7 APPARENT WEIGHT

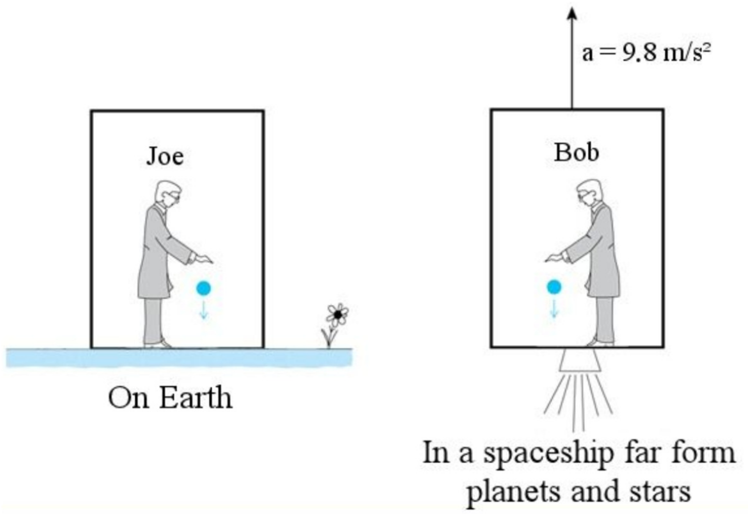
What are the apparent weight (magnitude and direction) and the number of g's experienced by a 70 kg fighter pilot when his aircraft is catapulted from an aircraft carrier knowing that the plane accelerates up to a speed of 77 m/s (150 knots) over a distance of 94.5 m (on the USS Nimitz)?

commons.wikimedia.org/wiki/File:US_Navy_071006-N-4166B-033_An_F-A-18_Hornet_attached_to_the_Warhawks_of_Strike_Fighter_Squadron_(VFA)_97_conducts_a_touch_and_go_landing_and_takeoff_aboard_the_Nimitz-class_aircraft_carrier_USS_Abraham_Lincoln_(CVN_72).jpg

Discover the answer to this question in this chapter.

7.1 APPARENT WEIGHT FORMULA

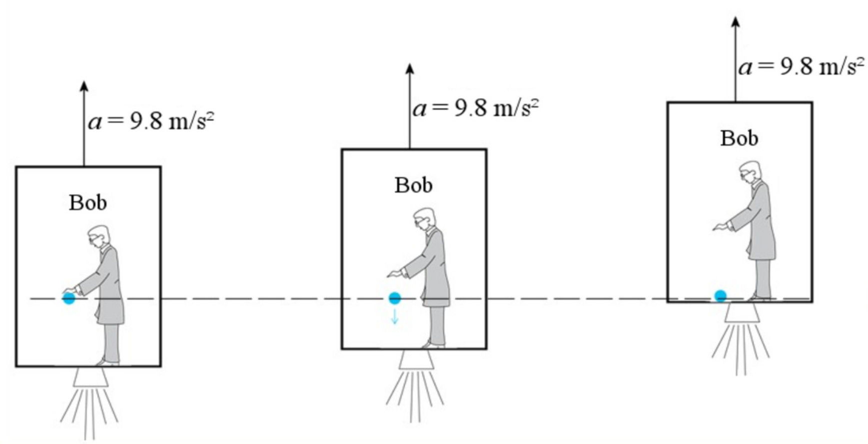
The starting point of Einstein's general relativity is the equivalence principle. To illustrate this principle, consider the two situations shown in the following diagram. In the diagram to the left, Joe is enclosed in a box placed on the surface of the Earth. In the diagram to the right, Bob is enclosed in a box in space, far from any large masses. This box accelerates in the direction indicated in the diagram with an acceleration of 9.8 m/s².

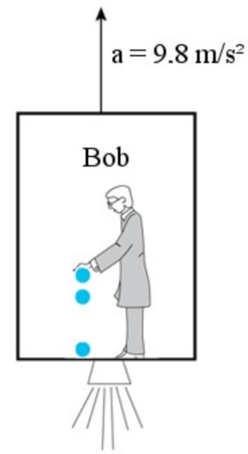


thinkingscifi.wordpress.com/2012/07/21/intelligence-and-imagination/

The equivalence principle says that everything that happens in the boxes must be absolutely identical for Bob and Joe. For example, consider what happens if Bob and Joe both drop a ball. When Joe drops his ball, the gravitational force makes the ball fall to the ground with an acceleration of 9.8 m/s².

When Bob drops his ball, there is no longer any force acting on the ball, and it stops accelerating while the Bob box's continues to accelerate upwards. To simplify, suppose that the speed of the box is zero when Bob drops his ball. The resulting motion of the ball and the box is shown in the following diagram.

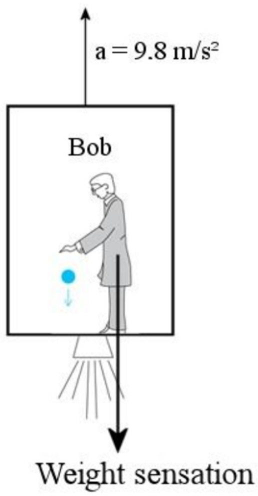




The ball stays in place since no force acts on it while Bob's box accelerates upwards.

From inside the box, Bob has the impression that the ball is moving downwards with an acceleration of 9.8 m/s². In fact, the motion of the ball observed by Bob is identical to the motion of the ball seen by Joe.

The equivalence principle also means that if you are locked inside a box and observe objects accelerating towards to the bottom of the box, it is impossible to know if this happens because there is a gravitational force or because you are inside an accelerating box. No experiment can help you determine if you are on Earth or accelerating at 9.8 m/s².

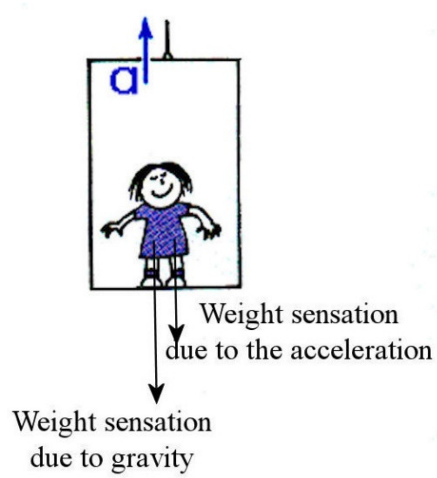


If everything is exactly the same for Bob and Joe, then Bob must feel a weight sensation exactly identical to the weight sensation felt by Joe due to the attraction of the Earth. As Joe feels a force directed towards the floor, Bob must also feel a force directed towards the floor, so in direction opposite to the acceleration. Also, if the gravitational force is proportional to the mass, then the effect on Bob must also be proportional to Bob's mass. Thus, the magnitude of this weight sensation on Bob must be *ma*.

If a person accelerates while a gravitational force acts on him, then the weight sensation is just the sum of these two effects. This sum is the apparent weight.

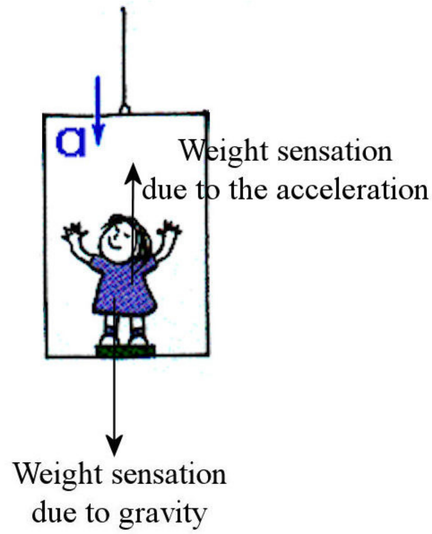
$$\vec{w}_{app} = m\vec{g} + \left(-m\vec{a}\right)$$

To understand the meaning of this equation, let's look at what happens when a person (we'll call him Tony) take an elevator to go up a few floors. When the elevator begins its motion, it accelerates upwards. Then, a weight sensation directed downwards is added to the downwards sensation made by the gravitational force, and Tony feels heavier.



www.ux1.eiu.edu/~cfadd/1150/05UCMGrav/Wt.html

When the elevator reaches its destination, it slows down and accelerates downwards. Then, a weight sensation directed upwards is added to the downwards sensation made by the gravitational force, and Tony then feels lighter.



To calculate the apparent weight, the x and y-components of this equation will be used (with a horizontal x-axis and a y-axis directed upwards). Thus, we have the following equations.

