives (using an *x*-axis directed downhill)

$$\begin{aligned} \sum F_y &= ma_y \\ 784N \cdot \sin\left(-60°\right) + F_N &= 0 \\ F_N &= 678.96N \end{aligned}$$

The magnitude of the friction force is then

$$\begin{aligned} F_f &= \mu_c F_N \\ &= 0.1 \cdot 678.96N \\ &= 67.896N \end{aligned}$$

The work done by the force of friction is therefore

$$\begin{aligned} W_f &= F_f \Delta s \cos\theta \\ &= 67.896N \cdot 300m \cdot \cos\left(-180°\right) \\ &= -20{,}369J \end{aligned}$$

c) As the normal force does not work, the net work is

$$\begin{aligned} W_{net} &= W_g + W_f \\ &= 117{,}600J + -20{,}369J \\ &= 97{,}231J \end{aligned}$$

5. Initially, the spring is compressed 50 cm. Therefore, $x = 0.5$ m. At instant 2, the spring is neither stretched nor compressed. Therefore, $x' = 0$ m. Thus, the work done by the spring is

$$\begin{aligned} W_{sp} &= -\frac{1}{2}k\left(x_2^2 - x_1^2\right) \\ &= -\frac{1}{2}\cdot 2000\tfrac{N}{m}\cdot\left((0m)^2 - (0.5m)^2\right) \\ &= 250J \end{aligned}$$

6. To find the work, it is necessary to divide the trajectory in three parts.

For the first part, the work is

$$\begin{aligned} W_1 &= F\Delta s\cos\theta \\ &= 40N\cdot 3m\cdot\cos 35^\circ \\ &= 98.30J \end{aligned}$$

For the second part, the work is

$$\begin{aligned} W_2 &= F\Delta s\cos\theta \\ &= 80N\cdot 2m\cdot\cos 145^\circ \\ &= -131.06J \end{aligned}$$

For the third part, the work is

$$\begin{aligned} W_3 &= F\Delta s\cos\theta \\ &= 100N\cdot 1m\cdot\cos 35^\circ \\ &= 81.92J \end{aligned}$$

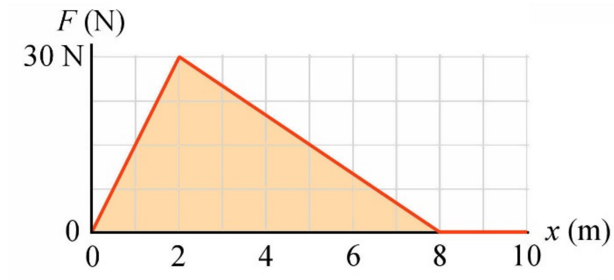
The total work is therefore

98.30 J + -131.06 J + 81.92 J = 49.15 J

7. To find the work, the area under the curve between x = 0 m and x = 6 m must be calculated. This area is the area of the whole triangle between x = 0 m and x = 8 m minus the area of the small triangle between x = 6 m and x = 8 m.

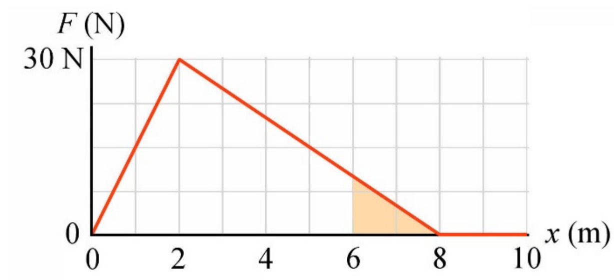
The area of the large triangle between x = 0 m and x = 8 m is

$$Area_1 = \frac{base \cdot height}{2} = \frac{8m \cdot 30N}{2} = 120J$$



The area of the small triangle between x = 6 m and x = 8 m is

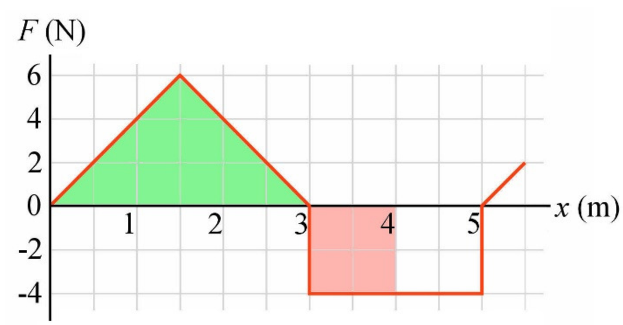
$$Area_2 = \frac{base \cdot height}{2} = \frac{2m \cdot 10N}{2} = 10J$$



Thus, the work is

$$\begin{aligned} W &= Area_1 - Area_2 \\ &= 120J - 10J \\ &= 110J \end{aligned}$$

8. The area under the curve between x = 0 and x = 4 m must be calculated. As the object goes towards the negative x, the area above the axis must be counted as a negative area and the area below the axis must be counted as a positive area. Therefore, the work is



W = area of the pink rectangle+ (- area of the green triangle)

The work is therefore

$$\begin{aligned} W &= 4N \cdot 1m + -\frac{3m \cdot 6N}{2} \\ &= -5J \end{aligned}$$

9. To find the work, it is necessary to divide the trajectory in three parts.

For the first part, the work is

$$\begin{aligned} W_1 &= F\Delta s\cos\theta \\ &= 10N\cdot 10m\cdot\cos 0° \\ &= 100J \end{aligned}$$

For the second part, the work is

$$\begin{aligned} W_2 &= F\Delta s\cos\theta \\ &= 15N\cdot 8m\cdot\cos 180° \\ &= -120J \end{aligned}$$

For the third part, the work is

$$\begin{aligned} W_3 &= F\Delta s\cos\theta \\ &= 20N\cdot 12m\cdot\cos 135° \\ &= -169.71J \end{aligned}$$

The total work is therefore

100 J + -120 J + -169.71 J = -189.71 J

10. The work is

$$\begin{aligned} W &= \int_x^{x'} F_x dx \\ &= \int_{-1m}^{3m} 18\tfrac{N}{m^2}\cdot x^2 dx \\ &= 18\tfrac{N}{m^2}\cdot\int_{-1m}^{3m} x^2 dx \\ &= 18\tfrac{N}{m^2}\cdot\left[\frac{x^3}{3}\right]_{-1m}^{3m} \\ &= 18\tfrac{N}{m^2}\cdot\left(\frac{(3m)^3}{3}-\frac{(-1m)^3}{3}\right) \\ &= 168J \end{aligned}$$

