Chapter 6 Solutions

1. a) The magnitude of the centripetal force is

$$\begin{aligned} F_c &= \frac{mv^2}{r} \\ &= \frac{200kg \cdot \left(34\tfrac{m}{s}\right)^2}{33m} \\ &= 7006N \end{aligned}$$

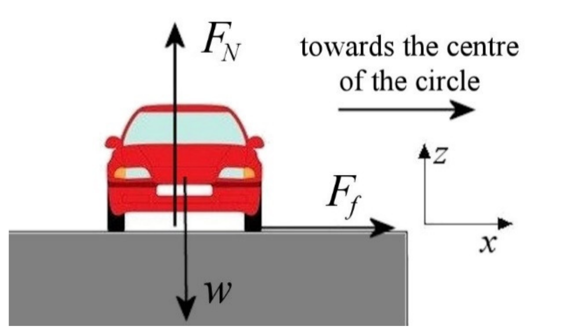
b) The magnitude of the centripetal force is

$$\begin{aligned} F_c &= \frac{mv^2}{r} \\ &= \frac{200kg \cdot \left(34\tfrac{m}{s}\right)^2}{24m} \\ &= 9633N \end{aligned}$$

2. *Forces Acting on the Object*

In the curve, there are 3 forces acting on the car.

1) The weight mg directed downwards.
2) A normal force (F_N) directed upwards.
3) A friction force (F_f) directed towards the centre of the circle.



Sum of the Forces

The sums of the forces are

$$\begin{aligned} \sum F_x &= F_f \\ \sum F_z &= -mg + F_N \end{aligned}$$

Newton's Second Law

Since the centripetal acceleration is towards the positive x-axis, we have

$$\sum F_x = ma_x \qquad \rightarrow \qquad F_f = m\frac{v^2}{r}$$
$$\sum F_z = ma_z \qquad \rightarrow \qquad -mg + F_N = 0$$

Solving the equations

The first equation gives $F_f = m\frac{v^2}{r}$

and the second equation gives $F_N = mg$.

Using this value in

$$F_f \le \mu_s F_N$$

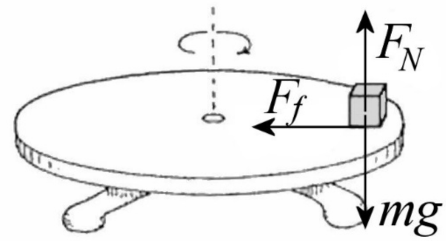
the following result is obtained

$$m\frac{v^2}{r} \le \mu_s mg$$
$$\frac{v^2}{r} \le \mu_s g$$
$$\frac{v^2}{rg} \le \mu_s$$

The minimum value of the coefficient of friction is therefore

$$\begin{aligned}\mu_{s\min} &= \frac{v^2}{rg} \\ &= \frac{\left(33.33\frac{m}{s}\right)^2}{100m \cdot 9.8\frac{N}{kg}} \\ &= 1.134\end{aligned}$$

3. *Forces Acting on the Object*



There are 3 forces exerted on the block.

1) The weight (mg) directed downwards.
2) A normal force (F_N) directed upwards.
3) A friction force (F_f) directed towards the centre of the circular path.

Sum of the Forces

The sums of the forces are (using an x-axis towards the centre of the circular path)

$$\sum F_x = F_f$$
$$\sum F_y = -mg + F_N$$

Newton's Second Law

Since the centripetal acceleration is towards the positive x-axis, we have

$$\sum F_x = ma_x \qquad \rightarrow \qquad F_f = m\frac{4\pi^2 r}{T^2}$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -mg + F_N = 0$$

As the period of rotation is sought (not the speed), the centripetal force formula with the period of rotation is used.

Solving the equations

The first equation gives $F_f = m\frac{4\pi^2 r}{T^2}$

and the second equation gives $F_N = mg$.

Using this value in

$$F_f \le \mu_s F_N$$

the following result is obtained

$$m\frac{4\pi^2 r}{T^2} \leq \mu_s mg$$
$$\frac{4\pi^2 r}{T^2} \leq \mu_s g$$
$$\sqrt{\frac{4\pi^2 r}{\mu_s g}} \leq T$$

The minimum period is therefore

$$\begin{aligned} T_{\min} &= \sqrt{\frac{4\pi^2 r}{\mu_s g}} \\ &= \sqrt{\frac{4\pi^2 \cdot 0.1m}{0.6 \cdot 9.8\frac{N}{kg}}} \\ &= 0.8194s \end{aligned}$$

4. Lowest point on the trajectory

Forces Acting on the Object

At the lowest point on the circular path, there are 2 forces exerted on the car.

1) The 9800 N weight directed downwards.
2) A normal force (F_N) directed upwards.

Sum of the Forces

The sum of the forces is (using a *y*-axis directed upwards)

$$\sum F_y = -9800N + F_N$$

Newton's Second Law

Since the centripetal acceleration is towards the positive *y*-axis, we have

$$\sum F_y = ma_y \qquad \rightarrow \qquad -9800N + F_N = m\frac{v^2}{r}$$

Solving the equations

The normal force is

$$-9800N + F_N = 1000kg \frac{\left(12\frac{m}{s}\right)^2}{5m}$$
$$-9800N + F_N = 28,800N$$
$$F_N = 38,600N$$

Highest point on the trajectory

Forces Acting on the Object

At the lowest point on the circular path, there are 2 forces exerted on the car.

1) The 9800 N weight directed downwards.
2) A normal force (F_N) directed downwards.