Apparent Weight From the Acceleration

$$\vec{w}_{app} = m\vec{g} + \left(-m\vec{a}\right)$$

In components:

$$w_{app\,x} = -ma_x$$
$$w_{app\,y} = -mg - ma_y$$

with a horizontal x-axis and a y-axis directed upwards

Using Newton's second law, another formula can also be obtained to calculate the apparent weight.

$$\vec{w}_{app} = m\vec{g} + \left(-m\vec{a}\right)$$
$$\vec{w}_{app} = m\vec{g} + \left(-\sum\vec{F}\right)$$

This leads to

Apparent Weight From the Forces

$$\vec{w}_{app} = -\left(\sum\vec{F} - m\vec{g}\right)$$

In components:

$$w_{app\,x} = -\sum F_x$$
$$w_{app\,y} = -\sum F_y - mg$$

with a horizontal x-axis and a y-axis directed upwards

This formula says that the apparent weight can be calculated with the sum of the forces acting on the object, without considering the gravitational force. The apparent weight is then equal to the sum of these forces but in the opposite direction. This is why some physicists define the apparent weight as the force that must be exerted on the object to support it.

Common Mistake: Using the Wrong Axes to Calculate the Components of the Apparent Weight.

The component equations obtained were made for a *y*-axis pointing upwards and a horizontal *x*-axis. You cannot rotate these axes.

Common Mistake: Putting Numbers in the Equations $\vec{w}_{app} = -\left(\sum \vec{F} - m\vec{g}\right)$ or $\vec{w}_{app} = m\vec{g} + \left(-m\vec{a}\right)$

These equations are vector equations. Thus, the correct answer is rarely obtained by directly substituting the values of *g, a* and *F* into these equations. You must work with the *x* and *y*-components of these equations.

The Number of *g*

It is often useful to compare the apparent weight to the actual weight on Earth to give a better idea of the sensation of weight. With the ratio of the magnitude of the apparent weight of the actual weight, the number of g's experienced is obtained. (This is usually called the *g-force*. This is a bad name since it is not a force. The term *number of g's* will therefore be used here.)

Number of g's

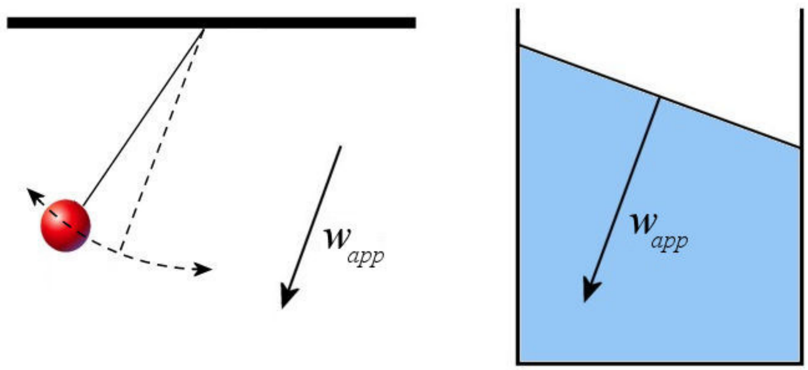
$$n_g = \frac{\left|w_{app}\right|}{w_{\text{actual on Earth}}} = \frac{\left|w_{app}\right|}{m \cdot 9.8 \frac{m}{s^2}}$$

Thus, if a person experienced 1.2 g's, then the person feels 1.2 times heavier than normal.

Everything Happens as if Gravity Were in the Direction of the Apparent Weight

The principle of equivalence tells us that it is impossible to tell the difference between the effect of gravitation and the effect of acceleration. Clearly, this means that everything happens exactly as if the gravitation were in the direction of the apparent weight.

If you drop an object, it will fall in the direction of the apparent weight. If there is a pendulum fixed to the ceiling, it will not oscillate from one side to the other of the vertical, but from one side to the other of the direction of apparent weight. If there is a container with water, the surface will not be horizontal, but it will be perpendicular to the direction of the apparent weight.



In addition, everything happens as if gravity were multiplied by n_g. For example, everything will happen as if g were 2 times larger if n_g = 2. If an object is dropped to the ground, the acceleration of the object will be 19.6 m/s² if n_g= 2. The normal of an object placed on the ground would be 2 times larger (if the ground is perpendicular to the direction of apparent weight). The buoyant force would be 2 times greater and it would be in the opposite direction to the apparent weight.

7.2 APPARENT WEIGHT WITH RECTILINEAR ACCELERATION

Example 7.2.1

What are the apparent weight and the number of g's experienced by a 60 kg person in an elevator in the following situations?

a) The elevator is moving upwards at a constant speed.

The only thing that can change the apparent weight is an acceleration. Since the acceleration is zero here, the apparent weight is equal to the weight.

$$\begin{aligned} w_{app\ y} &= -mg \\ &= -60kg \cdot 9.8 \tfrac{m}{s^2} \\ &= -588N \end{aligned}$$

As the value of $w_{app\ y}$ is negative, the apparent weight is directed downwards.

The number of g's experienced is

$$n_g = \frac{588N}{588N}$$
$$= 1$$

Thus, the person feels like its weight is the same as its normal weight on Earth in an elevator moving at constant speed.

b) The elevator is accelerating upwards at 2 m/s².

The apparent weight is

$$\begin{aligned} w_{app\ y} &= -mg - ma_y \\ &= -60kg \cdot 9.8\tfrac{m}{s^2} - 60kg \cdot 2\tfrac{m}{s^2} \\ &= -708N \end{aligned}$$

As the value of $w_{app\ y}$ is negative, the apparent weight is directed downwards.

The number of g's experienced is

$$n_g = \frac{708N}{588N}$$
$$= 1.204$$

Therefore, the person in this elevator feels like its weight is 1.204 times its normal weight on Earth. It is this kind of feeling that we get in an elevator that begins its upwards motion (which, therefore, accelerates upwards).

c) The elevator is accelerating downwards at 2 m/s².

The apparent weight is

$$\begin{aligned} w_{app\ y} &= -mg - ma_y \\ &= -60kg \cdot 9.8\tfrac{m}{s^2} - 60kg \cdot \left(-2\tfrac{m}{s^2}\right) \\ &= -468N \end{aligned}$$

As the value of $w_{app\ y}$ is negative, the apparent weight is directed downwards.

The number of g's experienced is

$$n_g = \frac{468N}{588N}$$
$$= 0.796$$

The person in this elevator thus feels like its weight is equal to 0.796 times its normal weight on Earth. It is this kind of feeling that we get in an elevator that stops its upwards motion (which, therefore, accelerates downwards).

d) The elevator is accelerating downwards at 9.8 m/s².

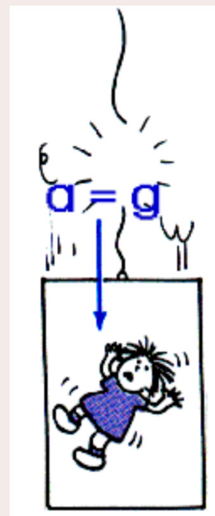
The apparent weight is

$$\begin{aligned} w_{app\,y} &= -mg - ma_y \\ &= -60kg \cdot 9.8\tfrac{m}{s^2} - 60kg \cdot \left(-9.8\tfrac{m}{s^2}\right) \\ &= 0N \end{aligned}$$

There is no apparent weight.

The number of g's experienced is

$$\begin{aligned} n_g &= \frac{0N}{588N} \\ &= 0 \end{aligned}$$



The person in this elevator thus feels like he has no weight. It feels like there is no more gravitation, and the person floats freely in the elevator (up to the moment the elevator arrives at the bottom of the elevator shaft…). In free fall, the effect of acceleration always exactly nullifies the effect of gravity and the apparent weight is always zero.

e) The elevator is accelerating downwards at 15 m/s².

The apparent weight is

$$\begin{aligned} w_{app} &= -mg - ma_y \\ &= -60kg \cdot 9.8\tfrac{m}{s^2} - 60kg \cdot \left(-15\tfrac{m}{s^2}\right) \\ &= 312N \end{aligned}$$

As the value of $w_{app\,y}$ is positive, the apparent weight is directed upwards.

The number of g's experienced is

$$\begin{aligned} n_g &= \frac{312N}{588N} \\ &= 0.531 \end{aligned}$$

The person in this elevator, therefore, feels like its weight is equal to 0.531 times the weight he normally has on Earth but directed upwards. Therefore, this person walks on the ceiling of the elevator.

(Note there is a rope that pulls the elevator down, because its acceleration cannot exceed 9.8 m/s² if only the gravitational force is pulling the elevator down.)

