Chapter 7 Solutions

1. a) Karl's weight is

$$\begin{aligned} w &= mg \\ &= 70kg \cdot 9.8\tfrac{N}{kg} \\ &= 686N \end{aligned}$$

Directed downwards.

b) With an acceleration of 6 m/s² upwards, the components of the apparent weight are

$$\begin{aligned} w_{app\ x} &= -ma_x \\ &\rightarrow P_{app\ x} = 0 \\ w_{app\ y} &= -mg - ma_y \\ &\rightarrow w_{app\ y} = -70kg \cdot 9.8\tfrac{N}{kg} - 70kg \cdot 6\tfrac{m}{s^2} \\ &\rightarrow w_{app\ y} = -1106N \end{aligned}$$

The apparent weight is thus 1106 N downwards.

c) The number of g is

$$\begin{aligned} n_g &= \frac{\left|w_{app}\right|}{w_{\text{actual on Earth}}} \\ &= \frac{1106N}{686N} \\ &= 1.612 \end{aligned}$$

2. a) Karl's weight is

$$\begin{aligned} w &= mg \\ &= 70kg \cdot 1.6\tfrac{N}{kg} \\ &= 112N \end{aligned}$$

Directed downwards.

b) With an acceleration of 6 m/s² upwards, the components of the apparent weight are

$$w_{app\ x} = -ma_x$$
$$\to P_{app\ x} = 0$$
$$w_{app\ y} = -mg - ma_y$$
$$\to w_{app\ y} = -70kg \cdot 1.6\tfrac{N}{kg} - 70kg \cdot 6\tfrac{m}{s^2}$$
$$\to w_{app\ y} = -532N$$

Therefore, the apparent weight is 532 N downwards.

c) The number of *g* is

$$n_g = \frac{|w_{app}|}{w_{\text{actual on earth}}} = \frac{532N}{686N} = 0.7755$$

3. To find the apparent weight, the acceleration is needed. Since the car goes from 0 to 100 km/h in 1.8 seconds, the acceleration is

$$v_x = v_{0x} + a_x t$$
$$27.78\tfrac{m}{s} = 0\tfrac{m}{s} + a \cdot 1.8s$$
$$a_x = 15.432\tfrac{m}{s^2}$$

The components of the apparent weight are therefore

$$w_{app\ x} = -ma_x$$
$$\to P_{app\ x} = -ma_x$$
$$w_{app\ y} = -mg - ma_y$$
$$\to w_{app\ y} = -mg - 0$$

Therefore, the magnitude of the apparent weight is

$$w_{app} = \sqrt{w_{appx}^2 + w_{appy}^2}$$
$$w_{app} = \sqrt{(-ma_x)^2 + (-mg)^2}$$
$$w_{app} = \sqrt{(ma_x)^2 + (mg)^2}$$

Then, the number of g is

$$\begin{aligned}
n_g &= \frac{|w_{app}|}{w_{\text{actual sur Terre}}} \\
&= \frac{\sqrt{(ma_x)^2 + (mg)^2}}{mg} \\
&= \frac{\sqrt{m^2 a_x^2 + m^2 g^2}}{mg} \\
&= \frac{\sqrt{m^2 (a_x^2 + g^2)}}{mg} \\
&= \frac{\sqrt{m^2}\sqrt{(a_x^2 + g^2)}}{mg} \\
&= \frac{\cancel{m}\sqrt{a_x^2 + g^2}}{\cancel{m}g} \\
&= \frac{\sqrt{(15.432\,\frac{m}{s^2})^2 + (9.8\,\frac{N}{kg})^2}}{9.8\,\frac{N}{kg}} \\
&= 1.865
\end{aligned}$$

4. a) The components of the acceleration are

$$a_x = 6\,\tfrac{m}{s^2} \cdot \cos 60° = 3\,\tfrac{m}{s^2}$$
$$a_y = 6\,\tfrac{m}{s^2} \cdot \sin 60° = 5.196\,\tfrac{m}{s^2}$$