11. *W_{net} Calculation*

During the climb, the force of gravity is the only force acting on the object. The work done by gravity is therefore equal to the net work.

$$\begin{aligned} W_{net} &= W_g \\ &= F_g \Delta s \cos\theta \\ &= \left(0.43kg \cdot 9.8\tfrac{N}{kg}\right) \cdot 20m \cdot \cos(180°) \\ &= -84.28J \end{aligned}$$

ΔE_k Calculation

The change in kinetic energy is

$$\Delta E_k = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$$

$$\Delta E_k = \frac{1}{2}mv'^2 - \frac{1}{2} \cdot 0.43kg \cdot \left(30\tfrac{m}{s}\right)^2$$

$$\Delta E_k = \frac{1}{2}mv'^2 - 193.5J$$

Application of the Work-Energy Theorem

$$W_{net} = \Delta E_k$$

$$-84.28J = \frac{1}{2}mv'^2 - 193.5J$$

$$109.22J = \frac{1}{2}mv'^2$$

$$109.22J = \frac{1}{2} \cdot 0.43kg \cdot v'^2$$

$$v' = 22.54\tfrac{m}{s}$$

12. *W_{net} Calculation*

During the descent, there are three forces exerted on Mara.

1) The 245 N weight directed downwards.
2) A normal force perpendicular to the surface of the slide.
3) A friction force directed towards the top of the slide.

Let's find the work done by each of these forces. To get there, the magnitude of the displacement is needed. This displacement is equal to the length of the slide. This length is

$$\sin(70°) = \frac{3m}{\Delta s}$$
$$\Delta s = 3.1925m$$

The work done by gravity is

$$\begin{aligned} W_g &= F_g \Delta s \cos\theta \\ &= 245 \cdot 3.1925m \cdot \cos(20°) \\ &= 735J \end{aligned}$$

The work done by the normal force vanishes, because there are 90° between the displacement and the force.

To find the work done by the friction force, the magnitude of the force of friction is needed. This force depends on the magnitude of the normal force. This magnitude is found with the equation for the *y*-components of the forces.

$$\sum F_y = ma_y$$
$$245N \cdot \sin(20°) + F_N = 0$$
$$F_N = 83.79N$$

The force of friction is therefore

$$\begin{aligned} F_f &= \mu_c F_N \\ &= 0.1 \cdot 83.79N \\ &= 8.379N \end{aligned}$$

The work done by the force of friction is therefore

$$\begin{aligned} W_f &= F_f \Delta s \cos\theta \\ &= 8.379N \cdot 3.1925m \cdot \cos(180°) \\ &= -26.75J \end{aligned}$$

The net work is thus

$$\begin{aligned} W_{net} &= W_g + W_N + W_f \\ &= 735J + 0J - 26.75J \\ &= 708.25J \end{aligned}$$

ΔE_k Calculation

The change in kinetic energy is

$$\Delta E_k = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$$

$$\Delta E_k = \frac{1}{2}mv'^2 - \frac{1}{2}\cdot 25kg\cdot\left(0\tfrac{m}{s}\right)^2$$

$$\Delta E_k = \frac{1}{2}mv'^2$$

Application of the Work-Energy Theorem

$$W_{net} = \Delta E_k$$

$$708.25J = \frac{1}{2}mv'^2$$

$$109.22J = \frac{1}{2}\cdot 25kg\cdot v'^2$$

$$v' = 7.527\tfrac{m}{s}$$

13. Full Stop

Knowing the work required to stop the skier, its mass can be found.

W_{net} Calculation

During the slowdown, only the friction force did a job on the skier and we know that the work done by the force is -3000 J.

ΔE_k Calculation

With a speed that goes from 10 m/s to 0 m/s, the change in kinetic energy is

$$\Delta E_k = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$$
$$\Delta E_k = \frac{1}{2}\cdot m\cdot\left(0\tfrac{m}{s}\right)^2 - \frac{1}{2}\cdot m\cdot\left(10\tfrac{m}{s}\right)^2$$
$$\Delta E_k = -\frac{1}{2}\cdot m\cdot\left(10\tfrac{m}{s}\right)^2$$

Application of the Work-Energy Theorem

$$W_{net} = \Delta E_k$$
$$-3000J = -\frac{1}{2}\cdot m\cdot\left(10\tfrac{m}{s}\right)^2$$
$$m = 60kg$$

Partial Stop

W_{net} Calculation

During the slowdown, only the friction force does a job on the skier and we know that the work done by the force is -1500 J.

ΔE_k Calculation

The change in kinetic energy is

$$\Delta E_k = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$$
$$\Delta E_k = \frac{1}{2}\cdot 60kg\cdot v'^2 - \frac{1}{2}\cdot 60kg\cdot\left(10\tfrac{m}{s}\right)^2$$
$$\Delta E_k = 30kg\cdot v'^2 - 3000J$$

Application of the Work-Energy Theorem

$$W_{net} = \Delta E_k$$
$$-1500J = 30kg\cdot v'^2 - 3000J$$
$$v' = 7.071\tfrac{m}{s}$$

14. *W_{net} Calculation*

During the fall, there are two forces exerted on René.

1) The 539 N weight directed downwards.
2) A friction force directed upwards.

The work done by the force of gravity is

$$\begin{aligned} W_g &= F_g \Delta s \cos\theta \\ &= 539N \cdot 300m \cdot \cos(0°) \\ &= 161,700J \end{aligned}$$

As to the work done by friction, nothing does allow us to find it for now. The net work is therefore

$$W_{net} = W_g + W_f$$
$$W_{net} = 161,700J + W_f$$

ΔE_k Calculation

With a speed that goes from 0 m/s at 39.4 m/s, the change in kinetic energy is

$$\begin{aligned} \Delta E_k &= \frac{1}{2} m v'^2 - \frac{1}{2} m v^2 \\ &= \frac{1}{2} \cdot 55kg \cdot \left(39.4 \tfrac{m}{s}\right)^2 - \frac{1}{2} \cdot 55kg \cdot \left(0 \tfrac{m}{s}\right)^2 \\ &= 42,689.9J \end{aligned}$$

Application of the Work-Energy Theorem

$$W_{net} = \Delta E_k$$
$$161,700J + W_f = 42,689.9J$$
$$W_f = -119,010.1J$$

15. *W_{net} Calculation*

During the descent of the block, there are two forces acting on the block:

1) The 1.96 N weight directed downwards.
2) The spring force directed upwards.