Sum of the Forces

The sum of the forces is (using a *y*-axis directed upwards)

$$\sum F_y = -9800N - F_N$$

Newton's Second Law

Since the centripetal acceleration is towards the negative *y*-axis, we have

$$\sum F_y = ma_y \qquad \rightarrow \qquad -9800N - F_N = -m\frac{v^2}{r}$$

Solving the equations

The normal force is

$$-9800N - F_N = -1000kg \frac{\left(12\frac{m}{s}\right)^2}{5m}$$
$$-9800N - F_N = -28,800N$$
$$F_N = 19,000N$$

5. If the car is touching the road, then there must be a normal force acting on the car. We will therefore look for the condition for a normal force to exist by looking at the forces acting on the car.

Forces Acting on the Object

At the highest point on the circular path, there are 2 forces exerted on the car.

1) The weight (mg) directed downwards.
2) A normal force (F_N) directed downwards.

Sum of the Forces

The sum of the forces is (using a y-axis directed upwards)

$$\sum F_y = -mg - F_N$$

Newton's Second Law

Since the centripetal acceleration is towards the negative y-axis, we have

$$\sum F_y = ma_y \qquad \rightarrow \qquad -mg - F_N = -m\frac{v^2}{r}$$

Solving the equations

The normal force is

$$-mg - F_N = -m\frac{v^2}{r}$$
$$F_N = m\frac{v^2}{r} - mg$$

If we want the car to be in contact with the road, the normal force must be positive. This means that

$$m\frac{v^2}{r} - mg \geq 0$$
$$m\frac{v^2}{r} \geq mg$$
$$\frac{v^2}{r} \geq g$$
$$v \geq \sqrt{rg}$$

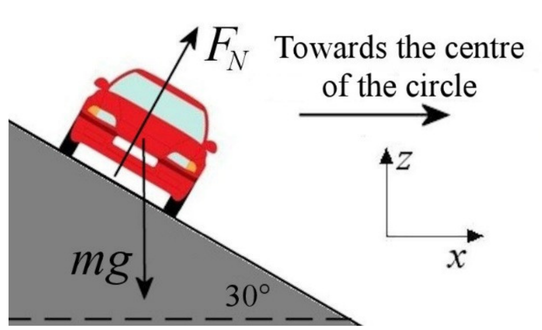
The minimum speed is therefore

$$\begin{aligned} v_{\min} &= \sqrt{rg} \\ &= \sqrt{5m \times 9.8 \tfrac{N}{kg}} \\ &= 7 \tfrac{m}{s} \end{aligned}$$

6. *Forces Acting on the Object*

There are 2 forces acting on the car.

1) The weight mg directed downwards.
2) A normal force F_N perpendicular to the road.



Sum of the Forces

The sums of the forces are (using an x-axis directed towards the right)

$$\begin{aligned} \sum F_x &= F_N \cos 70° \\ \sum F_z &= -mg + F_N \sin 70° \end{aligned}$$

Newton's Second Law

Since the centripetal acceleration is towards the positive x-axis, we have

$$\begin{aligned} \sum F_x = ma_x \quad &\rightarrow \quad F_N \cos 70° = m\frac{v^2}{r} \\ \sum F_z = ma_z \quad &\rightarrow \quad -mg + F_N \sin 70° = 0 \end{aligned}$$

Solving the equations

The normal force can be found with the sum of the y-component of the forces.

$$\begin{aligned} -mg + F_N \sin 70° &= 0 \\ F_N &= \frac{mg}{\sin 70°} \end{aligned}$$

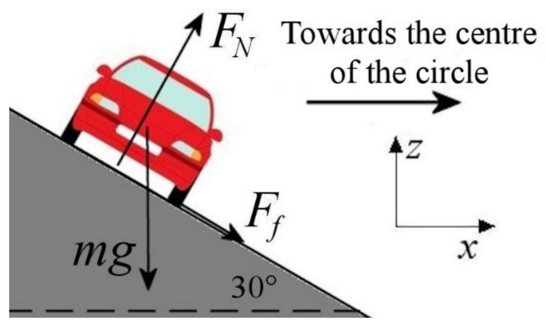
The sum of the x-component of the forces then becomes

$$F_N \cos 70° = m\frac{v^2}{r}$$
$$\frac{mg}{\sin 70°}\cos 70° = m\frac{v^2}{r}$$
$$\frac{g\cos 70°}{\sin 70°} = \frac{v^2}{r}$$
$$v = \sqrt{\frac{rg\cos 70°}{\sin 70°}}$$
$$v = \sqrt{\frac{rg}{\tan 70°}}$$
$$v = \sqrt{\frac{80m \cdot 9.8\frac{N}{kg}}{\tan 70°}}$$
$$v = 16.89\tfrac{m}{s}$$

7. *Forces Acting on the Object*

There are 3 forces exerted on the car.

1) The weight (mg) directed downwards.
2) A normal force (F_N) perpendicular to the road.
3) A friction force (F_f) parallel to the road.



Actually, the direction of the frictional force is not known. It may be directed uphill or downhill. Let's assume the direction is as shown in the figure. If, in the end, we have a positive frictional force, we will know that this direction is correct. If the frictional force is negative, the force is directed uphill instead.

Sum of the Forces

The sums of the forces are (using an x-axis directed towards the right)

$$\sum F_x = F_N \cos 60° + F_f \cos(-30°)$$
$$\sum F_z = -mg + F_N \sin 60° + F_f \sin\left(-30°\right)$$

Newton's Second Law

Since the centripetal acceleration is towards the positive x-axis, we have

$$\sum F_x = ma_x \qquad \rightarrow \qquad F_N \cos 60° + F_f \cos(-30°) = m\frac{v^2}{r}$$

$$\sum F_z = ma_z \qquad \rightarrow \qquad -mg + F_N \sin 60° + F_f \sin\left(-30°\right) = 0$$

Solving the equations

The normal force can be found with the sum of the *y*-component of the forces.