The components of the apparent weight are therefore

$$w_{app\,x} = -ma_x$$
$$\rightarrow w_{app\,x} = -70kg \cdot 3\tfrac{m}{s^2}$$
$$\rightarrow w_{app\,x} = -210N$$
$$w_{app\,y} = -mg - ma_y$$
$$\rightarrow w_{app\,y} = -70kg \cdot 9.8\tfrac{N}{kg} - 70kg \cdot 5.196\tfrac{m}{s^2}$$
$$\rightarrow w_{app\,y} = -1049.73N$$

Therefore, the magnitude of the apparent weight is

$$\begin{aligned} w_{app} &= \sqrt{w_{appx}^2 + w_{appy}^2} \\ &= \sqrt{(-210N)^2 + (-1049.73N)^2} \\ &= 1070.53N \end{aligned}$$

The direction of the apparent weight is

$$\begin{aligned} \theta &= \arctan\frac{w_{appy}}{w_{appx}} \\ &= \arctan\frac{-1049.73N}{-210N} \\ &= -101.3° \end{aligned}$$

(180° were removed to the answer given by the calculator because w_{appx} is negative.)

b) The number of g is

$$\begin{aligned} n_g &= \frac{|w_{app}|}{w_{\text{actual on Earth}}} \\ &= \frac{1070.53N}{686N} \\ &= 1.56 \end{aligned}$$

5. a)

At point A, the acceleration is v^2/r upwards. The components of the apparent weight are therefore

$$w_{app\,x} = -ma_x$$
$$\rightarrow w_{app\,x} = 0$$
$$w_{app\,y} = -mg - ma_y$$
$$\rightarrow w_{app\,y} = -mg - m\frac{v^2}{r}$$
$$\rightarrow w_{app\,y} = -120kg \cdot 9.8\tfrac{N}{kg} - 120kg \cdot \frac{\left(25\tfrac{m}{s}\right)^2}{10m}$$
$$\rightarrow P_{app\,y} = -8676N$$

The number of g is then

$$n_g = \frac{\left|w_{app}\right|}{w_{\text{actual on Earth}}} = \frac{8676N}{1176N} = 7.378$$

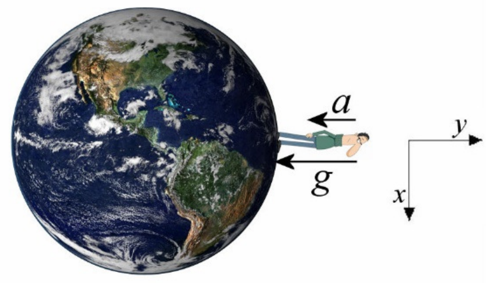
b) At point B, the acceleration is v^2/r downwards. The components of the apparent weight are therefore

$$w_{app\,x} = -ma_x$$
$$\rightarrow w_{app\,x} = 0$$
$$w_{app\,y} = -mg - ma_y$$
$$\rightarrow w_{app\,y} = -mg - m\cdot\left(-\frac{v^2}{r}\right)$$
$$\rightarrow w_{app\,y} = -120kg \cdot 9.8\tfrac{N}{kg} + 120kg \cdot \frac{\left(10\tfrac{m}{s}\right)^2}{15m}$$
$$\rightarrow w_{app\,y} = -376N$$

The number of g is then

$$n_g = \frac{\left|w_{app}\right|}{w_{\text{actual on earth}}} = \frac{376N}{1176N} = 0.32$$

6. With an acceleration of $4\pi^2 r/T^2$ towards the centre of the Earth, the components of the apparent weight are therefore



$$w_{app\ x} = -ma_x$$
$$\to P_{app\ x} = 0$$
$$w_{app\ y} = -mg - ma_y$$
$$\to w_{app\ y} = -mg - m\left(-\frac{4\pi^2 r}{T^2}\right)$$

If we want a vanishing apparent weight, we must have

$$0 = -mg - m\left(-\frac{4\pi^2 r}{T^2}\right)$$
$$\frac{4\pi^2 r}{T^2} = g$$
$$T = \sqrt{\frac{4\pi^2 r}{g}}$$
$$T = \sqrt{\frac{4\pi^2 \cdot 6.378\times10^6 m}{9.8\frac{N}{kg}}}$$
$$T = 5069s = 84.48\min$$