Let's call the stretch of the spring *d*. This stretching is equal to the displacement of the block.

The work done by gravity is

$$W_g = F_g \Delta s \cos\theta$$
$$W_g = 1.96N \cdot d \cdot \cos(0°)$$
$$W_g = 1.96N \cdot d$$

The work done by the spring is

$$W_R = -\frac{1}{2}k\left(x'^2 - x^2\right)$$
$$W_R = -\frac{1}{2} \cdot 50\tfrac{N}{m} \cdot \left(d^2 - (0m)^2\right)$$
$$W_R = -25\tfrac{N}{m} \cdot d^2$$

The net work is thus

$$W_{net} = W_g + W_R$$
$$W_{net} = 1.96N \cdot d - 25\tfrac{N}{m} \cdot d^2$$

ΔE_k Calculation

Initially, the speed is zero and it becomes zero again when the spring reaches the maximum stretch. The change in kinetic energy is therefore

$$\begin{aligned}\Delta E_k &= \frac{1}{2}mv'^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \cdot 0.2kg \cdot \left(0\tfrac{m}{s}\right)^2 - \frac{1}{2} \cdot 0.2kg \cdot \left(0\tfrac{m}{s}\right)^2 \\ &= 0J\end{aligned}$$

Application of the Work-Energy Theorem

$$W_{net} = \Delta E_k$$
$$1{,}96N \cdot d - 25\tfrac{N}{m} \cdot d^2 = 0$$

We note that there is a solution $d = 0$ m. This position is actually the initial position. We have this solution because the theorem will give us all positions where the speed is zero. As the speed was zero at the start, we found this solution. This is, however, not the solution that interests us. The other solution is

$$1.96N \cdot d - 25\tfrac{N}{m} \cdot d^2 = 0$$
$$1.96N - 25\tfrac{N}{m} \cdot d = 0$$
$$d = 0.0784m = 7.84cm$$

16. *W_{net} Calculation*

During the ascent, there are two forces acting on the ball.

1) The 4.9 N weight directed downwards (that is always acting).
2) A force made by the spring (which acts only at the beginning).

The total displacement of the ball is

$$\Delta s = 0.1m + h_{\max}$$

The work done by gravity is

$$W_g = F_g \Delta s \cos\theta$$
$$W_g = 4.9N \cdot (0.1m + h_{\max}) \cdot \cos(180°)$$
$$W_g = -4.9N \cdot (0.1m + h_{\max})$$

The work done by the spring is

$$\begin{aligned} W_R &= -\frac{1}{2}k\left(x'^2 - x^2\right) \\ &= -\frac{1}{2} \cdot 500\tfrac{N}{m} \cdot \left((0m)^2 - (0.10m)^2\right) \\ &= 2.5J \end{aligned}$$

The net work is thus

$$W_{net} = W_g + W_R$$
$$W_{net} = -4.9N \cdot (0.1m + h_{\max}) + 2.5J$$

ΔE_k Calculation

Initially, the speed is zero and it becomes zero again when the ball reaches its highest point. The change in kinetic energy is therefore

$$\begin{aligned}\Delta E_k &= \frac{1}{2}mv'^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2}\cdot 0.5kg\cdot\left(0\tfrac{m}{s}\right)^2 - \frac{1}{2}\cdot 0.5kg\cdot\left(0\tfrac{m}{s}\right)^2 \\ &= 0J\end{aligned}$$

Application of the Work-Energy Theorem

$$W_{net} = \Delta E_k$$
$$-4.9N\cdot\left(0.1m + h_{\max}\right) + 2.5J = 0$$
$$h_{\max} = 0.4102m = 41.02cm$$

17. *W_{net} Calculation*

During the motion uphill, there are three forces on the block:

1) The 4.9 N weight directed downwards (which is always acting).
2) A normal force perpendicular to the slope (which is always acting).
3) A force made by the spring (which acts only at the beginning).

The total displacement of the ball is

$$\Delta s = 0.5m + L$$

The work done by gravity is

$$W_g = F_g \Delta s \cos\theta$$
$$W_g = 4.9N\cdot\left(0.5m + L\right)\cdot\cos\left(110°\right)$$

The work done by the spring is

$$\begin{aligned}W_R &= -\frac{1}{2}k\left(x'^2 - x^2\right) \\ &= -\frac{1}{2}\cdot 100\tfrac{N}{m}\cdot\left(\left(0m\right)^2 - \left(0.50m\right)^2\right) \\ &= 12.5J\end{aligned}$$

The net work is thus

$$W_{net} = W_g + W_R$$
$$W_{net} = 4.9N \cdot (0.5m + L) \cdot \cos(110°) + 12.5J$$

ΔE_k Calculation

Initially, the speed is zero and it becomes zero again when the ball reaches its highest point. The change in kinetic energy is therefore

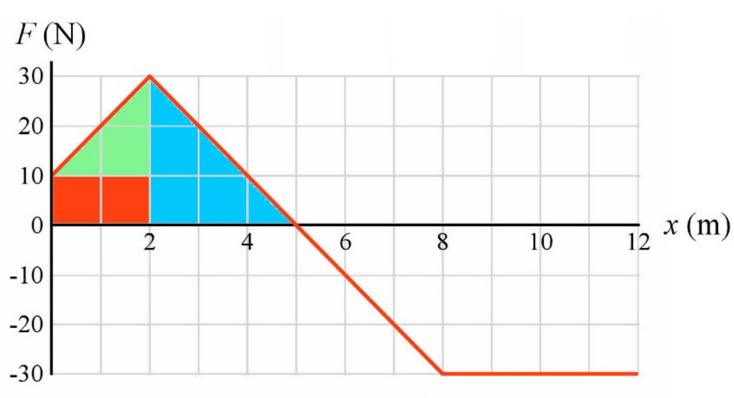
$$\begin{aligned}\Delta E_k &= \frac{1}{2}mv'^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \cdot 0.5kg \cdot \left(0\tfrac{m}{s}\right)^2 - \frac{1}{2} \cdot 0.5kg \cdot \left(0\tfrac{m}{s}\right)^2 \\ &= 0J\end{aligned}$$

Application of the Work-Energy Theorem

$$W_{net} = \Delta E_k$$
$$4.9N \cdot (0.5m + L) \cdot \cos(110°) + 12.5J = 0$$
$$L = 6.959m$$

18. a) *W_{net} Calculation*

Here, the work is found by calculating the area under the curve. This area is calculated by slitting the area (the splitting used is not the only way to split).