$$-mg + F_N \sin 60° + F_f \sin\left(-30°\right) = 0$$

$$F_N \sin 60° = mg - F_f \sin\left(-30°\right)$$

$$F_N \sin 60° = mg + F_f \sin 30°$$

$$F_N = \frac{mg + F_f \sin 30°}{\sin 60°}$$

The sum of the *x*-component of the forces then becomes

$$F_N \cos 60° + F_f \cos 30° = m\frac{v^2}{r}$$

$$\frac{mg + F_f \sin 30°}{\sin 60°}\cos 60° + F_f \cos 30° = m\frac{v^2}{r}$$

If we solve the equation for the friction force, we obtain

$$\frac{mg + F_f \sin 30°}{\sin 60°}\cos 60° + F_f \cos 30° = m\frac{v^2}{r}$$

$$\frac{mg}{\sin 60°}\cos 60° + \frac{F_f \sin 30°}{\sin 60°}\cos 60° + F_f \cos 30° = m\frac{v^2}{r}$$

$$\frac{F_f \sin 30°}{\sin 60°}\cos 60° + F_f \cos 30° = m\frac{v^2}{r} - \frac{mg}{\sin 60°}\cos 60°$$

$$F_f\left(\frac{\sin 30°}{\sin 60°}\cos 60° + \cos 30°\right) = m\frac{v^2}{r} - \frac{mg}{\sin 60°}\cos 60°$$

$$F_f\left(1.1547\right) = m\frac{v^2}{r} - \frac{mg}{\sin 60°}\cos 60°$$

a) If the speed is 100 m/s, we have

$$F_f\left(1.1547\right)=m\frac{v^2}{r}-\frac{mg}{\sin 60°}\cos 60°$$
$$F_f\left(1.1547\right)=1200kg\cdot\frac{\left(100\frac{m}{s}\right)^2}{80m}-\frac{1200kg\cdot 9.8\frac{N}{kg}}{\sin 60°}\cos 60°$$
$$F_f\left(1.1547\right)=143,210N$$
$$F_f=124,023N$$

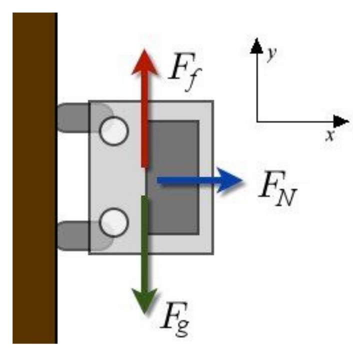
The friction force is thus directed downhill.

b) If the speed is 10 m/s, we have

$$F_f\left(1.1547\right)=m\frac{v^2}{r}-\frac{mg}{\sin 60°}\cos 60°$$
$$F_f\left(1.1547\right)=1200kg\cdot\frac{\left(10\frac{m}{s}\right)^2}{80m}-\frac{1200kg\cdot 9.8\frac{N}{kg}}{\sin 60°}\cos 60°$$
$$F_f\left(1.1547\right)=-5290N$$
$$F_f=-4581N$$

The friction force is thus directed uphill.

8. *Forces Acting on the Object*



There are 3 forces exerted on the car.

1) The weight (mg) directed downwards.
2) A normal force (F_N) directed towards the right.
3) A friction force (F_f) directed upwards.

Sum of the Forces

The sums of the forces are

$$\sum F_x=F_N$$
$$\sum F_y=-mg+F_f$$

Newton's Second Law

Since the centripetal acceleration is towards the positive x-axis, we have

$$\sum F_x = ma_x \qquad \rightarrow \qquad F_N = m\frac{v^2}{r}$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -mg + F_f = 0$$

(There is no *y*-component for the acceleration since the car is not sliding down the wall.)

Solving the equations

The second equation gives $F_f = mg$

and the first equation gives $F_N = m\frac{v^2}{r}$.

Using these values in

$$F_f \leq \mu_s F_N$$

the following result is obtained.

$$mg \leq \mu_s m\frac{v^2}{r}$$
$$g \leq \mu_s \frac{v^2}{r}$$
$$\sqrt{\frac{rg}{\mu_s}} \leq v$$

Therefore, the minimum speed is

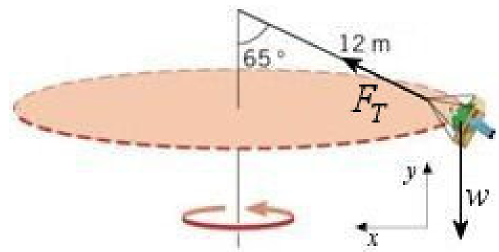
$$\begin{aligned} v_{\min} &= \sqrt{\frac{5m \cdot 9.8\frac{N}{kg}}{0.8}} \\ &= 7.826\tfrac{m}{s} = 28.2\tfrac{km}{h} \end{aligned}$$

and the normal force is

$$\begin{aligned} F_N &= m\frac{v^2}{r} \\ &= 1200kg \cdot \frac{\left(7.826\frac{m}{s}\right)^2}{5m} \\ &= 14,700N \end{aligned}$$

9. *Forces Acting on the Object*

There are 2 forces exerted on the person.



1) The weight (mg) directed downwards.
2) A tension force (F_T) directed at 25°.

Sum of the Forces

The sums of the forces are (using an axis towards the centre of the circular path)

$$\sum F_x = F_T \cos 25°$$
$$\sum F_y = -mg + F_T \sin 25°$$

Newton's Second Law

Since the centripetal acceleration is towards the positive x-axis, we have

$$\sum F_x = ma_x \qquad \rightarrow \qquad F_T \cos 25° = m\frac{4\pi^2 r}{T^2}$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -mg + F_T \sin 25° = 0$$

As the period of rotation is sought (not the speed), the centripetal force formula with the period of rotation is used.