7. a) With an acceleration of v^2/r downwards, the components of the apparent weight are

$$w_{app\ x} = -ma_x$$
$$\to P_{app\ x} = 0$$
$$w_{app\ y} = -mg - ma_y$$
$$\to w_{app\ y} = -mg - m\left(-\frac{v^2}{r}\right)$$

The only remaining component is the y-component. It is

$$w_{app\ y} = -mg + m\frac{v^2}{r}$$
$$= -60kg \cdot 9.8\tfrac{N}{kg} + 60kg \cdot \frac{\left(250\frac{m}{s}\right)^2}{5000m}$$
$$= 162N$$

Therefore, the apparent weight is 162 N upwards. Juliette could thus walk on the ceiling of the aircraft.

b) The number of g is

$$\begin{aligned} n_g &= \frac{\left|w_{app}\right|}{w_{\text{actual on Earth}}} \\ &= \frac{162N}{588N} \\ &= 0.2755 \end{aligned}$$

8. With an acceleration of v^2/r downwards, the components of the apparent weight are

$$\begin{aligned} w_{app\ x} &= -ma_x \\ &\rightarrow w_{app\ x} = 0 \\ w_{app\ y} &= -mg - ma_y \\ &\rightarrow w_{app\ y} = -mg - m\left(-\frac{v^2}{r}\right) \end{aligned}$$

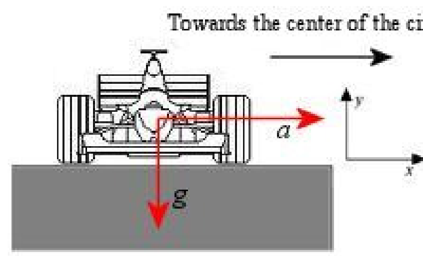
As Victor's weight is 50 kg x 9.8 N/kg = 490 N, the magnitude of Victor's apparent weight must be 980 N.

With an apparent weight upwards, we have

$$\begin{aligned} w_{app\ y} &= -mg + m\frac{v^2}{r} \\ 980N &= -50kg \cdot 9.8\tfrac{N}{kg} + 50kg \cdot \frac{v^2}{10m} \\ v &= 17.146\tfrac{m}{s} \end{aligned}$$

9. With an acceleration of v^2/r towards the right, the components of the apparent weight are

$$w_{app\ x} = -ma_x$$
$$\rightarrow w_{app\ x} = -m\frac{v^2}{r}$$
$$w_{app\ y} = -mg - ma_y$$
$$\rightarrow w_{app\ y} = -mg$$



www.draftsperson.net/index.php?title=Formula_1_-_Free_AutoCAD_Blocks

Therefore, the magnitude of the apparent weight is

$$w_{app} = \sqrt{w_{app\ x}^2 + w_{app\ y}^2}$$
$$w_{app} = \sqrt{\left(-m\frac{v^2}{r}\right)^2 + (-mg)^2}$$
$$w_{app} = \sqrt{\left(m\frac{v^2}{r}\right)^2 + (mg)^2}$$

The number of g is

$$n_g = \frac{|w_{app}|}{w_{\text{actuel on Earth}}}$$
$$= \frac{\sqrt{\left(m\frac{v^2}{r}\right)^2 + (mg)^2}}{mg}$$
$$= \frac{\cancel{m}\sqrt{\left(\frac{v^2}{r}\right)^2 + (g)^2}}{\cancel{m}g}$$
$$= \frac{\sqrt{\left(\frac{v^2}{r}\right)^2 + (g)^2}}{g}$$

As the pilot experiences 4 g, we have

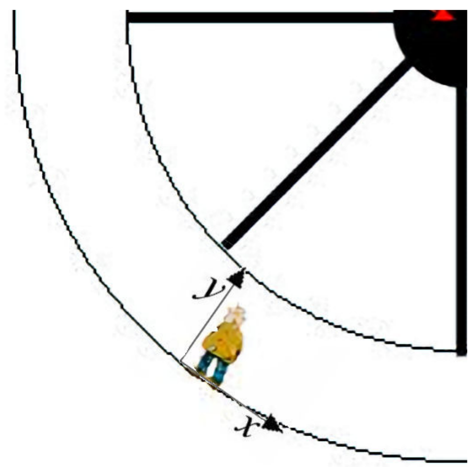
$$4 = \frac{\sqrt{\left(\frac{v^2}{r}\right)^2 + (g)^2}}{g}$$