The area of the red rectangle is 10 N · 2 m = 20 J.

The area of the green triangle is ½ (2 m · 20 N) = 20 J.

The area of the blue triangle is ½ (3 m · 30 N) = 45 J.

The total area is therefore 20 J + 20 J + 45 J = 85 J.

The net work is thus

$$W_{net} = 85J$$

<u>*ΔE_k Calculation*</u>

The change in kinetic energy is

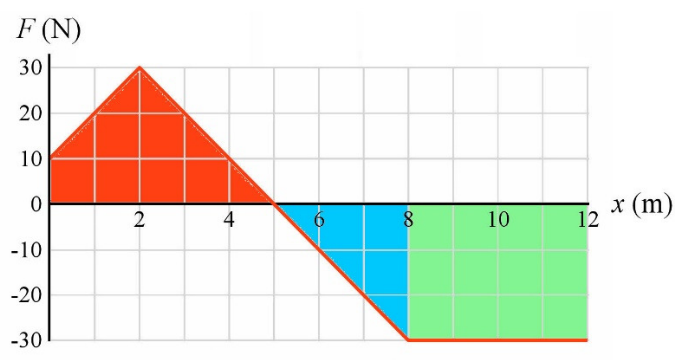
$$\Delta E_k = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$$
$$\Delta E_k = \frac{1}{2}\cdot 5kg \cdot v'^2 - \frac{1}{2}\cdot 5kg \cdot \left(2\tfrac{m}{s}\right)^2$$
$$\Delta E_k = 2.5kg \cdot v'^2 - 10J$$

<u>*Application of the Work-Energy Theorem*</u>

$$W_{net} = \Delta E_k$$
$$85J = 2.5kg \cdot v'^2 - 10J$$
$$v' = 6.164\tfrac{m}{s}$$

b) <u>*W_{net} Calculation*</u>

Here, the work is found by calculating the area under the curve. This area is calculated by slitting the area (the splitting used is not the only way to split).



The area of red part is 85 J (calculated in a).

The area of the blue triangle is -½ (3 m · 30 N) = -45 J.

The area of the green rectangle is - (4 m · 30 N) = -120 J.

The total area is therefore 85 J + -45 J + -120 J = -80 J.

The net work is thus

$$W_{net} = -80J$$

ΔE_k Calculation

The change in kinetic energy is

$$\Delta E_k = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$$
$$\Delta E_k = \frac{1}{2}\cdot 5kg\cdot v'^2 - \frac{1}{2}\cdot 5kg\cdot \left(2\tfrac{m}{s}\right)^2$$
$$\Delta E_k = 2.5kg\cdot v'^2 - 10J$$

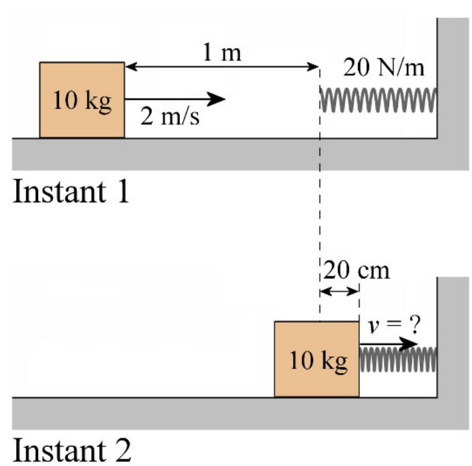
Application of the Work-Energy Theorem

$$W_{net} = \Delta E_k$$
$$-80J = 2.5kg\cdot v'^2 - 10J$$
$$-70J = 2.5kg\cdot v'^2$$

There's no solution to this equation. This means that the object cannot reach x = 12 m. The negative forces that acts on the object after x = 5 m prevent the object from reaching this place. In fact, it could be calculated that the object cannot go beyond x = 9.667 m.

19. a) *W_{net} Calculation*

We have the following positions at the instants 1 and 2.



There are 4 forces acting on the block.

1) The weight directed upwards.
2) A normal force directed upwards.
3) A friction force directed towards the left.
4) The spring force directed towards the left. This force will act only after an initial displacement of 1 m.

The work done by weight is zero.

The work done by the normal force is zero.

The friction force acts throughout the displacement (1.2 m). The work done by the friction force is then

$$\begin{aligned} W_f &= F_f \Delta s \cos\theta \\ &= \mu_c F_N \Delta s \cos\theta \\ &= \mu_c mg \Delta s \cos\theta \\ &= 0.1 \cdot 10kg \cdot 9.8\tfrac{N}{kg} \cdot 1.2m \cdot \cos 180° \\ &= -11.76J \end{aligned}$$

where it has used the fact that $F_N = mg$ (since the sum of the y-components of the forces is $F_N - mg = 0$).

The work done by the spring is

$$W_R = -\frac{k}{2}\left(x'^2 - x^2\right)$$

Initially, the compression of the spring is zero ($x = 0$) and at the end it is 0.2 m ($x' = 0.2$ m). So the work is

$$\begin{aligned} W_R &= -\frac{20\frac{N}{m}}{2} \cdot \left((0.2m)^2 - 0^2\right) \\ &= -0.4J \end{aligned}$$

The net work is thus

$$\begin{aligned} W_{net} &= W_f + W_R \\ &= -11.76J + -0.4J \\ &= -12.16J \end{aligned}$$

<u>*ΔE_k Calculation*</u>

The change in kinetic energy is

$$\Delta E_k = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$$

$$\Delta E_k = \frac{1}{2} \cdot 10kg \cdot v'^2 - \frac{1}{2} \cdot 10kg \cdot \left(2\tfrac{m}{s}\right)^2$$