Solving the equations

The tension can be found with the sum of the y-component of the forces.

$$-mg + F_T \sin 25° = 0$$
$$F_T = \frac{mg}{\sin 25°}$$
$$F_T = \frac{60kg \cdot 9.8\frac{N}{kg}}{\sin 25°}$$
$$F_T = 1391.3N$$

The sum of the x-component of the forces then becomes

$$F_T \cos 25° = m\frac{4\pi^2 r}{T^2}$$
$$1391.3N \cdot \cos 25° = m\frac{4\pi^2 r}{T^2}$$

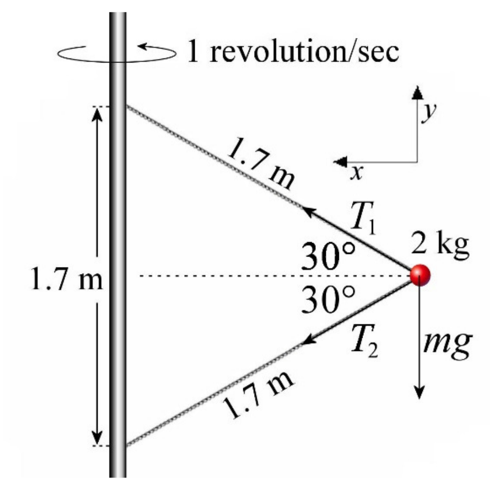
Since the radius of the circular path is

$$\frac{r}{12m} = \sin 65°$$
$$r = 12m \cdot \sin 65°$$

the period of rotation is

$$1391.3N \cdot \cos 25° = 60kg \cdot \frac{4\pi^2 \cdot 12m \cdot \sin 65°}{T^2}$$
$$T = 4.52s$$

10. *Forces Acting on the Object*



There are 3 forces exerted on the 2 kg object.

1) The 19.6 N weight directed downwards.
2) A tension force T_1 directed at 30°.
3) A tension force T_2 directed at -30°.

(We have 30° angles because the strings and the vertical post form an equilateral triangle, whose angle at each vertex is 60°.)

Sum of the Forces

The sums of the forces are

$$\sum F_x = T_1 \cos 30° + T_2 \cos(-30°)$$
$$\sum F_y = -19.6N + T_1 \sin 30° + T_2 \sin(-30°)$$

Newton's Second Law

Since the centripetal acceleration is towards the positive x-axis, we have

$$\sum F_x = ma_x \qquad \rightarrow \qquad T_1 \cos 30° + T_2 \cos\left(-30°\right) = m\frac{4\pi^2 r}{T^2}$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -19.6N + T_1 \sin 30° + T_2 \sin\left(-30°\right) = 0$$

As the period of rotation is known (1 s) instead of the speed, the centripetal force formula with the period of rotation is used.

Solving the equations

If we solve for T_1 in the sum of the y-component of the forces, we obtain

$$-19.6N + T_1 \sin 30° + T_2 \sin\left(-30°\right) = 0$$
$$T_1 = \frac{19.6N - T_2 \sin\left(-30°\right)}{\sin 30°}$$
$$T_1 = \frac{19.6N + T_2 \sin 30°}{\sin 30°}$$
$$T_1 = \frac{19.6N}{\sin 30°} + T_2$$
$$T_1 = 39.2N + T_2$$

We then substitute this value in the sum of the x-component of the forces to obtain

$$T_1 \cos 30° + T_2 \cos 30° = m\frac{4\pi^2 r}{T^2}$$
$$T_1 + T_2 = m\frac{4\pi^2 r}{T^2 \cos 30°}$$
$$\left(39.2N + T_2\right) + T_2 = m\frac{4\pi^2 r}{T^2 \cos 30°}$$
$$39.2N + 2T_2 = m\frac{4\pi^2 r}{T^2 \cos 30°}$$
$$2T_2 = m\frac{4\pi^2 r}{T^2 \cos 30°} - 39.2N$$
$$T_2 = m\frac{2\pi^2 r}{T^2 \cos 30°} - 19.6N$$

Since the radius of the circular path is (dotted line in the diagram)

$$\frac{r}{1,7m} = \cos 30°$$
$$r = 1,7m \cdot \cos 30°$$

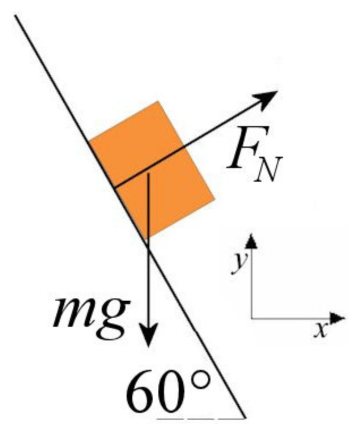
we have

$$T_2 = m\frac{2\pi^2 r}{T^2\cos 30°} - 19.6N$$
$$T_2 = m\frac{2\pi^2 \cdot 1.7m \cdot \cos 30°}{T^2\cos 30°} - 19.6N$$
$$T_2 = m\frac{2\pi^2 \cdot 1.7m}{T^2} - 19.6N$$
$$T_2 = 2kg \cdot \frac{2\pi^2 \cdot 1.7m}{(1s)^2} - 19.6N$$
$$T_2 = 47.51N$$

Thus, the tension T_1 is

$$T_1 = 39.2N + T_2$$
$$T_1 = 86.71N$$

11. *Forces Acting on the Object*



There are 2 forces acting on the small block.

1) The weight (mg) directed downwards.
2) A normal force (F_N) directed at 30°.