According to the equation of the apparent weight from the forces, the change of apparent weight can be measured by standing on a scale in an elevator. Then, there is only one force acting on the person if the weight is excluded: the normal force directed upwards. This means that the normal force has the same magnitude (but opposite direction) as the apparent weight in such a case since

$$\begin{aligned} w_{app\ y} &= -\sum F_y - mg \\ &= -\left(-mg + F_N\right) - mg \\ &= -F_N \end{aligned}$$

As a scale measures the contact force between the person and the scale (which is the normal force), the change of apparent weight can be seen on the scale. The value indicated by the scale change at the start and at the end of the elevator ride (so when there is an acceleration), as shown in this clip.
http://www.youtube.com/watch?v=z42xuQLkkGQ

Example 7.2.2

What is the number of g's experienced by a person in a rocket at takeoff if the acceleration of the rocket is 6 m/s² directed upwards?

The apparent weight is

$$\begin{aligned} w_{app\ y} &= -mg - ma_y \\ &= -m\left(g + a_y\right) \end{aligned}$$

It is impossible to calculate the apparent weight because the mass of the astronaut is not known. Despite this, the number of g's can be found.

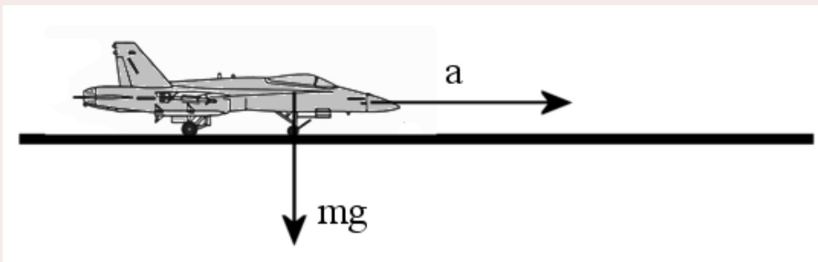
$$\begin{aligned} n_g &= \frac{\left|w_{app}\right|}{mg} \\ &= \frac{\left|-m\left(g + a_y\right)\right|}{mg} \end{aligned}$$

$$=\frac{\left|-\left(g+a_y\right)\right|}{g}$$
$$=\frac{\left(9.8\frac{m}{s^2}+6\frac{m}{s^2}\right)}{9.8\frac{m}{s^2}}$$
$$=1.61$$

Contrary to popular belief, astronauts do not experience a very large number of *g* on takeoff. The force exerted by the rocket is very important, but it does not give such a great acceleration to the rocket. Everyone could undergo this number of *g* without any training.

Example 7.2.3

What are the apparent weight (magnitude and direction) and the number of g's experienced by a 70 kg fighter pilot when his aircraft is catapulted from an aircraft carrier knowing that the plane accelerates up at a speed of 77 m/s (150 knots) over a distance of 94.5 m (on the USS Nimitz)?



www.fas.org/programs/ssp/man/uswpns/air/fighter/f18.html

The two components of the apparent weight are

$$w_{app\ x}=-ma_x$$
$$w_{app\ y}=-mg$$

To calculate the *x*-component, the acceleration must be known. As it is known that the speed changes from 0 to 77 m/s over a distance of 94.5 m, the acceleration is

$$2a_x(x-x_0)=v^2-v_0^2$$
$$2a_x(94.5m-0m)=\left(77m/s\right)^2-0$$
$$a_x=31.4\frac{m}{s^2}$$

Therefore, the components of the apparent weight are

$$w_{app\ x}=-ma_x=-70kg\cdot 31.4\frac{m}{s^2}=-2198N$$

$$w_{app\ y}=-mg=-70kg\cdot 9.8\frac{m}{s^2}=-686N$$

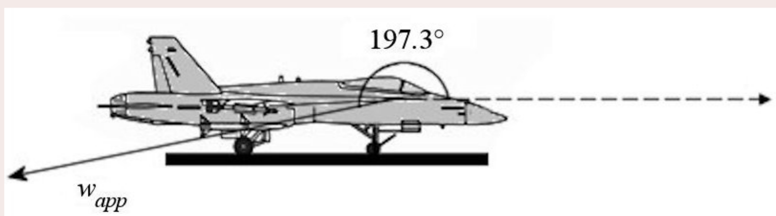
The magnitude of the apparent weight is

$$\begin{aligned} w_{app} &= \sqrt{w_{app\,x}^{2} + w_{app\,y}^{2}} \\ &= \sqrt{(-2198N)^2 + (-686N)^2} \\ &= 2302N \end{aligned}$$

The direction of the apparent weight is

$$\begin{aligned} \theta &= \arctan \frac{w_{app\,y}}{w_{app\,x}} \\ &= \arctan \frac{-686N}{-2198N} \\ &= 197.3° \end{aligned}$$

The following diagram illustrates this direction.



The pilot feels a weight directed towards the back of the plane and a bit downwards.

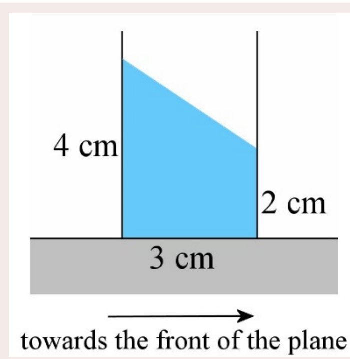
The number of g's experienced by the pilot is

$$\begin{aligned} n_g &= \frac{w_{app}}{mg} \\ &= \frac{2302N}{686N} \\ &= 3.36 \end{aligned}$$

The pilot feels 3.36 times heavier than normal.

Example 7.2.4

A glass of water is firmly fixed on a table in a plane taking off. If the water surface is oriented as shown in the diagram, what is the acceleration of the plane?

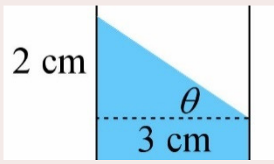


Knowing that the surface is perpendicular to the apparent weight, the direction of the apparent weight can be found. Then the components of the apparent weight can be obtained since the direction of a vector is given by

$$\tan\theta = \frac{P_{app\ y}}{P_{app\ x}}$$

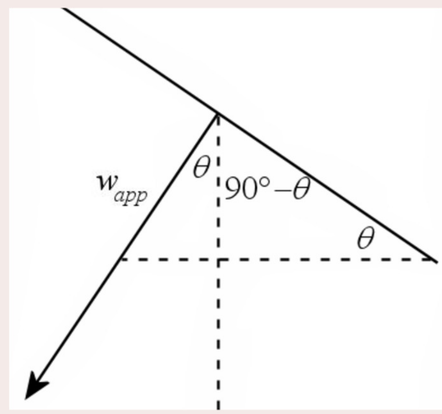
With these components, the acceleration can be calculated.

Let's start by finding the angle of the surface. According to the diagram to the left, this angle is



$$\theta = \arctan\frac{2cm}{3cm}$$
$$= 33.7°$$

This angle is also the angle between the vertical and the apparent weight as can be seen in the diagram to the right. The angle between the apparent weight and the positive x-axis is thus



$$-(90° + 33.7°) = -123.7°$$

The components of the apparent weight are

$$w_{app\ x} = -ma_x$$
$$w_{app\ y} = -mg$$

The acceleration can be found with the vector direction formula.

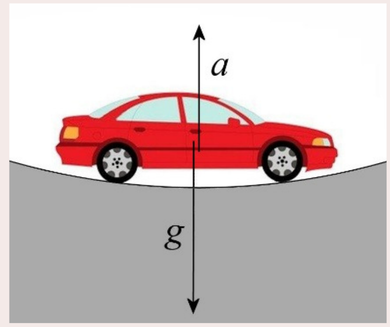
$$\tan\theta = \frac{w_{app\ y}}{w_{app\ x}}$$
$$\tan\theta = \frac{-\cancel{m}g}{-\cancel{m}a_x}$$
$$\tan(-123.7°) = \frac{9.8\frac{m}{s^2}}{a_x}$$
$$a_x = 6.53\tfrac{m}{s^2}$$

7.3 APPARENT WEIGHT WITH CIRCULAR MOTIONS

As soon as an object is undergoing a circular motion, the apparent weight of the object is no longer equal to its actual weight since there is an acceleration.

Example 7.3.1

Leo, who has a mass of 60 kg, is in a car that passes through a dip at 90 km/h. The radius of the dip is 125 m.



a) What is Leo's apparent weight ?