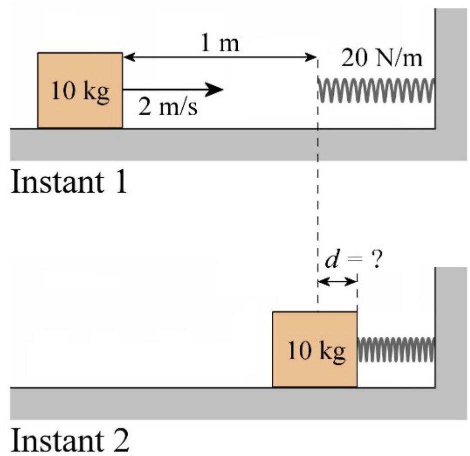
$$\Delta E_k = 5kg \cdot v'^2 - 20J$$

Application of the Work-Energy Theorem

$$W_{net} = \Delta E_k$$
$$-12.16J = 5kg \cdot v'^2 - 20J$$
$$v' = 1.252 \tfrac{m}{s}$$

b) *W_{net} Calculation*

We have the following positions at the instants 1 and 2.



The friction force acts throughout the displacement (1 m + *d*). The work done by the friction force is then

$$\begin{aligned} W_f &= F_f \Delta s \cos\theta \\ &= \mu_c F_N \Delta s \cos\theta \\ &= \mu_c mg \Delta s \cos\theta \\ &= 0.1 \cdot 10kg \cdot 9.8 \tfrac{N}{kg} \cdot (1m + d) \cdot \cos 180° \\ &= -9.8N \cdot (1m + d) \end{aligned}$$

The work done by the spring is

$$W_R = -\frac{k}{2}\left(x'^2 - x^2\right)$$

Initially, the compression of the spring is zero (x = 0) and at the end she is d (x' = d). The work is then

$$W_R = -\frac{k}{2}\left(d^2 - 0^2\right)$$
$$W_R = -10 \tfrac{N}{m} \cdot d^2$$

The net work is thus

$$\begin{aligned} W_{net} &= W_f + W_R \\ &= -9.8N \cdot (1m + d) - 10 \tfrac{N}{m} \cdot d^2 \end{aligned}$$

ΔE_k Calculation

As the block is at rest at maximum compression, the speed of the block decreases from 2 m/s (v) to 0 m/s (v'). The change in kinetic energy is therefore

$$\begin{aligned}\Delta E_k &= \frac{1}{2}mv'^2 - \frac{1}{2}mv^2\\ &= \frac{1}{2}\cdot 10kg\cdot\left(0\tfrac{m}{s}\right)^2 - \frac{1}{2}\cdot 10kg\cdot\left(2\tfrac{m}{s}\right)^2\\ &= -20J\end{aligned}$$

Application of the Work-Energy Theorem

$$\begin{aligned}W_{net} &= \Delta E_k\\ -9.8N\cdot\left(1m+d\right) - 10\tfrac{N}{m}\cdot d^2 &= -20J\\ 10\tfrac{N}{m}\cdot d^2 + 9.8N\cdot d - 10.2J &= 0\end{aligned}$$

If this quadratic equation is solved, we have d = 0.6325 m. (We also have d = - 1.6125 m, which corresponds to a stretching of the spring here, which makes no sense.)

20. While riding on the flat, the force made by the engine must be equal to the force of friction acting on the car.

The drag force is

$$\begin{aligned}F_d &= \frac{1}{2}\left(C_x A\right)\rho v^2\\ &= \frac{1}{2}\cdot\left(0.632m^2\right)\cdot 1.3\tfrac{kg}{m^3}\cdot\left(33.33\tfrac{m}{s}\right)^2\\ &= 456.44N\end{aligned}$$

The force made by the engine is thus 456.44 N forwards.

The power of this force is therefore

$$\begin{aligned}P &= Fv\cos\theta\\ &= 456.44N\cdot 33.33\tfrac{m}{s}\cdot\cos\left(0°\right)\\ &= 15.215W = 20.4hp\end{aligned}$$

21. If the piano moves up 2 m, then the work done by gravity is

$$\begin{aligned} W_g &= F_g \Delta s \cos\theta \\ &= 980N \cdot 2m \cdot \cos(180°) \\ &= -1960N \end{aligned}$$

As speed is zero at the beginning and at the end of the motion, the kinetic energy has not changed. Then, according to the work-energy theorem, we have

$$\begin{aligned} W_{net} &= 0 \\ W_g + W_{ropes} &= 0 \\ -1960J + W_{ropes} &= 0 \\ W_{ropes} &= 1960J \end{aligned}$$

As Laura is pulling on the rope to do this work, the work done by Laura is 1960 J.

As she did this work in 20 seconds, Laura's average power is

$$\begin{aligned} \overline{P} &= \frac{W}{\Delta t} \\ &= \frac{1960W}{20s} \\ &= 98W = 0.1314hp \end{aligned}$$

22. The tension force is the only force doing work on the crate. This work is thus the net work. According to the work-energy theorem, this net work is

$$\begin{aligned} W_{net} &= \Delta E_k \\ &= \frac{1}{2}mv'^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \cdot 5kg \cdot \left(20\tfrac{m}{s}\right)^2 - \frac{1}{2} \cdot 5kg \cdot \left(0\tfrac{m}{s}\right)^2 \\ &= 1000J \end{aligned}$$

It is the work done by the tension of the rope, so by the winch. The average power of the winch is therefore

$$\begin{aligned}\overline{P} &= \frac{W}{\Delta t}\\ &= \frac{1000J}{10s}\\ &= 100W\end{aligned}$$

The winch must then accelerate a mass of 100 kg to give it a speed of 10 m/s. Then the work done by the winch is

$$\begin{aligned}W_{net} &= \Delta E_k\\ &= \frac{1}{2}mv'^2 - \frac{1}{2}mv^2\\ &= \frac{1}{2}\cdot 100kg\cdot\left(10\tfrac{m}{s}\right)^2 - \frac{1}{2}\cdot 100kg\cdot\left(0\tfrac{m}{s}\right)^2\\ &= 5000J\end{aligned}$$

Since the average power is the same, we have

$$\begin{aligned}\overline{P} &= \frac{W}{\Delta t}\\ 100W &= \frac{5000J}{\Delta t}\\ \Delta t &= 50s\end{aligned}$$

23. a)

As the power of the winch is sought, the power of the tension force must be found. Let's find the tension first.