Sum of the Forces

The sums of the forces are

$$\sum F_x = F_N\cos 30°$$
$$\sum F_y = -mg + F_N\sin 30°$$

Newton's Second Law

Since the centripetal acceleration is towards the positive x-axis, we have

$$\sum F_x = ma_x \qquad \rightarrow \qquad F_N\cos 30° = m\frac{4\pi^2 r}{T^2}$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -mg + F_N\sin 30° = 0$$

<u>*Solving the equations*</u>

If we solve for F_N in the sum of the y-component of the forces, we obtain

$$F_N = \frac{mg}{\sin 30^\circ}$$

We then substitute this value in the sum of the x-component of the forces to obtain

$$\frac{mg}{\sin 30^\circ}\cos 30^\circ = m\frac{4\pi^2 r}{T^2}$$

It only remains to solve for r.

$$\frac{g}{\sin 30^\circ}\cos 30^\circ = \frac{4\pi^2 r}{T^2}$$
$$r = \frac{gT^2}{4\pi^2 \tan 30^\circ}$$

Thus, the value of r is

$$\begin{aligned} r &= \frac{9.8\tfrac{N}{kg}\cdot(0.5s)^2}{4\pi^2 \tan 30^\circ} \\ &= 0.1075m \end{aligned}$$

Therefore, the value of x, which is the radius of the circular path, is 10.75 cm

12. a) We'll first find the tangential acceleration. It is calculated with

$$\begin{aligned} F_t &= ma_t \\ 1.5N &= 3kg\cdot a_t \\ a_t &= 0.5\tfrac{m}{s^2} \end{aligned}$$

To find centripetal acceleration, the speed of the object is required. This speed is

$$\begin{aligned} v &= v_0 + a_t t \\ &= 0\tfrac{m}{s} + 0.5\tfrac{m}{s^2}\cdot 2s \\ &= 1\tfrac{m}{s} \end{aligned}$$

Therefore, the centripetal acceleration is

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{\left(1\frac{m}{s}\right)^2}{1.2m} \\ &= 0.8333\frac{m}{s^2} \end{aligned}$$

Therefore, the total acceleration is

$$\begin{aligned} a &= \sqrt{a_c^2 + a_t^2} \\ &= \sqrt{\left(0.8333\frac{m}{s^2}\right)^2 + \left(0.5\frac{m}{s^2}\right)^2} \\ &= 0.9718\frac{m}{s^2} \end{aligned}$$

b) The tube is the only thing that exerts a centripetal force. Its tension must therefore be equal to the centripetal force. This force is

$$\begin{aligned} T &= ma_c \\ &= 3kg \cdot 0.8333\frac{m}{s^2} \\ &= 2.5N \end{aligned}$$

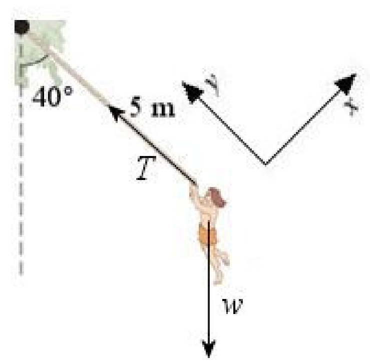
13. a) The centripetal acceleration is

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{\left(10\frac{m}{s}\right)^2}{5m} \\ &= 20\frac{m}{s^2} \end{aligned}$$

b) *Forces Acting on the Object*

There are 2 forces exerted on Gontran.

1) The 637 N weight directed downwards.
2) The tension force (T).



Sum of the Forces

The sums of the forces are

$$\sum F_x = mg\cos(-130°)$$
$$\sum F_y = T + mg\sin(-130°)$$

Newton's Second Law

Since the centripetal acceleration is towards the positive y-axis, we have

$$\sum F_x = ma_x \qquad \rightarrow \qquad mg\cos(-130°) = ma_t$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad T + mg\sin(-130°) = m\frac{v^2}{r}$$

Solving the equations

The tangential acceleration can be found with the sum of the x-component of the forces.

$$mg\cos(-130°) = ma_t$$
$$a_t = g\cos(-130°)$$
$$a_t = -6.299\tfrac{m}{s^2}$$

c) The acceleration is therefore

$$\begin{aligned} a &= \sqrt{a_c^2 + a_t^2} \\ &= \sqrt{\left(20\tfrac{m}{s^2}\right)^2 + \left(6.299\tfrac{m}{s^2}\right)^2} \\ &= 20.969\tfrac{m}{s^2} \end{aligned}$$

d) The tension can be found with the sum of the x-component of the forces.

$$T + mg\sin(-130°) = m\frac{v^2}{r}$$
$$T = m\frac{v^2}{r} - mg\sin(-130°)$$
$$T = m\frac{v^2}{r} + mg\sin 130°$$
$$T = 65kg \cdot \frac{\left(10\tfrac{m}{s}\right)^2}{5m} + 65kg \cdot 9.8\tfrac{N}{kg} \cdot \sin 130°$$
$$T = 1300N + 488N$$
$$T = 1788N$$

14. The field is

$$\begin{aligned} g &= \frac{GM}{r^2} \\ &= \frac{6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 5.972\times10^{24}\,kg}{\left(7381\times10^3\,m\right)^2} \\ &= 7.336\,\tfrac{N}{kg} \end{aligned}$$

15. a) The field is

$$\begin{aligned} g &= \frac{GM}{r^2} \\ &= \frac{6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 6.4185\times10^{23}\,kg}{\left(3386\times10^3\,m\right)^2} \\ &= 3,736\,\tfrac{N}{kg} \end{aligned}$$

b) The weight of the person is

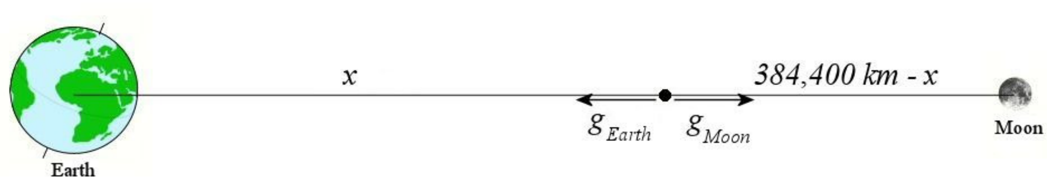
$$\begin{aligned} w &= mg \\ &= 70kg\cdot 3.736\,\tfrac{N}{kg} \\ &= 261.5N \end{aligned}$$

c) As the weight of this person on Earth is 70 kg · 9.8 N/kg = 686 N, the ratio between the weight on Mars and the weight on the Earth is

$$\frac{261.5N}{686N} = 0.381$$

The weight on Mars is therefore only 38.1% of the weight of the person on Earth.