$$4g = \sqrt{\left(\frac{v^2}{r}\right)^2 + (g)^2}$$

$$4 \cdot 9.8 \tfrac{N}{kg} = \sqrt{\left(\frac{\left(50 \tfrac{m}{s}\right)^2}{r}\right)^2 + \left(9.8 \tfrac{N}{kg}\right)^2}$$

$$r = 65.867m$$

10. When there's no gravity, the axes can have any orientation. Using the axes shown in the figure, the components of apparent weight are



$$w_{app\ x} = -ma_x$$

$$\rightarrow w_{app\ x} = 0$$

$$w_{app\ y} = -mg - ma_y$$

$$\rightarrow w_{app\ y} = -0 - m\frac{4\pi^2 r}{T^2}$$

The number of g is thus

$$n_g = \frac{\left|w_{app}\right|}{w_{\text{actual on Earth}}} = \frac{m\frac{4\pi^2 r}{T^2}}{mg} = \frac{4\pi^2 r}{T^2 g}$$

If the person must experience 1 g, we have

$$1=\frac{4\pi^2 r}{T^2 g}$$

$$1=\frac{4\pi^2\cdot 12m}{T^2\cdot 9.8\frac{N}{kg}}$$

$$T=6.953s$$

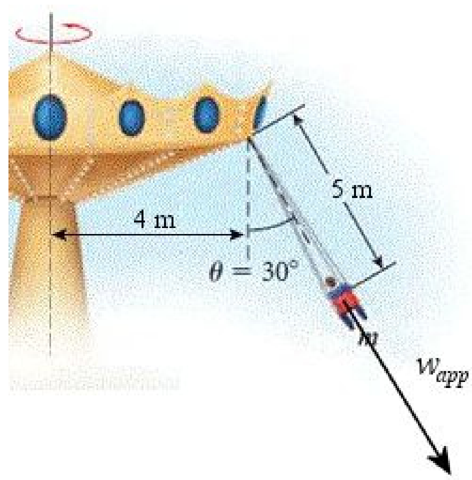
11. The only forces acting on the person are the weight and the tension force of the rope. Therefore

$$\vec{w}_{app}=-\left(\sum\vec{F}-m\vec{g}\right)$$

$$\vec{w}_{app}=-\left(\left(\vec{T}+m\vec{g}\right)-m\vec{g}\right)$$

$$\vec{w}_{app}=-\vec{T}$$

The apparent weight is in the direction opposite to the tension force of the rope.



The direction of the apparent weight is therefore -60°.

To find the direction of the apparent weight, the components must be known. These components are

$$w_{app\,x}=-ma_x$$

$$\rightarrow w_{app\,x}=-m\left(-\frac{4\pi^2 r}{T^2}\right)$$

$$w_{app\,y}=-mg-ma_y$$

$$\rightarrow w_{app\,y}=-mg$$

The direction is therefore

$$\begin{aligned}\tan\theta &= \frac{w_{app\ y}}{w_{app\ x}} \\ &= \frac{-mg}{m\frac{4\pi^2 r}{T^2}} \\ &= \frac{-T^2 g}{4\pi^2 r}\end{aligned}$$

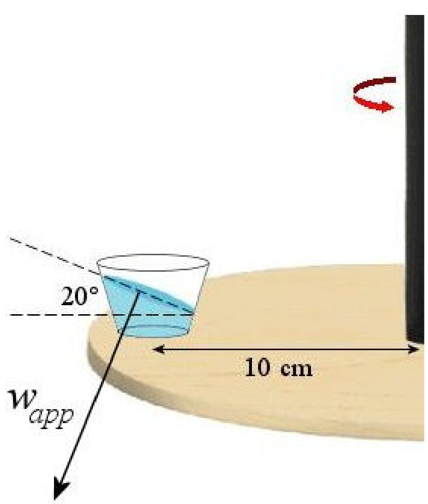
Before finding the solution to this equation, the radius of the circular path (i.e. the distance between the person and the axis of rotation of the merry-go-round) must be known. This distance is

$$\begin{aligned}r &= 4m + 5m\cdot\sin 30^\circ \\ &= 6.5m\end{aligned}$$

The equation then becomes

$$\begin{aligned}\tan\theta &= \frac{-T^2 g}{4\pi^2 r} \\ \tan\left(-60^\circ\right) &= \frac{-T^2\cdot 9.8\frac{N}{kg}}{4\pi^2\cdot 6.5m} \\ T &= 6.734s\end{aligned}$$