Since there is only a vertical acceleration (y-component), the components of Leo's apparent weight are

$$w_{app\,x} = 0$$
$$w_{app\,y} = -mg - ma_y$$

Leo's acceleration is needed. The acceleration is

$$\begin{aligned} a_y &= \frac{v^2}{r} \\ &= \frac{\left(25\frac{m}{s}\right)^2}{125m} \\ &= 5\frac{m}{s^2} \end{aligned}$$

Therefore, Leo's apparent weight is

$$\begin{aligned} w_{app\,y} &= -mg - ma_y \\ &= -60kg \cdot 9.8\frac{m}{s^2} - 60kg \cdot 5\frac{m}{s^2} \\ &= -888N \end{aligned}$$

b) What is the number of g's experienced by Leo?

The number of g is

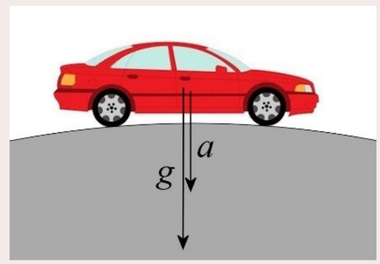
$$\begin{aligned} n_g &= \frac{\left|w_{app}\right|}{mg} \\ &= \frac{888N}{588N} \\ &= 1.51 \end{aligned}$$

The apparent weight remains directed downwards, and Leo feels like he is weighing 151% of his usual weight.

An identical motion can be made in an airplane by following a trajectory that curves upwards. In this case, the passengers of the aircraft will have a downwards feeling of weight (since the apparent weight is negative) and they will feel heavier than usual (since the magnitude of the apparent weight is greater than the magnitude of the weight.).

Example 7.3.2

Leo, who has a mass of 60 kg, is in a car that passes over a bump at 90 km/h. The radius of the bump is 125 m.



a) What is Leo's apparent weight ?

Since there is only a vertical acceleration (y-component), the components of Leo's apparent weight are

$$w_{app\ x} = 0$$
$$w_{app\ y} = -mg - ma_y$$

Leo's acceleration is needed. The acceleration is

$$\begin{aligned} a_y &= -\frac{v^2}{r} \\ &= -\frac{\left(25\frac{m}{s}\right)^2}{125m} \\ &= -5\tfrac{m}{s^2} \end{aligned}$$

Therefore, Leo's apparent weight is

$$\begin{aligned} w_{app\ y} &= -mg - ma_y \\ &= -60kg \cdot 9.8\tfrac{m}{s^2} - 60kg \cdot \left(-5\tfrac{m}{s^2}\right) \\ &= -288N \end{aligned}$$

b) What is the number of g's experienced by Leo?

The number of g is

$$\begin{aligned} n_g &= \frac{\left|w_{app}\right|}{mg} \\ &= \frac{288N}{588N} \\ &= 0.49 \end{aligned}$$

The apparent weight remains directed downwards, but Leo feels like he weighs only 49% of his usual weight.

An identical motion can be made in an airplane by following a trajectory that curves downwards. In this case, the passengers of the aircraft will still have a feeling of weight directed downwards (if the magnitude of the acceleration does not exceed 9.8 m/s²), but they will feel less heavy than usual (since the magnitude of the apparent weight is smaller than the magnitude of the weight.).

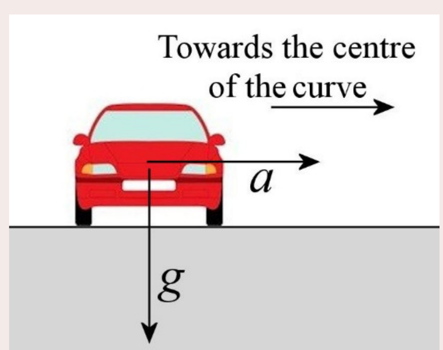
The aircraft could follow this trajectory and have an acceleration whose magnitude is greater than 9.8 m/s². For example, the aircraft could have an acceleration of -12 m/s². In this case, the apparent weight would be

$$\begin{aligned} w_{app\ y} &= -mg - ma_y \\ &= -m \cdot 9.8 \tfrac{m}{s^2} - m\left(-12 \tfrac{m}{s^2}\right) \\ &= m \cdot 2.2 \tfrac{m}{s^2} \end{aligned}$$

We see that if the downwards acceleration has a magnitude greater than 9.8 m/s², then the apparent weight becomes positive, which means that the apparent weight will be directed directly towards the ceiling of the aircraft. According to the principle of equivalence, everything will happen as if gravity were directed towards the ceiling of the aircraft.
https://www.youtube.com/watch?app=desktop&v=CtnXWwzn368
That's a bit disgusting...
https://www.youtube.com/watch?v=NGcojK436LQ
During turbulence, an important downwards acceleration can quickly transform a downwards apparent weight into an upwards apparent weight.
https://www.youtube.com/watch?v=envRYX6JnXo

Example 7.3.3

Gladys, who has a mass of 60 kg, is in a car that takes a curve at 90 km/h. The radius of the curve is 100 m.



a) What is the magnitude of Gladys' apparent weight ?

Since there is only a horizontal acceleration (x-component), the components of Gladys' apparent weight are

$$w_{app\ x} = -ma_x$$
$$w_{app\ y} = -mg$$

Gladys' acceleration is needed. The acceleration is

$$\begin{aligned} a_x &= \frac{v^2}{r} \\ &= \frac{\left(25\frac{m}{s}\right)^2}{100m} \\ &= 6.25\tfrac{m}{s^2} \end{aligned}$$

Therefore, the components of the apparent weight are

$$\begin{aligned} w_{app\,x} &= -ma_x \\ &= -60kg \cdot 6.25\tfrac{m}{s^2} \\ &= -375N \end{aligned} \qquad \begin{aligned} w_{app\,y} &= -mg \\ &= -60kg \cdot 9.8\tfrac{m}{s^2} \\ &= -588N \end{aligned}$$

The magnitude of Gladys' apparent weight is therefore

$$\begin{aligned} w_{app} &= \sqrt{w_{app\,x}{}^2 + w_{app\,y}{}^2} \\ &= \sqrt{\left(-375N\right)^2 + \left(-588N\right)^2} \\ &= 697.4N \end{aligned}$$

b) What is the number of g's experienced by Gladys?

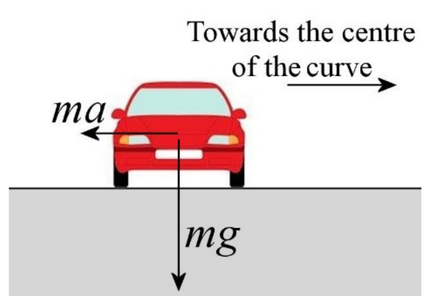
The number of g is

$$\begin{aligned} n_g &= \frac{\left|w_{app}\right|}{mg} \\ &= \frac{697.4N}{588N} \\ &= 1.186 \end{aligned}$$

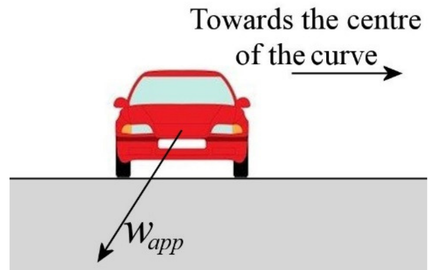
Let's look at the direction of the apparent weight in the car during this turn. The direction of the apparent weight is given by this vector sum.

$$\vec{w}_{app} = m\vec{g} + \left(-m\vec{a}\right)$$

Here is the direction of the two vectors $m\vec{g}$ and $-m\vec{a}$ (which is a vector of magnitude ma in the direction opposite to the acceleration.)



If these 2 vectors are added, the direction of apparent weight shown in the figure on the right is obtained.



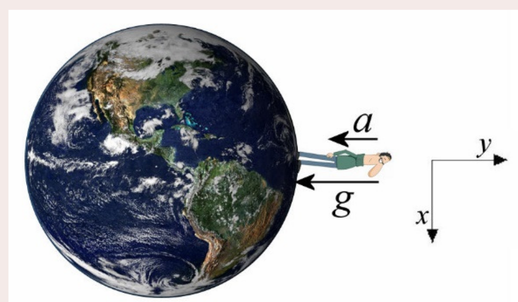
The person in the car therefore feels attracted down and out of the corner. (It is this outward component of weight that people will falsely attribute to centrifugal force.)