16. We have the following configuration.



The sum of the fields is

$$g = -\frac{GM_{Earth}}{x^2} + \frac{GM_{Moon}}{\left(3.844\times10^8 m - x\right)^2}$$

Since we want a vanishing field, we must have

$$0 = -\frac{GM_{Earth}}{x^2} + \frac{GM_{Moon}}{\left(3.844\times10^8 m - x\right)^2}$$

$$\frac{GM_{Earth}}{x^2} = \frac{GM_{Moon}}{\left(3.844\times10^8 m - x\right)^2}$$

$$\frac{M_{Earth}}{x^2} = \frac{M_{Moon}}{\left(3.844\times10^8 m - x\right)^2}$$

$$\left(3.844\times10^8 m - x\right)^2 = \frac{M_{Moon}}{M_{Earth}} x^2$$

$$\left(3.844\times10^8 m - x\right)^2 = \frac{7.35\times10^{22} kg}{5.98\times10^{24} kg} \cdot x^2$$

$$\left(3,844\times10^8 m - x\right)^2 = 0.01229 \cdot x^2$$

$$1.4776\times10^{17} m^2 - 7.688\times10^8 m \cdot x + x^2 = 0.01229 \cdot x^2$$

$$1.4776\times10^{17} m^2 - 7.688\times10^8 m \cdot x + 0.9877 \cdot x^2 = 0$$

The solution of this quadratic equation is

$$x = 3.46 \times 10^8 \text{ m}$$

The field, therefore, vanishes 346,000 km from the Earth.

(There is another solution at 432,000 km from the Earth. There, both fields have the same magnitude, but the total field is non-zero because fields are in the same direction.)

17. An x-axis towards the right and a y-axis upwards will be used. The field made by the Moon is towards the left, so towards the negative x. The field made by the Moon is

$$\begin{aligned} g_{Mx} &= -\frac{GM}{r^2} \\ &= -\frac{6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 7.35\times10^{22}\,kg}{\left(150\,000\times10^3\,m\right)} \\ &= -2.1801\times10^{-4}\,\tfrac{N}{kg} \end{aligned}$$

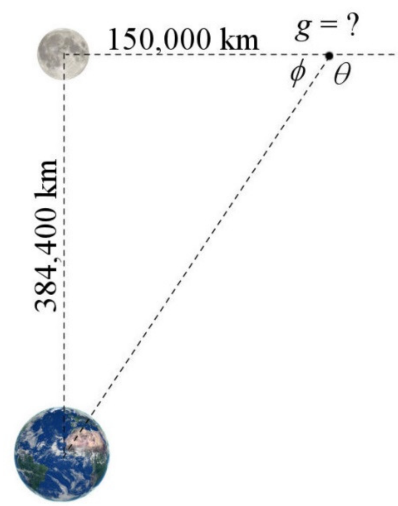
To find the magnitude of the field made by the Earth, the distance between the Earth and the place where the field is sought must be known. This distance is

$$\begin{aligned} r &= \sqrt{\left(384\,400\times10^3\,m\right)^2+\left(150\,000\times10^3\,m\right)^2} \\ &= 4.126\times10^8\,m \end{aligned}$$

Therefore, the magnitude of the field made by the Earth is

$$\begin{aligned} g_E &= \frac{GM}{r^2} \\ &= \frac{6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 5.98\times10^{24}\,kg}{\left(4.126\times10^8\,m\right)} \\ &= 2.344\times10^{-3}\,\tfrac{N}{kg} \end{aligned}$$

To resolve this field into components, the angle θ must be known. The angle ϕ is found first



$$\begin{aligned} \tan\phi &= \frac{384,400}{150,000} \\ \phi &= 68.68° \end{aligned}$$

Therefore, the angle θ is

$$\begin{aligned} \theta &= 180° - \phi \\ &= 180 - 68.68° \\ &= 111.32° \end{aligned}$$

Therefore, the components of the field of the Earth are

$$\begin{aligned} g_{Ex} &= 2.344\times10^{-3}\,\tfrac{N}{kg}\cdot\cos(-111.32°) \\ &= -8.521\times10^{-4}\,\tfrac{N}{kg} \end{aligned}$$

$$\begin{aligned} g_{Ey} &= 2.344\times10^{-3}\,\tfrac{N}{kg}\cdot\sin(-111.32°) \\ &= -2.184\times10^{-3}\,\tfrac{N}{kg} \end{aligned}$$

Thus, the components of the total field are

$$\begin{aligned} g_x &= g_{Mx} + g_{Mx} \\ &= -2.180\times10^{-4}\,\tfrac{N}{kg} + -8.521\times10^{-4}\,\tfrac{N}{kg} \\ &= -1.070\times10^{-3}\,\tfrac{N}{kg} \end{aligned}$$

$$\begin{aligned} g_y &= g_{My} + g_{Ey} \\ &= 0 + -2.184\times10^{-3}\,\tfrac{N}{kg} \\ &= -2.184\times10^{-3}\,\tfrac{N}{kg} \end{aligned}$$

Therefore, the magnitude of the field is

$$\begin{aligned} g &= \sqrt{g_x^2 + g_y^2} \\ &= \sqrt{\left(1.070\times10^{-3}\,\tfrac{N}{kg}\right)^2 + \left(2.184\times10^{-3}\,\tfrac{N}{kg}\right)^2} \\ &= 2.432\times10^{-3}\,\tfrac{N}{kg} \end{aligned}$$