12. Since the water surface is perpendicular to the direction of the apparent weight, the direction of the apparent weight is -110° as shown in this figure.



To find the direction of the apparent weight, the components must be known. These components are

$$w_{app\,x} = -ma_x$$

$$\rightarrow w_{app\,x} = -m\frac{4\pi^2 r}{T^2}$$

$$w_{app\,y} = -mg - ma_y$$

$$\rightarrow w_{app\,y} = -mg$$

The direction is

$$\begin{aligned}\tan\theta &= \frac{w_{app\,y}}{w_{app\,x}} \\ &= \frac{-mg}{-m\frac{4\pi^2 r}{T^2}} \\ &= \frac{T^2 g}{4\pi^2 r}\end{aligned}$$

This gives

$$\tan(-110°) = \frac{T^2 \cdot 9.8\frac{N}{kg}}{4\pi^2 \cdot 0.1m}$$

$$T = 1.052s$$

If the glass is now 6 cm from the axis, the direction of the apparent weight becomes

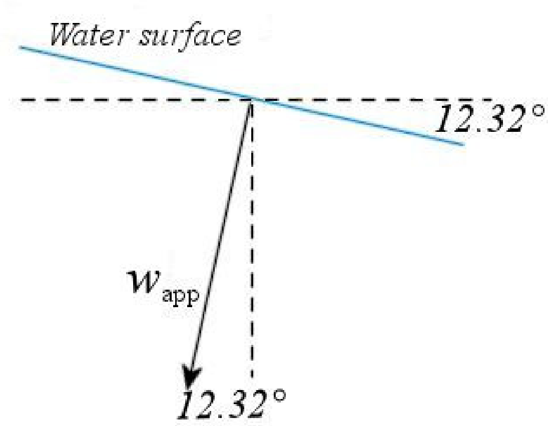
$$\tan\theta = \frac{T^2 g}{4\pi^2 r}$$

$$\tan\theta = \frac{(1.052s)^2 \cdot 9.8\frac{N}{kg}}{4\pi^2 \cdot 0.06m}$$

$$\tan\theta = 4.579$$

$$\theta = 77.68° \quad \text{or} \quad -102.32°$$

The first response (an apparent weight almost pointing upwards) has no meaning. The second answer is the right answer. Since the apparent weight is tilted 12.32° from the vertical, the water surface is inclined at 12.32° relative to the horizontal.



13. a) The initial y-component of the velocity can be found with

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

As the plane returns to the same height, we have $y = y_0$. Therefore,

$$\begin{gathered} y_0 = y_0 + v_{0y}t - \frac{1}{2}gt^2 \\ 0 = v_{0y}t - \frac{1}{2}gt^2 \\ 0 = v_{0y} - \frac{1}{2}gt \\ \frac{1}{2}gt = v_{0y} \\ \frac{1}{2}\cdot 9.8\frac{m}{s^2}\cdot 25s = v_{0y} \\ 122.5\frac{m}{s} = v_{0y} \end{gathered}$$

As the y-component of the velocity is $v_{0y} = v_0 \sin\theta$, and since v_0 = 200 m/s, we have

$$\begin{gathered} 122.5\frac{m}{s} = v_0 \sin\theta \\ 122.5\frac{m}{s} = 200\frac{m}{s}\cdot\sin\theta \\ 0.6125 = \sin\theta \\ \theta = 37.77° \end{gathered}$$

b) At the highest point, the y-component of the velocity is zero. Therefore,

$$2a_y\left(y-y_0\right)=v_y^2-v_{0y}^2$$
$$2\cdot\left(-9.8\tfrac{m}{s^2}\right)\cdot\Delta y=0-\left(122.5\tfrac{m}{s}\right)^2$$
$$\Delta y=765.6m$$