If there's no friction, there are 3 forces exerted on the crate (using an *x*-axis directed uphill).

1) The 490 N weight directed downwards.
2) The normal force perpendicular to the surface.
3) The tension force directed uphill.

The sum of the *x*-components of the forces is

$$\begin{aligned}\sum F_x &= ma_x\\ 490N\cdot\cos(-120°) + T &= 0\\ T &= 245N\end{aligned}$$

The power of the tension forces is then

$$\begin{aligned} P &= Fv\cos\theta \\ &= 245N \cdot 10\tfrac{m}{s} \cdot \cos(0°) \\ &= 2450W \end{aligned}$$

b) Once again, we have to find the power of the tension force. Let's find the tension first.

With friction, the forces exerted on the crate are (using an *x*-axis directed uphill):

1) The 490 N weight directed downwards.
2) The normal force perpendicular to the surface.
3) The tension force directed uphill.
4) A friction force directed downhill.

The equations of forces are

$$\sum F_x = ma_x$$
$$\rightarrow 490N \cdot \cos(-120°) + T - \mu_k F_N = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow 490N \cdot \sin(-120°) + F_N = 0$$

The normal force can be found with the sum of the *y*-components of the forces.

$$\begin{aligned} 490N \cdot \sin(-120°) + F_N &= 0 \\ F_N &= 424.35N \end{aligned}$$

This value is then substituted in the sum of the *x*-components of the forces.

$$\begin{aligned} 490N \cdot \cos(-120°) + T - \mu_k F_N &= 0 \\ 490N \cdot \cos(-120°) + T - 0.3 \cdot 424.35N &= 0 \\ T &= 372.3N \end{aligned}$$

The power is therefore

$$\begin{aligned} P &= Fv\cos\theta \\ &= 372.3N \cdot 10\tfrac{m}{s}\cos(0°) \\ &= 3723W \end{aligned}$$

24. a) According to the work-energy theorem, we have

$$\begin{aligned} W_{net} &= \Delta E_k \\ W_g + W_{engine} &= \Delta E_k \\ W_{engine} &= \Delta E_k - W_g \end{aligned}$$

This gives

$$W_{engine} = \Delta E_k - F_g \Delta s \cos\theta$$

To calculate this energy, we need the altitude of the rocket. Since the acceleration is constant, we have

$$\begin{aligned} y &= y_0 + \frac{1}{2}\left(v_{0y} + v_y\right)t \\ &= 0 + \frac{1}{2}\left(0 + 300\tfrac{m}{s}\right)\cdot 60s \\ &= 9000m \end{aligned}$$

Thus, the work done is

$$\begin{aligned} W_{engine} &= \Delta E_k - F_g \Delta s \cos\theta \\ &= \left(\tfrac{1}{2}mv'^2 - \tfrac{1}{2}mv^2\right) - mg\Delta s\cos\theta \\ &= \left(\tfrac{1}{2}\cdot 30kg\cdot\left(300\tfrac{m}{s}\right)^2 - \tfrac{1}{2}\cdot 30kg\cdot\left(0\tfrac{m}{s}\right)^2\right) - 30kg\cdot 9.8\tfrac{N}{kg}\cdot 9000m\cdot\cos 180° \\ &= 1,350,000J - -2,646,000J \\ &= 3,996,000J \end{aligned}$$

b) The average power of the engine is

$$\begin{aligned} P &= \frac{W}{\Delta t} \\ &= \frac{3,996,000J}{60s} \\ &= 66,600W = 89.28hp \end{aligned}$$

25. a) The work is

$$\begin{aligned} W &= \int P dt \\ &= \int_0^{10s} 12\tfrac{W}{s^2} \cdot t^2 dt \\ &= \left[4\tfrac{W}{s^2} \cdot t^3 \right]_{0s}^{10s} \\ &= 4000J \end{aligned}$$

b) The speed is found with the work-energy theorem.

$$\begin{aligned} W &= \Delta E_k \\ W &= \tfrac{1}{2} m v'^2 - \tfrac{1}{2} m v^2 \\ 4000J &= \tfrac{1}{2} \cdot 20kg \cdot v'^2 - 0 \\ v' &= 20\tfrac{m}{s} \end{aligned}$$

c) There are two options here. First, the force can be found with

$$P = Fv\cos\theta$$

at t = 10 s, the power is

$$\begin{aligned} P &= 12\tfrac{W}{s^2} \cdot t^2 \\ &= 1200W \end{aligned}$$

Therefore, the force is given by

$$1200W = F \cdot 20\tfrac{m}{s} \cdot \cos\theta$$

As the power is positive, the angle must be of 0°. Thus,

$$\begin{aligned} 1200W &= F \cdot 20\tfrac{m}{s} \cdot \cos 0° \\ F &= 60N \end{aligned}$$

Therefore, the acceleration is

$$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{60N}{20kg} \\ &= 3\tfrac{m}{s^2} \end{aligned}$$

The other option is to find the formula for the speed as a function of time. Ultimately, this calculation must be done for e).

The work at time *t* is

$$\begin{aligned} W &= \int P dt \\ &= \int_0^t 12 \tfrac{W}{s^2} \cdot t^2 dt \\ &= \left[4 \tfrac{W}{s^2} \cdot t^3 \right]_{0s}^{t} \\ &= 4 \tfrac{W}{s^2} \cdot t^3 \end{aligned}$$

With the work-energy theorem, the formula for the velocity as a function time is found.

$$\begin{aligned} W &= \Delta E_k \\ W &= \tfrac{1}{2} m v'^2 - \tfrac{1}{2} m v^2 \\ 4 \tfrac{W}{s^2} \cdot t^3 &= \tfrac{1}{2} \cdot 20kg \cdot v'^2 - 0 \\ v' &= \sqrt{0.4 \tfrac{m^2}{s^5}} \cdot t^{3/2} \end{aligned}$$

Acceleration is the derivative of velocity, the acceleration is

$$\begin{aligned} a &= \frac{dv'}{dt} \\ &= \sqrt{0.4 \tfrac{m^2}{s^5}} \cdot \frac{d\left(t^{3/2}\right)}{dt} \\ &= \frac{3}{2} \sqrt{0.4 \tfrac{m^2}{s^5}} \cdot t^{1/2} \end{aligned}$$