This latter effect can be seen in this video showing how the orientation of the surface of a liquid changes in a car in a curve.
http://www.youtube.com/watch?v=yOFERQMtGNM

Generally, the number of g is not very large when taking a curve in the car. It could hardly exceed 1.5 g (if the road is not tilted). However, in Formula 1, the number of g in a curve can go up to 6 or even up to 7.
https://www.youtube.com/watch?v=WGoVT1J9yMk

Example 7.3.4

Because the Earth rotates, every person on Earth makes a circular motion. This ensures that the apparent weight on Earth is not equal to our true weight. Knowing this, what is the number of g experienced by a person at the equator?



Since there is only an acceleration along the y-axis (remember, the y-axis must be in the opposite direction to gravity), the components of the apparent weight are

$$w_{app\,x} = 0$$
$$w_{app\,y} = -mg - ma_y$$

The acceleration of the person must be found to determine the number of g. As the person makes on rotation with a radius of 6380 km in 24 h (actually a little less), the centripetal acceleration towards the centre of the Earth is

$$\begin{aligned} a_y &= -\frac{4\pi^2 r}{T^2} \\ &= -\frac{4\pi^2 \left(6.38\times 10^6 m\right)}{\left(86,400 s\right)^2} \\ &= -0.03374 \tfrac{m}{s^2} \end{aligned}$$

The number of g's experienced is therefore

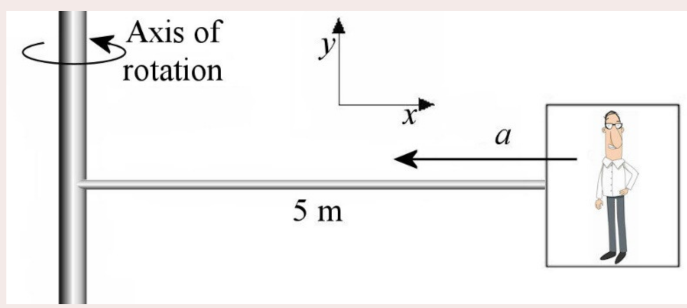
$$\begin{aligned} n_g &= \frac{|w_{app}|}{mg} \\ &= \frac{|-mg - ma_y|}{mg} \\ &= \frac{|-g - a_y|}{g} \\ &= \frac{\left|\left(-9.8\frac{m}{s^2} - -0.03374\frac{m}{s^2}\right)\right|}{9.8\frac{m}{s^2}} \\ &= 0.9965 \end{aligned}$$

Thus, at the equator, a person feels like its weight is only 99.65% of what it would be if he were at the pole (where there is no acceleration due to the Earth's rotation).

A person can experience a large number of g in a centrifuge. Essentially, the person sits in a box making a circular motion at the end of a long beam. As there is an acceleration in a circular motion, the apparent weight increases. If the box rotates faster, the acceleration is larger, and the apparent weight also gets larger. Here is a centrifuge of this type in action.
http://www.youtube.com/watch?v=sG6PPWxjgu0

Example 7.3.5

A centrifuge with a 5 m radius rotates with a 1.5 second period. What is the number of g experienced by the person in the centrifuge?



Since there is only an acceleration along the x-axis, the components of the apparent weight are

$$\begin{aligned} w_{app\,x} &= -ma_x \\ w_{app\,y} &= -mg \end{aligned}$$

The magnitude of the apparent weight is

$$w_{app} = \sqrt{(mg)^2 + (ma_x)^2}$$

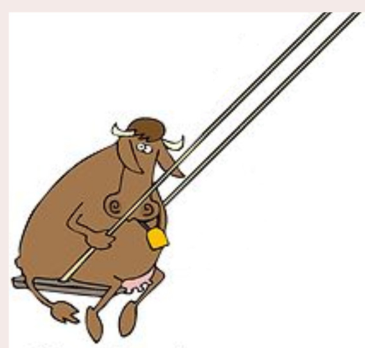
The magnitude of the acceleration must be found. In a uniform circular motion, it can be found with

$$\begin{aligned} a_x &= -\frac{4\pi^2 r}{T^2} \\ &= -\frac{4\pi^2 5m}{(1.5s)^2} \\ &= -87.73\tfrac{m}{s^2} \end{aligned}$$

Even if the mass of the person is not known, the number of g's experienced can be found.

$$\begin{aligned} n_g &= \frac{w_{app}}{mg} \\ &= \frac{\sqrt{(mg)^2 + (ma_x)^2}}{mg} \\ &= \frac{\sqrt{m^2 g^2 + m^2 a_x^2}}{mg} \\ &= \frac{\sqrt{m^2\left(g^2 + a_x^2\right)}}{mg} \\ &= \frac{\cancel{m}\sqrt{g^2 + a_x^2}}{\cancel{m}g} \\ &= \frac{\sqrt{\left(9.8\tfrac{m}{s^2}\right)^2 + \left(-87.73\tfrac{m}{s^2}\right)^2}}{9.8\tfrac{m}{s^2}} \\ &= 9.01 \end{aligned}$$

Example 7.3.6



What is the magnitude and direction of the apparent weight on this cow? The rope makes a 10° angle with a vertical line, the speed of the cow is 5 m/s, the cow has a mass of 720 kg, and the ropes have a length of 6 m. (We neglect air friction.)

www.clipartof.com/portfolio/djart/illustration/cow-elephant-and-pig-swinging-together-on-a-playground-39760.html

The solution to this problem is quite complicated if the following formulas are used

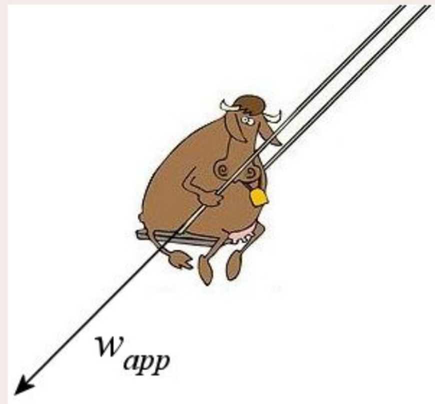
$$\begin{aligned} w_{app\ x} &= -ma_x \\ w_{app\ y} &= -mg - ma_y \end{aligned}$$

It would then be necessary to find the centripetal and tangential accelerations, find the magnitude and direction of the total acceleration and then find the components of the total acceleration using horizontal and vertical axes.

However, the solution is much simpler if the formula with the sum of the forces is used.

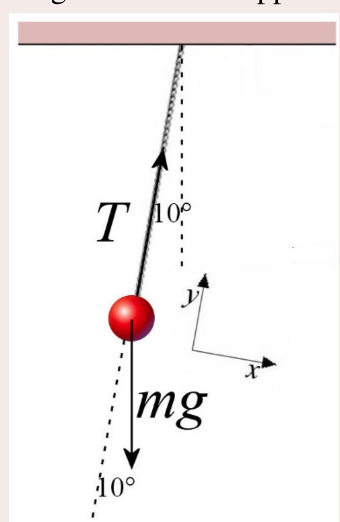
$$\vec{w}_{app} = -\left(\sum \vec{F} - m\vec{g}\right)$$

This formula tells us that the apparent weight is in the opposite direction of the sum of the forces, without considering the force of gravitation. As here there is only one force (the tension force) if the force of gravitation is excluded, the apparent weight is in the opposite direction to the tension force. The direction of the apparent weight is then shown is the diagram.



If this cow could hold a glass of water (if it had fingers), the surface of the water would always be perpendicular to the direction of the rope.

It remains only to find the tension of the rope to find the magnitude of the apparent weight.



The sum of the y-component of the forces is

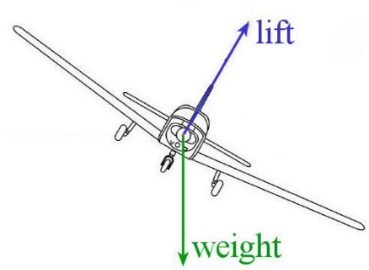
$$\sum F_y = T + mg\sin(-80°) = m\frac{v^2}{r}$$

Thus, the tension is

$$\begin{aligned} T &= m\frac{v^2}{r} - mg\sin(-80°) \\ &= 720kg \cdot \frac{\left(5\frac{m}{s}\right)^2}{6m} - 720kg \cdot 9.8\tfrac{N}{kg} \cdot \sin(-80°) \\ &= 9949N \end{aligned}$$

The magnitude of the apparent weight is, therefore, 9949 N. (It could then be calculated that the cow undergoes 1.41 g.)

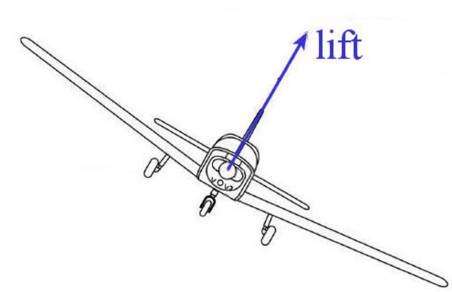
When turning on an airplane, the apparent weight always remains perpendicular to the floor of the aircraft.