14. a)

As this force is in the direction opposite to the apparent weight, the direction of the apparent weight must be found. We find this direction with the components of the apparent weight

$$w_{app\,x}=-ma_x$$
$$\rightarrow w_{app\,x}=-ma_x$$
$$w_{app\,y}=-mg-ma_y$$
$$\rightarrow w_{app\,y}=-mg$$

The direction of the apparent weight is

$$\begin{aligned}\theta&=\arctan\frac{w_{app\,y}}{w_{app\,x}}\\&=\arctan\frac{-mg}{-ma_x}\\&=\arctan\frac{-g}{-a_x}\\&=\arctan\frac{-9.8\frac{N}{kg}}{-2\frac{m}{s^2}}\\&=-101.53^\circ\end{aligned}$$

The buoyant force in the opposite direction, at 78.47°.

The number of g is also required to calculate the magnitude of the buoyant force. The magnitude of the apparent weight is

$$w_{app}=\sqrt{w_{app\,x}^2+w_{app\,y}^2}$$
$$w_{app}=\sqrt{\left(-ma\right)^2+\left(-mg\right)^2}$$
$$w_{app}=\sqrt{\left(ma\right)^2+\left(mg\right)^2}$$

The number of g is

$$\begin{aligned} n_g &= \frac{|w_{app}|}{w_{\text{actual on Earth}}} \\ &= \frac{\sqrt{(ma)^2 + (mg)^2}}{mg} \\ &= \frac{\cancel{m}\sqrt{(a)^2 + (g)^2}}{\cancel{m}g} \\ &= \frac{\sqrt{\left(2\frac{m}{s^2}\right)^2 + \left(9.8\frac{N}{kg}\right)^2}}{9.8\frac{N}{kg}} \\ &= 1.0206 \end{aligned}$$

The magnitude of buoyant force is therefore

$$\begin{aligned} F_A &= \rho n_g \cdot 9.8\tfrac{N}{kg} \cdot V_f \\ &= 1000\tfrac{kg}{m^3} \cdot 1.0206 \cdot 9.8\tfrac{N}{kg} \cdot 0.0004m^3 \\ &= 4.0008N \end{aligned}$$

b) To find the tension, the sum of the forces on the block will be found. The forces on the block of cedar are:

1) The weight, 3.43 N downwards.
2) The tension force *T*.
3) The buoyant force, 4.0008 N at 78.47°.

The equations of forces are

$$\begin{aligned} &\sum F_x = ma_x \\ &\qquad \to T_x + 4.0008N\cos(78.47°) = 0.35kg \cdot 2\tfrac{m}{s^2} \\ &\sum F_y = ma_y \\ &\qquad \to -3.43N + T_y + 4.0008N\sin(78.47°) = 0 \end{aligned}$$

The sum of the *x*-component of the forces gives us

$$\begin{aligned} T_x + 4.0008N \cdot \cos(78.47°) &= 0.35kg \cdot 2\tfrac{m}{s^2} \\ T_x + 0.79997N &= 0.7N \\ T_x &= -0.09997N \end{aligned}$$

The sum of the y-component of the forces gives us

$$-3.43N + T_y + 4.0008N \cdot \sin(78.47°) = 0$$
$$-3.43N + T_y + 3.92N = 0$$
$$T_y = -0.49N$$

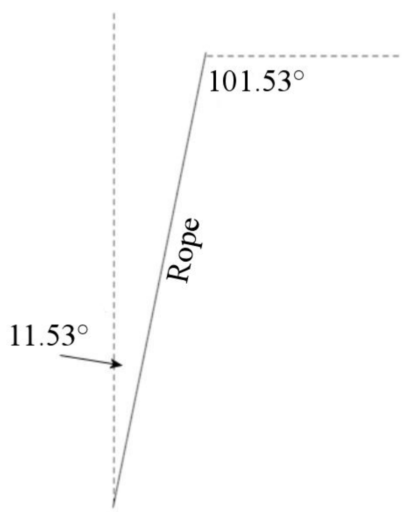
The tension is therefore

$$\begin{aligned} T &= \sqrt{T_x^2 + T_y^2} \\ &= \sqrt{(-0.09997N)^2 + (-0.49N)^2} \\ &= 0.50009N \end{aligned}$$

c) The direction of the tension force is

$$\begin{aligned} \theta &= \arctan \frac{T_y}{T_y} \\ &= \arctan \frac{-0.49N}{-0.09997N} \\ &= -101.53° \end{aligned}$$