At $t = 10$ s, the acceleration is

$$\begin{aligned} a &= \frac{3}{2} \sqrt{0.4 \tfrac{m^2}{s^5}} \cdot \sqrt{10s} \\ &= 3 \tfrac{m}{s^2} \end{aligned}$$

d) There are two options here. First, the force can be found as done in c) with

$$P = Fv \cos \theta$$

At t = 10 s, the power is

$$\begin{aligned} P &= 12\tfrac{W}{s^2} \cdot t^2 \\ &= 1200W \end{aligned}$$

Therefore, the force is given by

$$1200W = F \cdot 20\tfrac{m}{s} \cos\theta$$

As the power is positive, the angle must be 0°. Thus, the force is

$$\begin{aligned} 1200W &= F \cdot 20\tfrac{m}{s} \cos 0° \\ F &= 60N \end{aligned}$$

If the acceleration was directly found in c), then the force is

$$\begin{aligned} F &= ma \\ &= 20kg \cdot 3\tfrac{m}{s^2} \\ &= 60N \end{aligned}$$

e) To find the displacement, the formula of the velocity as a function of time find in option 2 in c) is used

$$v' = \sqrt{0.4\tfrac{m^2}{s^5}} \cdot t^{3/2}$$

Thus, the displacement is

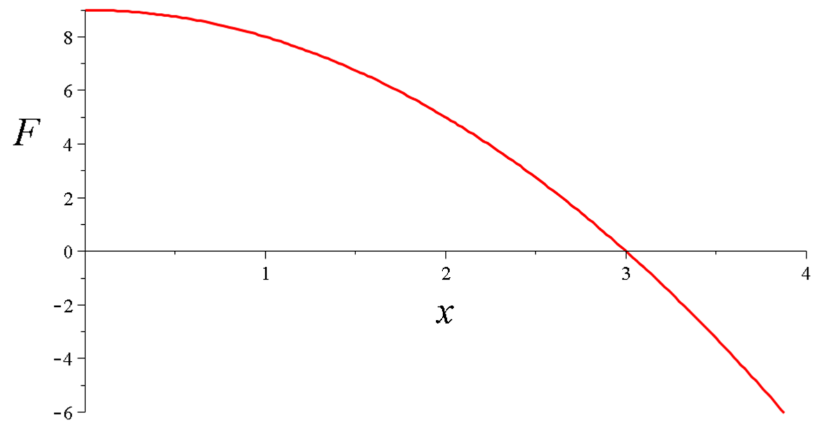
$$\Delta x = \int_{0s}^{10s} \sqrt{0.4\tfrac{m^2}{s^5}} \cdot t^{3/2} dt$$

This gives

$$\begin{aligned} \Delta x &= \sqrt{0.4\tfrac{m^2}{s^5}} \int_{0s}^{10s} t^{3/2} dt \\ &= \sqrt{0.4\tfrac{m^2}{s^5}} \left[\frac{2t^{5/2}}{5} \right]_{0s}^{10s} \\ &= \sqrt{0.4\tfrac{m^2}{s^5}} \cdot \frac{2 \cdot (10s)^{5/2}}{5} \\ &= 80m \end{aligned}$$

Note: the displacement cannot be found with $W = F\Delta x\cos\theta$ since the force is not constant.

26. Let's start by finding the position where the speed is maximum. The graph of the function is



At $x = 0$ m, the force is positive, and the object will accelerate towards the positive x-axis. As long as $x < 3$ m, the velocity is positive, which means that the velocity and the acceleration have the same sign and the object goes faster and faster towards the positive x-axis. At $x = 3$ m, the force becomes negative. Then, the velocity is positive whereas the acceleration is negative. This means that the object starts to slow down at $x = 3$ m. The maximum speed is thus reach at $x = 3$ m.
To find the speed, the work done on the object between $x = 0$ m and $x = 3$ m can be calculated. The work is

$$\begin{aligned}
W &= \int_{0m}^{3m} F dx \\
&= \int_{0m}^{3m} \left(9N - 1\tfrac{N}{m^2}\cdot x^2\right) dx \\
&= \left[9N\cdot x - \frac{1\frac{N}{m^2}}{3}\cdot x^3\right]_{0m}^{3m} \\
&= 9N\cdot 3m - \frac{1\frac{N}{m^2}}{3}\cdot (3m)^3 \\
&= 18J
\end{aligned}$$

The speed can then be found with the work-energy theorem.

$$W = \Delta E_k$$
$$18J = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

Since the initial velocity is zero, the equation becomes

$$18J = \frac{1}{2} \cdot 1kg \cdot v'^2 - 0$$
$$v' = 6\tfrac{m}{s}$$

27. a)

This trajectory is divided into two parts. First, there is the horizontal part (from (1,1) to (2,1)). Along this trajectory, *x* changes, but *y* is constant (*y* = 1 m). Thus, *dy* = 0 for this part. Therefore, the work done along this part of the trajectory is

$$\begin{aligned} W &= \int \vec{F} \cdot \overrightarrow{ds} \\ &= \int \left(\left(3\tfrac{N}{m^2} x^2 + 1\tfrac{N}{m} y \right) \vec{i} + \left(2\tfrac{N}{m} y + 1\tfrac{N}{m} x \right) \vec{j} \right) \cdot dx\vec{i} \\ &= \int_{1m}^{2m} \left(3\tfrac{N}{m^2} x^2 + 1\tfrac{N}{m} y \right) dx \\ &= \left[1\tfrac{N}{m^2} x^3 + 1\tfrac{N}{m} xy \right]_{1m}^{2m} \\ &= \left[1\tfrac{N}{m^2} \cdot (2m)^3 + 1\tfrac{N}{m} \cdot 2m \cdot 1m \right] - \left[1\tfrac{N}{m^2} \cdot (1m)^3 + 1\tfrac{N}{m} \cdot 1m \cdot 1m \right] \\ &= 10J - 2J \\ &= 8J \end{aligned}$$