Let's see why. We have seen that the centripetal force is made by lift (the force made by the air on the wings) during a turn.

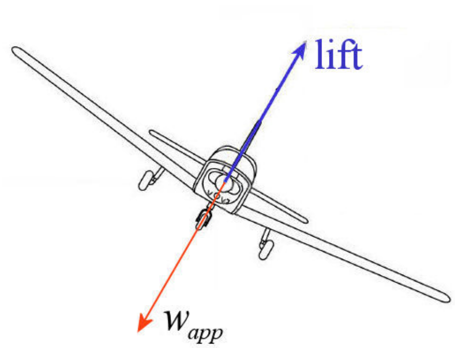
The direction of the apparent weight in the aircraft can then be found with this formula.

$$\vec{w}_{app} = -\left(\sum \vec{F} - m\vec{g}\right)$$



Not counting the weight, there is only one force left (figure on the right).

The sum of the forces except the weight is therefore equal to the lift. Since the apparent weight is equal to the opposite of this sum, the apparent weight must be opposite to the lift.



As the lift force is always perpendicular to the wings, the direction of the apparent weight is, therefore, also always perpendicular to the wings. This means that the occupants of the airplane always feel drawn directly towards the floor of the aircraft, even when the aircraft is tilted. They do not feel attracted to one side of the aircraft, even if the angle of inclination in the turn is very large.

Since the equivalence principle says that one cannot distinguish between the effect of gravitation and the effect of acceleration, everything happens as if gravity were always directed exactly towards the floor of the aircraft. If you drop an object, it will fall directly towards the floor of the plane even if the plane is tilted. If there is a ball on the floor, it will not roll to one side of the plane. If there is a glass of water in the plane, the surface of the water will remain parallel to the floor and will not tilt into the glass.

Everything happens as if gravity were directed exactly towards the floor of the plane. You can even pour yourself a glass of water normally during a turn.
https://www.youtube.com/watch?v=g99ho_ExApU
https://www.youtube.com/watch?v=_7n0hflAxD4

In short, it is impossible to feel the inclination on the plane! There is only one difference during the turn: gravity seems greater in a turn because the number of g is greater than 1 (and the increase is not very large for commercial flights). Apart from this difference, everything happens as if gravity were perpendicular to the floor of the aircraft.

It is therefore easy to be disoriented on a plane. When it's nighttime and the horizon cannot be seen anymore, a pilot may not realize that the plane is slowly tilting to one side if they are not attentive. Several plane crashes started like this.

By combining turns in different directions, the apparent weight can be constantly changed, to the delight (or not) of the passengers.
https://www.youtube.com/watch?v=aR-fA6OG21w

7.4 WEIGHTLESSNESS

It is possible that the weight sensation due to the acceleration completely cancels the weight sensation due to gravity, as it was the case with the free-falling elevator. Then the apparent weight becomes zero and the people then float as if there were no gravity. These people are in a state of weightlessness.

In fact, as soon as a person accelerates downwards with the same acceleration as the acceleration due to gravity, the apparent weight becomes zero. Of course, this includes all free-fall motion.

In orbit

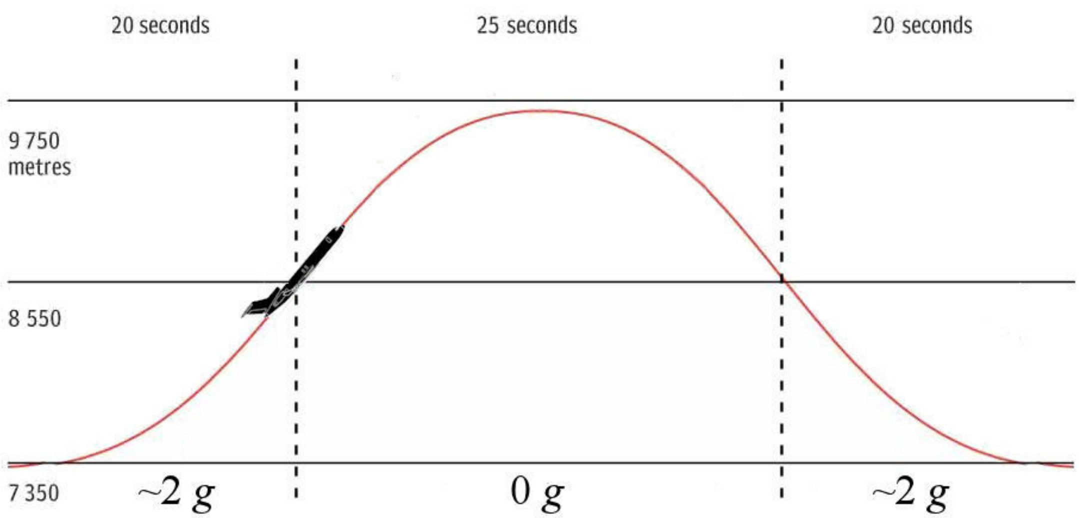
This is what is happening to the astronauts in the space station. Since the space station in orbit around the Earth, it is endlessly free-falling towards Earth. Thus, the acceleration of the space station is the same as the acceleration due to gravity, and the apparent weight of the astronauts is zero.

Common Mistake: Thinking that the Force of Gravity Acting on Astronauts in Orbit is Zero.

It is not the weight (the force of gravity) that is zero; it is the apparent weight. Actually, the weight of the astronauts in orbit is barely less than what it is on Earth. Moreover, if the weight were zero, there would be no force acting on the astronauts, and they could not make a circular motion around the Earth because there would be no centripetal force.

Parabolic Flight

There is no need to be in the space station to be in a state of weightlessness; you just have to accelerate downwards with an acceleration of 9.8 m/s². Obviously, you can lock yourself in a free-falling box, but there are a few problems: the drag force that will reduce the acceleration and, obviously, the impact with the ground. It is safer to do this free-falling motion in a plane following a parabolic path.



www.telegraph.co.uk/travel/travel-truths/how-do-zero-gravity-planes-work-parabolic-flights/

The top curved part of the trajectory is a parabola, the same parabola that would follow a free-falling object. This means that for about 25 seconds, the plane has a 9.8 m/s² downwards acceleration and the apparent weight becomes zero during this part of the trajectory. Here is a video showing the occupants of an aircraft making such a trajectory.
http://www.youtube.com/watch?v=Lhu198E8z2U

You can even do it as a tourist
http://www.youtube.com/watch?v=pH2TCEiYwKs

or in your own private plane.
https://www.youtube.com/watch?v=bsdr3-dey2Y

An OK go music video was made during such a flight.
https://www.youtube.com/watch?v=LWGJA9i18Co

7.5 MAXIMUM NUMBER OF *g*'s THAT A HUMAN BODY CAN WITHSTAND

Without training, 3 or 4 *g*'s can quickly become uncomfortable. The displacements of fluids in the body that deprive the brain of blood, and the muscle effort required to stay in place can quickly exhaust someone. This is why the maximum number of *g*'s that can be experienced in a roller coaster is about 4 *g* downwards and 0.5 *g* upwards. (Upwards *g*'s are not supported as well as downwards g's since the blood accumulates in the head then.)

For people practising activities that will lead to more than 4 *g*, some training must be done. For example, the formula 1 drivers experience up to 7 *g*'s in very fast curves and they must therefore be trained to be able to withstand these large apparent weight.

Fighter pilots can experience a large number of *g*'s during high acceleration maneuvers such as very tight turns. Therefore, they must be able to withstand large g-forces, a rather difficult thing to do. For example, if a person experiences 10 *g*'s directed upwards, the blood in his body accumulates in the legs, thereby depriving the brain of blood. This could result in loss of consciousness which is not a good thing if this happens while you are piloting a jet fighter. To withstand a high number of *g*'s, pilots must contract all the muscles of their legs at the same time to prevent blood from accumulating there. They have even developed pilot suits that squeeze the legs when there are significant accelerations to prevent blood from accumulating there.

Aspiring American pilots must pass a test before becoming actual fighter pilots. They must withstand 9 *g*'s for 10 seconds. They have only two chances to pass the test, otherwise, they are rejected. Here are some of these pilots.
http://www.youtube.com/watch?v=jKNDhEdHoBc
http://www.youtube.com/watch?v=dUkUC8QWa8g

But can a human body withstand more than that? During the 40's and 50's, John Stapp sought to answer this question with a series of tests in which he experienced an increasingly large number of *g*'s. Here is one of these tests.
http://www.youtube.com/watch?v=3UEYxf4fl_A
Stapp survived a 46.2 *g* test. Since he had his head forward in this test, blood accumulated in his head. This shattered the capillaries in his eyes, thereby filling his eyes with blood and making him blind for a few days.