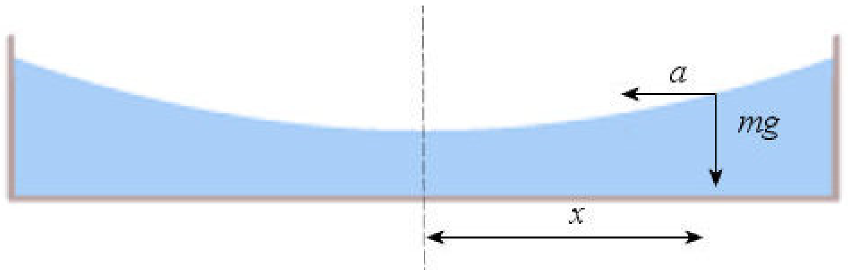
We then have



The angle is therefore 11.53°.

The tension force of the rope is directly opposed to buoyancy.

15. Let's find the direction of the apparent weight on the surface at a distance x from the axis of rotation. At this distance, the weight of a molecule of water is directed downwards and its acceleration is towards the axis of rotation.



Therefore, the components of the apparent weight are

$$w_{app\,x} = -ma_x$$

$$\rightarrow w_{app\,x} = -m\left(-\frac{4\pi^2 x}{T^2}\right)$$

$$w_{app\,y} = -mg - ma_y$$

$$\rightarrow w_{app\,y} = -mg$$

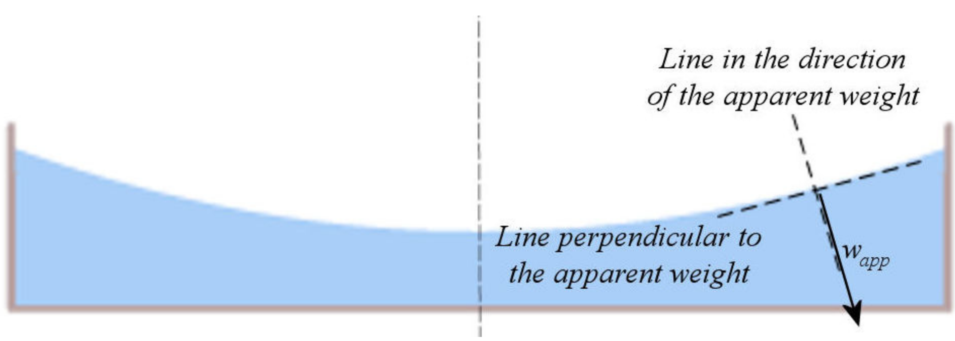
Therefore, the direction of apparent weight is

$$\tan\theta = \frac{w_{app\,y}}{w_{app\,x}}$$

$$= \frac{-mg}{m\left(\frac{4\pi^2 x}{T^2}\right)}$$

$$= \frac{-gT^2}{4\pi^2 x}$$

This tangent is the slope of a line in the direction of the apparent weight.



As the surface is perpendicular to this line, the slope of a line parallel to the surface is

$$slope = \frac{4\pi^2 x}{gT^2}$$

(Since the link between the slopes of two perpendicular lines is $m_1 m_2$ = -1.)

Then, we have the slope of the surface at the distance x. As the slope is the derivative, we have

$$\frac{dy}{dx} = \frac{4\pi^2 x}{gT^2}$$

Integrating this equation, we obtain

$$y = \frac{2\pi^2 x^2}{gT^2} + cst$$

If the height of the liquid at $x = 0$ is y_0, then the constant is y_0. Therefore,

$$y = \frac{2\pi^2 x^2}{gT^2} + y_0$$

This is the equation of the surface. It is a parabola.