Then there is the vertical part (from (2,1) to (2,2)). Along this path, *y* changes, but *x* is constant (*x* = 2 m). Thus, *dx* = 0 for this part. Therefore, the work done along this part of the trajectory is

$$\begin{aligned} W &= \int \vec{F} \cdot \overrightarrow{ds} \\ &= \int \left(\left(3\tfrac{N}{m^2} x^2 + 1\tfrac{N}{m} y \right) \vec{i} + \left(2\tfrac{N}{m} y + 1\tfrac{N}{m} x \right) \vec{j} \right) \cdot dy\vec{j} \\ &= \int_{1m}^{2m} \left(2\tfrac{N}{m} y + 1\tfrac{N}{m} x \right) dy \\ &= \left[1\tfrac{N}{m^2} y^2 + 1\tfrac{N}{m} xy \right]_{1m}^{2m} \\ &= \left[1\tfrac{N}{m^2} \cdot (2m)^2 + 1\tfrac{N}{m} \cdot 2m \cdot 2m \right] - \left[1\tfrac{N}{m^2} \cdot (1m)^2 + 1\tfrac{N}{m} \cdot 2m \cdot 1m \right] \\ &= 8J - 3J \\ &= 5J \end{aligned}$$

Thus, the total work for these two parts is 13 J.

b) This trajectory is divided into two parts. First, there is the vertical part (from (1,1) (1,2)). Along this path, y change, but x is constant (x = 1 m). Thus, dx = 0 for this part. Therefore, the work done along this part of the trajectory is

$$\begin{aligned}
W &= \int \vec{F} \cdot \overrightarrow{ds} \\
&= \int \left(\left(3\tfrac{N}{m^2} x^2 + 1\tfrac{N}{m} y\right)\vec{i} + \left(2\tfrac{N}{m} y + 1\tfrac{N}{m} x\right)\vec{j} \right) \cdot dy\vec{j} \\
&= \int_{1m}^{2m} \left(2\tfrac{N}{m} y + 1\tfrac{N}{m} x\right) dy \\
&= \left[1\tfrac{N}{m^2} y^2 + 1\tfrac{N}{m} xy\right]_{1m}^{2m} \\
&= \left[1\tfrac{N}{m^2} \cdot (2m)^2 + 1\tfrac{N}{m} \cdot 1m \cdot 2m\right] - \left[1\tfrac{N}{m^2} \cdot (1m)^2 + 1\tfrac{N}{m} \cdot 1m \cdot 1m\right] \\
&= 6J - 2J \\
&= 4J
\end{aligned}$$

Then there is the horizontal part (from (1,2) to (2,2)). Along this path, x changes, but y is constant (y = 2 m). Thus, dy = 0 for this part. Therefore, the work done along this part of the trajectory is

$$\begin{aligned}
W &= \int \vec{F} \cdot \overrightarrow{ds} \\
&= \int \left(\left(3\tfrac{N}{m^2} x^2 + 1\tfrac{N}{m} y\right)\vec{i} + \left(2\tfrac{N}{m} y + 1\tfrac{N}{m} x\right)\vec{j} \right) \cdot dx\vec{i} \\
&= \int_{1m}^{2m} \left(3\tfrac{N}{m^2} x^2 + 1\tfrac{N}{m} y\right) dx \\
&= \left[1\tfrac{N}{m^2} x^3 + 1\tfrac{N}{m} xy\right]_{1m}^{2m} \\
&= \left[1\tfrac{N}{m^2} \cdot (2m)^3 + 1\tfrac{N}{m} \cdot 2m \cdot 2m\right] - \left[1\tfrac{N}{m^2} \cdot (1m)^3 + 1\tfrac{N}{m} \cdot 1m \cdot 2m\right] \\
&= 12J - 3J \\
&= 9J
\end{aligned}$$

Thus, the total work for these two parts is 13 J.

c) Along this trajectory, both x and y vary. Thus, the work is

$$\begin{aligned}
W &= \int \vec{F} \cdot \overrightarrow{ds} \\
&= \int \left(\left(3\tfrac{N}{m^2} x^2 + 1\tfrac{N}{m} y\right)\vec{i} + \left(2\tfrac{N}{m} y + 1\tfrac{N}{m} x\right)\vec{j} \right) \cdot \left(dx\vec{i} + dy\vec{j}\right) \\
&= \int \left[\left(3\tfrac{N}{m^2} x^2 + 1\tfrac{N}{m} y\right) dx + \left(2\tfrac{N}{m} y + 1\tfrac{N}{m} x\right) dy \right]
\end{aligned}$$

As $x = y$ along this path, we can write

$$\begin{aligned}
W &= \int\left[\left(3\tfrac{N}{m^2}x^2 + 1\tfrac{N}{m}y\right)dx + \left(2\tfrac{N}{m}y + 1\tfrac{N}{m}x\right)dy\right] \\
&= \int\left[\left(3\tfrac{N}{m^2}x^2 + 1\tfrac{N}{m}x\right)dx + \left(2\tfrac{N}{m}y + 1\tfrac{N}{m}y\right)dy\right] \\
&= \int\left[\left(3\tfrac{N}{m^2}x^2 + 1\tfrac{N}{m}x\right)dx + \left(3\tfrac{N}{m}y\right)dy\right] \\
&= \int_{1m}^{2m}\left(3\tfrac{N}{m^2}x^2 + 1\tfrac{N}{m}x\right)dx + \int_{1m}^{2m}\left(3\tfrac{N}{m}y\right)dy \\
&= \left[1\tfrac{N}{m^2}x^3 + \frac{1\frac{N}{m}}{2}x^2\right]_{1m}^{2m} + \left[\frac{3\frac{N}{m}}{2}y^2\right]_{1m}^{2m} \\
&= \left[1\tfrac{N}{m^2}\cdot(2m)^3 + \frac{1\frac{N}{m}}{2}\cdot(2m)^2\right] - \left[1\tfrac{N}{m^2}\cdot(1m)^3 + \frac{1\frac{N}{m}}{2}\cdot(1m)^2\right] + \left[\frac{3\frac{N}{m}}{2}\cdot(2m)^2\right] - \left[\frac{3\frac{N}{m}}{2}\cdot(1m)^2\right] \\
&= 10J - \tfrac{3}{2}J + 6J - \tfrac{3}{2}J \\
&= 13J
\end{aligned}$$

Note:

The result always seems to be 13 J. In fact, it is the case here. Regardless of the path, the work is always 13 J between these two points.

The work is the same for all trajectories with this force, but this is not always the case. For some forces, the work may be different depending on the trajectory.

We'll talk about this in chapter 9.