18. We have

$$T = 2\pi\sqrt{\frac{r^3}{GM_c}}$$

$$27.32\cdot24\cdot60\cdot60s = 2\pi\sqrt{\frac{\left(384\,400\times10^3 m\right)^3}{6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot M_{Earth}}}$$

$$M_{Earth} = 6.03\times10^{24} kg$$

19. a) With what is known about Io, the mass of Jupiter can be found.

$$T = 2\pi\sqrt{\frac{r^3}{GM_c}}$$

$$1.796 \cdot 24 \cdot 60 \cdot 60 s = 2\pi\sqrt{\frac{\left(421\,700 \times 10^3 m\right)^3}{6.674 \times 10^{-11} \frac{Nm^2}{kg^2} \cdot M_{Jupiter}}}$$

$$M_{Jupiter} = 1.8422 \times 10^{27} kg$$

This information is then used to find the period of revolution of Ganymede.

$$\begin{aligned} T &= 2\pi\sqrt{\frac{r^3}{GM_c}} \\ &= 2\pi\sqrt{\frac{\left(1\,070\,400 \times 10^3 m\right)^3}{6.674 \times 10^{-11} \frac{Nm^2}{kg^2} \cdot 1.8422 \times 10^{27} kg}} \\ &= 627{,}529 s = 7.263 j \end{aligned}$$

b) The speed of Ganymede is

$$\begin{aligned} v &= \sqrt{\frac{GM_c}{r}} \\ &= \sqrt{\frac{6.674 \times 10^{-11} \frac{Nm^2}{kg^2} \cdot 1.8422 \times 10^{27} kg}{1\,070\,400 \times 10^3 m}} \\ &= 10{,}717 \tfrac{m}{s} = 10.717 \tfrac{km}{s} \end{aligned}$$

20. The period of revolution is

$$\begin{aligned} T &= 2\pi\sqrt{\frac{r^3}{GM_c}} \\ &= 2\pi\sqrt{\frac{\left(1837 \times 10^3 m\right)^3}{6.674 \times 10^{-11} \frac{Nm^2}{kg^2} \cdot 7.35 \times 10^{22} kg}} \\ &= 7063 s = 1.962 h \end{aligned}$$

21. The distance is found with

$$T = 2\pi\sqrt{\frac{r^3}{GM_c}}$$

$$2\cdot 24\cdot 60\cdot 60s = 2\pi\sqrt{\frac{r^3}{6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot 5.97\times10^{24}kg}}$$

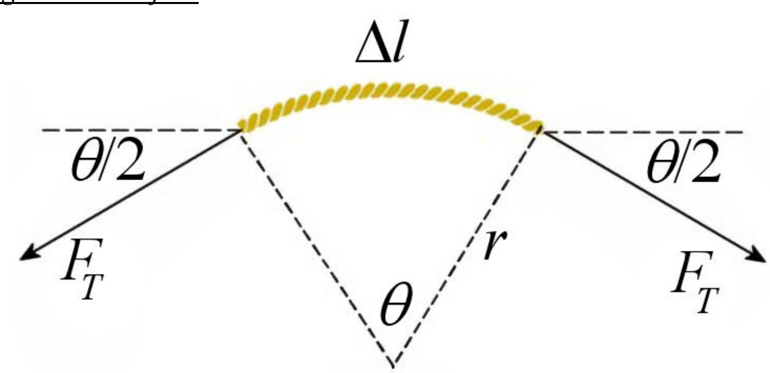
$$r = 6.7044\times10^{7}m = 67,044km$$

This is the distance from the centre of the Earth. Therefore, the distance from the surface is

$$dist = 67,044km - 6371km = 60,673km$$

22. Let's take a small piece of rope and examine the forces on this piece. (The angle θ is small on the figure.)

Forces Acting on the Object



Sum of the Forces

The sum of the y-component of the force is (using a y-axis directed upwards)

$$\sum F_y = F_T\sin\left(-\frac{\theta}{2}\right) + F_T\sin\left(180° + \frac{\theta}{2}\right)$$

Newton's Second Law

Since this piece makes a circular motion and since the centripetal acceleration is towards the negative y-axis, we have

$$\sum F_y = ma_y \qquad \rightarrow \qquad F_T\sin\left(-\frac{\theta}{2}\right) + F_T\sin\left(180° + \frac{\theta}{2}\right) = -m\frac{4\pi^2 r}{T^2}$$

Solving the Equations

Since sin -x = -sin x and sin (180°+x) = -sin x, it becomes

$$-F_T \sin\left(\frac{\theta}{2}\right) - F_T \sin\left(\frac{\theta}{2}\right) = -m\frac{4\pi^2 r}{T^2}$$
$$2F_T \sin\left(\frac{\theta}{2}\right) = m\frac{4\pi^2 r}{T^2}$$

Since the angle is small, sin $x = x$. (This means that we are now working with angles in radians.)

$$2F_T \frac{\theta}{2} = m\frac{4\pi^2 r}{T^2}$$
$$F_T\theta = m\frac{4\pi^2 r}{T^2}$$

The mass of the piece depends on the angle. The proportion of the mass of the rope in the small piece is the same as that of the angle compared to 2π radians.

$$\frac{m}{6kg} = \frac{\theta}{2\pi}$$
$$m = \frac{\theta}{2\pi} 6kg$$

The force equation then becomes

$$F_T\theta = m\frac{4\pi^2 r}{T^2}$$
$$F_T\theta = \frac{\theta}{2\pi} 6kg \frac{4\pi^2 r}{T^2}$$
$$F_T = 6kg \frac{2\pi r}{T^2}$$

With the values of r and T, the tension is

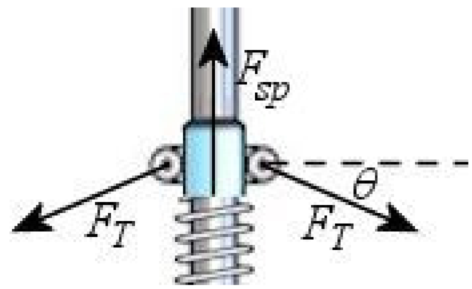
$$F_T = 6kg \cdot \frac{2\pi \cdot 0.5m}{(0.25s)^2}$$
$$= 301.6N$$

23. To obtain the spring compression, we will look at the forces exerted by the spring on the fixtures at the end of the spring. Then, the force exerted by the spring will be found, and the compression of the spring will be found from this force.