If the acceleration lasts for a very short period, fluids do not have time to move and no muscular effort is required to stay in place. Then, it is possible to withstand much larger g-forces. It is not uncommon for people to survive car crashes while experiencing up to 50 g's. Over 50 g's, serious injury may occur but is it possible to survive a 100 g's crash. Kenny Bräck even survived this accident in which he experienced, during a very short time, 214 g's! (We know this because there are accelerometers in race cars.)
http://www.youtube.com/watch?v=Hy8fgGiI1WA
Bräck returned to racing 2 years later…but he was not quite the same pilot.

SUMMARY OF EQUATIONS

Apparent Weight From the Acceleration

$$\vec{w}_{app} = m\vec{g} + \left(-m\vec{a}\right)$$

In components:

$$w_{app\,x} = -ma_x$$
$$w_{app\,y} = -mg - ma_y$$

with a horizontal x-axis and a y-axis directed upwards

Apparent Weight From the Forces

$$\vec{w}_{app} = -\left(\sum \vec{F} - m\vec{g}\right)$$

In components:

$$w_{app\,x} = -\sum F_x$$
$$w_{app\,y} = -\sum F_y - mg$$

with a horizontal x-axis and a y-axis directed upwards

Number of *g's*

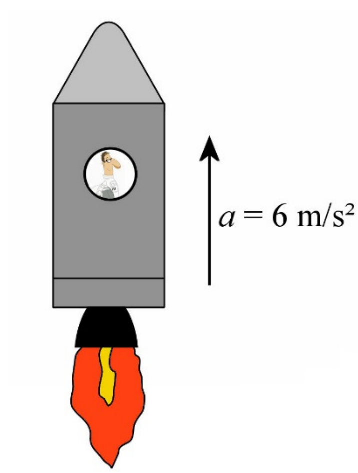
$$n_g = \frac{\left|w_{app}\right|}{w_{\text{actual on Earth}}} = \frac{\left|w_{app}\right|}{m \cdot 9.8\frac{m}{s^2}}$$

EXERCISES

7.2 Apparent Weight with Rectilinear Accelerations

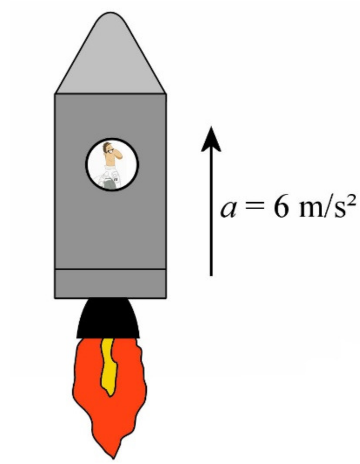
1. Karl, whose mass is 70 kg, is in a rocket. At takeoff, the rocket has a 6 m/s² acceleration directed upwards.

   a) What is Karl's weight (magnitude and direction)?
   b) What is Karl's apparent weight (magnitude and direction)?
   c) What is the number of g's experienced by Karl?



2. Karl, whose mass is 70 kg, is in a rocket. This time, the rocket takes off vertically from the surface of the Moon (where g is only 1.6 N/kg). At takeoff, the rocket has a 6 m/s² acceleration directed upwards.

   a) What is Karl's weight (magnitude and direction)?
   b) What is Karl's apparent weight (magnitude and direction)?
   c) What is the number of g's experienced by Karl?

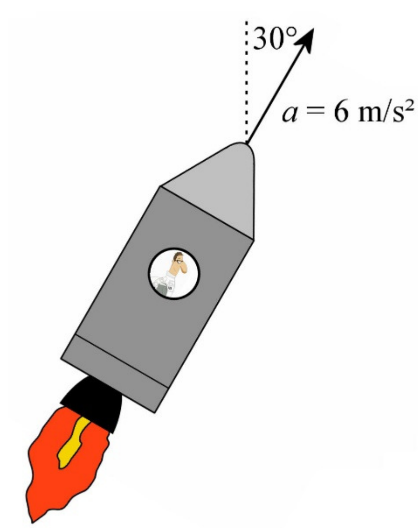


3. White Zombie is a modest-looking electric car, but it is capable of some extraordinary performances. It can reach a speed of 100 km/h in only 1.8 s, leaving cars such as Ferraris far behind. Assuming that the acceleration is constant, what is the number of g's experienced by the driver during this acceleration?

theelectricautoreview.com/2010/03/25/electric-drag-racing-white-zombie/

4. Karl, whose mass is 70 kg, is in a rocket. Shortly after liftoff from Earth, the rocket has an acceleration of 6 m/s² in a direction at 30° with the vertical.

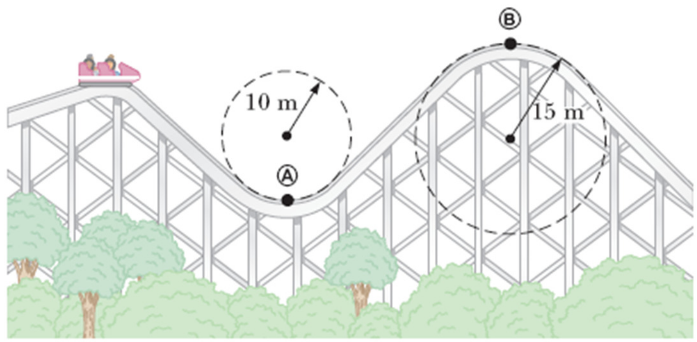
   a) What is Karl's apparent weight (magnitude and direction)?
   b) What is the number of g's experienced by Karl?



7.3 Apparent Weight in Circular Motion

5. Odette, whose mass is 120 kg, is in a roller-coaster cart following the track shown in this diagram.

   a) What are the apparent weight and the number of *g*'s experienced by Odette when the carriage is at point A if the speed of the carriage is 25 m/s at this location?
   b) What are the apparent weight and the number of *g*'s experienced by Odette when the carriage is at point B if the speed of the carriage is 10 m/s at this location?

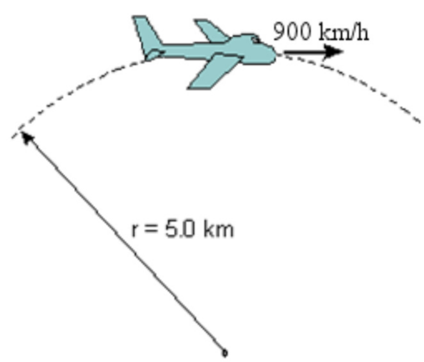


www.chegg.com/homework-help/questions-and-answers/physics-archive-2011-november-02

6. What should be the period of rotation of the Earth so that the apparent weight of a person vanishes at the equator? (Use 6378 km for the radius of the Earth.)

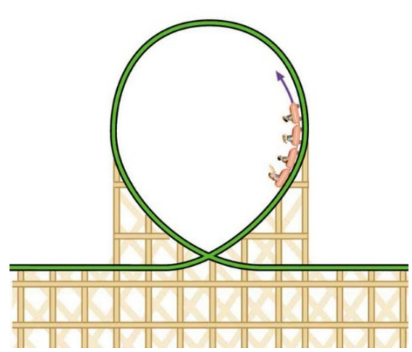
7. Juliet, whose mass is 60 kg, is in a plane following this path at a constant speed.

   a) What is Juliet's apparent weight (magnitude and direction)?
   b) What is the number of *g*'s experienced by Juliet?



www.physics.fsu.edu/users/ng/Courses/phy2053c/HW/Ch05/ch05.htm

8. Victor, whose mass is 50 kg, is on a roller-coaster ride. Somewhere on the track, there is a loop as shown in this diagram. At the top of the loop, the radius of curvature of the track is 10 m. What should be the speed of the carriage so that Victor has an apparent weight directed upwards whose magnitude equals twice the magnitude of its weight?