Fixtures and the end of the spring

Forces Acting on the Object

The diagram shows the forces exerted.



Sum of the Forces

The sum of the y-component of the force is (using a y-axis directed upwards)

$$\sum F_y = F_T \sin(-\theta) + F_T \sin(-\theta) + F_{sp}$$

Newton's Second Law

Since there is no acceleration at equilibrium, we have

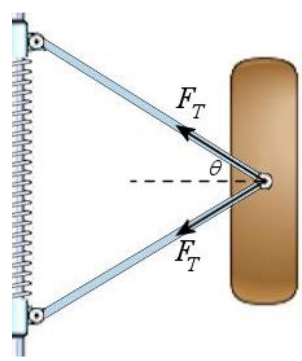
$$\sum F_y = ma_y \qquad \to \qquad F_T \sin(-\theta) + F_T \sin(-\theta) + F_{sp} = 0$$

To obtain the force made by the spring, the tension of the rods is needed. This tension can be found by looking at the forces exerted at the other end of those rods.

Masses

Forces acting on the Object

The diagram shows the forces exerted.



Sum of the Forces

The sum of the x-component of the force is (using a x-axis directed towards the left)

$$\sum F_x = F_T \cos\theta + F_T \cos\theta$$

Newton's Second Law

Since the centripetal acceleration is towards the positive *x*-axis, we have

$$\sum F_x = ma_x \qquad \rightarrow \qquad F_T \cos\theta + F_T \cos\theta = m\frac{4\pi^2 r}{T^2}$$

Solving the equations

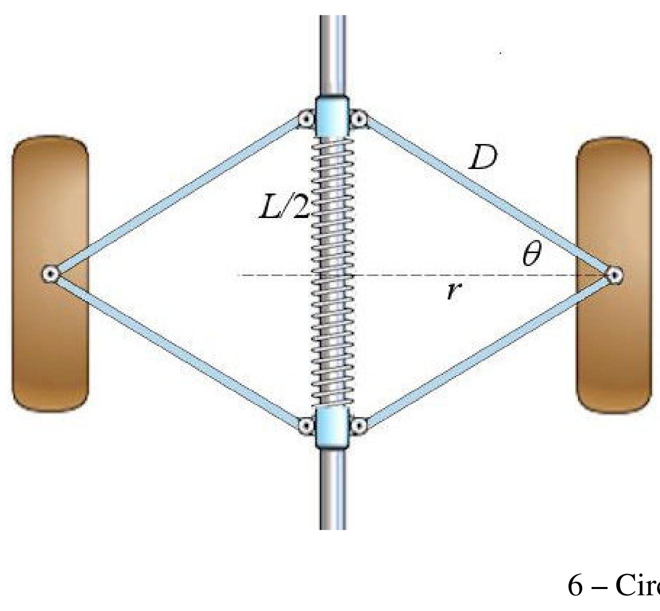
The sum of the *x*-components of the forces on the masses gives

$$F_T \cos\theta + F_T \cos\theta = m\frac{4\pi^2 r}{T^2}$$
$$2F_T \cos\theta = m\frac{4\pi^2 r}{T^2}$$
$$F_T = \frac{2m\pi^2 r}{T^2 \cos\theta}$$

This value can the be used in the equation of the forces on the fixtures

$$F_T \sin(-\theta) + F_T \sin(-\theta) + F_{sp} = 0$$
$$-2F_T \sin\theta + F_{sp} = 0$$
$$-2\frac{2m\pi^2 r}{T^2 \cos\theta}\sin\theta + F_{sp} = 0$$
$$F_{sp} = \frac{4m\pi^2 r}{T^2 \cos\theta}\sin\theta$$

Now, let's make the connection between a few variables.



Firstly,

$$D\cos\theta = r$$

so that

$$\begin{aligned} F_{sp} &= \frac{4m\pi^2 r}{T^2\cos\theta}\sin\theta \\ &= \frac{4m\pi^2 D}{T^2}\sin\theta \end{aligned}$$

Secondly,

$$D\sin\theta = \frac{L}{2}$$

so that

$$\begin{aligned} F_{sp} &= \frac{4m\pi^2 D}{T^2}\sin\theta \\ &= \frac{2m\pi^2 L}{T^2} \end{aligned}$$

Since the force exerted by the spring is

$$F_{sp} = k\left(x_0 - L\right)$$

where x_0 is the length of the spring when it is neither stretched nor compressed, the equation becomes

$$k\left(x_0 - L\right) = \frac{2m\pi^2 L}{T^2}$$

It only remains to solve this equation for *L*.

$$kx_0 - kL = \frac{2m\pi^2 L}{T^2}$$

$$kx_0 = \left(\frac{2m\pi^2}{T^2} + k\right)L$$

$$L = \frac{x_0}{1 + \frac{2m\pi^2}{kT^2}}$$

With the values given, the length is

$$\begin{aligned} L &= \frac{x_0}{1 + \frac{2m\pi^2}{kT^2}} \\ &= \frac{80cm}{1 + \frac{2 \cdot 1kg \cdot \pi^2}{2000\frac{N}{m} \cdot (0.1s)^2}} \\ &= 40.263cm \end{aligned}$$