cnx.org/content/m42086/latest/?collection=col11406/latest

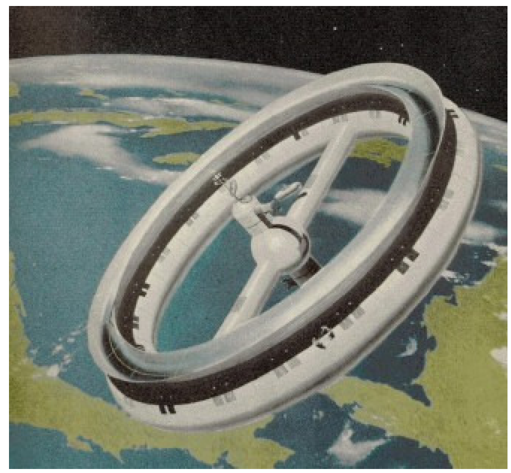
9. This formula 1 driver is experiencing 4 g's in this curve when the speed of the race car is 180 km/h. What is the radius of curvature of the track?

digitalcatharsis.wordpress.com/tag/crowne-plaza-hotel/

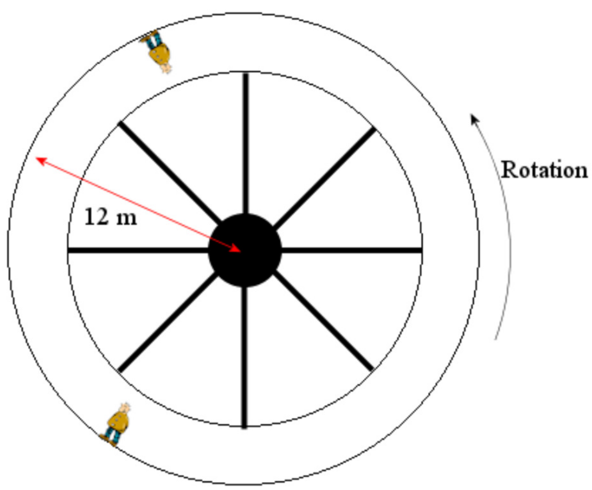
10. To avoid ending up in weightless conditions in space, it was proposed to build a rotating space station with this shape.

With the rotation of the station, astronauts make a circular motion that gives them an acceleration and, therefore, an apparent weight.

Astronauts would walk on the outer wall of the station as shown in this diagram.



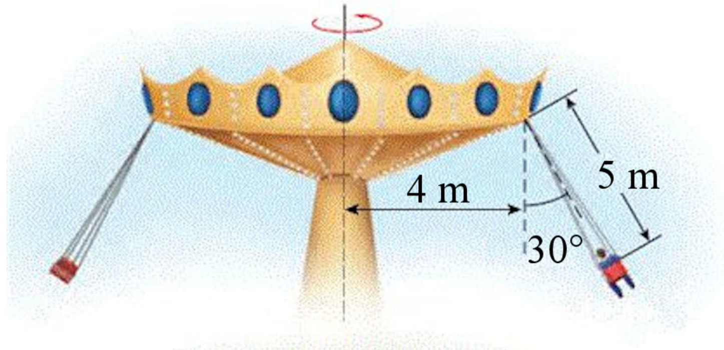
www.rogersrocketships.com/page_view.cfm?id=24



www.batesville.k12.in.us/Physics/PhyNet/Mechanics/Circular%20Motion/answers/assign_4_answers.htm

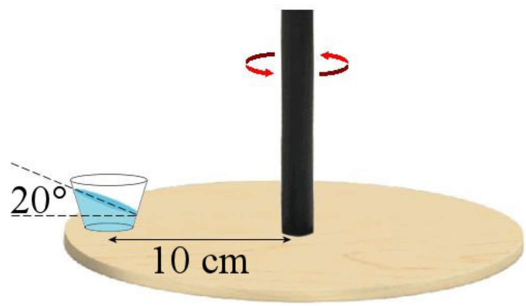
With the dimensions shown in the diagram, what should be the period of rotation of the station so that peoples in the station have an apparent weight equal to their weight on Earth if this station is in space, far from any planets or stars?

11. What is the period of rotation of this ride that is rotating at constant speed?



www.chegg.com/homework-help/questions-and-answers/popular-carnival-ride-consists-seats-attached-central-disk-cables-passengers-travel-unifor-q1568766

12. The surface of the water in this glass is tilted at 20°. What would be the angle of inclination of the water surface if the glass was 6 cm from the axis of rotation and the plate is still rotating with the same period?

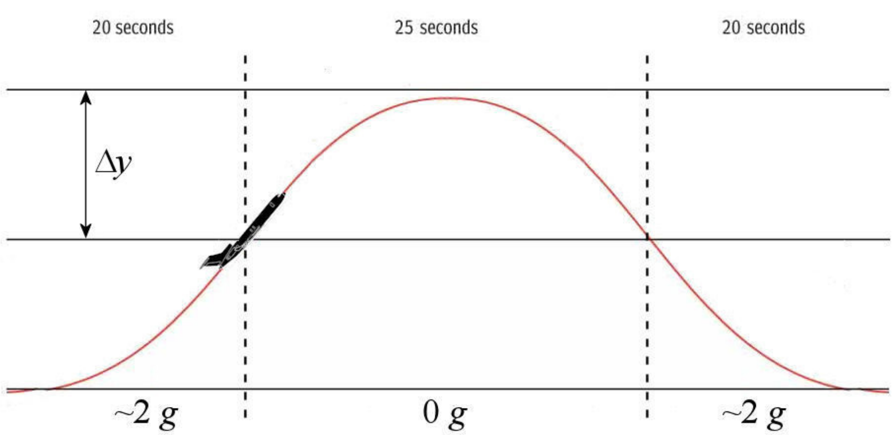


www.foundalis.com/phy/Mach-bucket.htm

7.4 Weightlessness

13. When the apparent weight vanishes in an aircraft for 25 seconds, it means that the aircraft makes a projectile motion during those 25 seconds. Here, the aircraft travels at 720 km/h at the beginning of the parabolic movement.

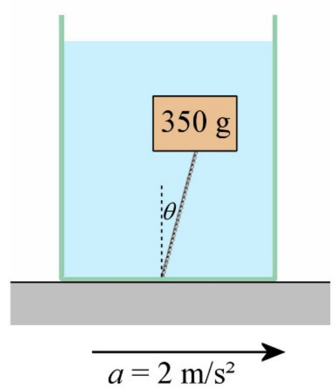
a) What is the angle of climb of the aircraft when this part of the flight begins?
b) What is the altitude variation during the ascending part of the projectile motion?



Challenges

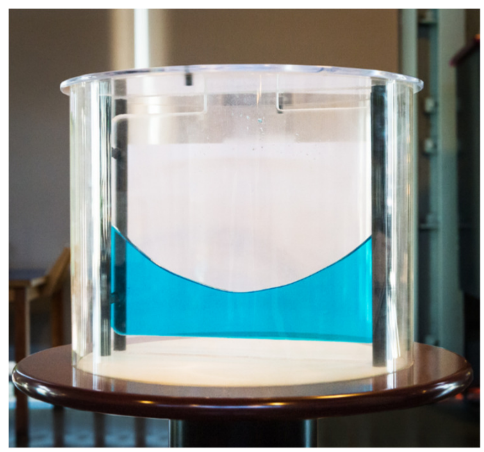
(Questions more difficult than the exam questions.)

14. A 350 g block of cedar having a 400 cm³ volume is tied to the bottom of a container filled with water as shown in the diagram. This container is in a car that accelerates to the right with an acceleration of 2 m/s².



a) What is the buoyant force (magnitude and direction) on this block of cedar? (The density of water is 1000 kg/m³)
b) What is the tension of the rope?
c) What is the angle θ in this diagram?

15. There is some water in a spinning container. The period of rotation is T. What is the equation $y = f(x)$ of the shape traced by the surface of the water? (In other words, we're looking for a formula that gives the height of the water as a function of the distance from the axis of rotation.)



www.exploratorium.edu/snacks/water-spinner

ANSWERS

7.2 Apparent Weight with Rectilinear Accelerations

1. a) 686 N downwards b) 1106 N downwards c) 1.612
2. a) 112 N downwards b) 532 N downwards c) 0.7755
3. 1.865
4. a) 1070.5 N at -101,3° b) 1.56

7.3 Apparent Weight in Circular Motion

5. a) 8676 N downwards, $n_g = 7.378$ b) 376 N downwards $n_g = 0.32$
6. 84.48 min
7. a) 162 N upwards b) 0.2755
8. 17.15 m/s
9. 65.87 m
10. 6.953 s
11. 6.734 s
12. 12.32°

7.5 Direction of the Apparent Weight

13. a) 37.8° b) 765,6 m

Challenges

14. a) 4 N at 78.47° b) 0.5 N c) 11.5°
15. $y = \frac{2\pi^2 x^2}{gT^2} + y_0$ (It's a parabola)