Chapter 5 Solutions

1. *Forces acting on the Object*

There are 3 forces exerted on the 10 kg object.

1) The 98 N weight directed downwards.
2) A normal force (F_N) directed upwards.
3) A friction force ($F_f = \mu_k F_N$) directed towards the left.

Sum of the Forces

The sum of the forces is (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\sum F_x = -\mu_k F_N$$
$$\sum F_y = -mg + F_N$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \quad \rightarrow \quad -\mu_k F_N = ma$$
$$\sum F_y = ma_y \quad \rightarrow \quad -mg + F_N = 0$$

Solving the Equations

The friction coefficient is found with

$$-\mu_k F_N = ma$$

However, the normal force and the acceleration are needed to obtain the coefficient.

The normal force is found with the equation for the y-component of the force.

$$-mg + F_N = 0$$
$$F_N = mg$$

As the block stops in 4 seconds, the acceleration is

$$v_x = v_{0x} + a_x t$$
$$0\tfrac{m}{s} = 10\tfrac{m}{s} + a_x \cdot 4s$$
$$a_x = -2.5\tfrac{m}{s^2}$$

Therefore, the coefficient is

$$-\mu_k F_N = ma$$
$$-\mu_k mg = ma$$
$$-\mu_k g = a$$
$$-\mu_k \cdot 9.8\tfrac{N}{kg} = -2.5\tfrac{m}{s^2}$$
$$\mu_k = 0{,}2551$$

2. To answer those questions, the acceleration must be known. This acceleration is found with Newton's second law.

Forces acting on the Object

There are 3 forces acting on Guy.

1) The weight (mg) directed downwards (at -60°).
2) A normal force (F_N) perpendicular to the slope.
3) A friction force ($F_f = \mu_c F_N$) opposed to the motion so directed uphill.

Sum of the Forces

The table of force is (using an x-axis directed downhill)

Forces	*x*	*y*
Weight	mg cos (-60°)	mg sin (-60°)
Normal force	0	F_N
Friction force	$-\mu_k F_N$	0

The sums of the forces are then

$$\sum F_x = mg\cos(-60°) - \mu_k F_N$$
$$\sum F_y = mg\sin(-60°) + F_N$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad mg\cos(-60°) - \mu_k F_N = ma$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad mg\sin(-60°) + F_N = 0$$

Solving the Equations

The acceleration can be obtained with the equation for the x-components. However, the normal force must be known to calculate a.

The normal force can be found with the sum of the y-component of the forces.

$$mg\sin(-60°) + F_N = 0$$
$$F_N = -mg\sin(-60°)$$
$$F_N = mg\sin(60°)$$

This value is then substituted in the sum of the x-component of the forces.

$$mg\cos(-60°) - \mu_k mg\sin(60°) = ma_x$$
$$g\cos(-60°) - \mu_k g\sin(60°) = a_x$$
$$9.8\tfrac{N}{kg}\cos(-60°) - 0.1 \cdot 9.8\tfrac{N}{kg}\sin(60°) = a_x$$
$$a_x = 4.051\tfrac{m}{s^2}$$

With the acceleration, the 2 questions can now be answered.

a) The time required to reach 50 m/s is

$$v_x = v_{0x} + a_x t$$
$$50\tfrac{m}{s} = 10\tfrac{m}{s} + 4.051\tfrac{m}{s^2} \cdot t$$
$$t = 9.873s$$

b) The distance travelled in 5 seconds is

$$\begin{aligned} x &= x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \\ &= 0m + 10\tfrac{m}{s} \cdot 5s + \tfrac{1}{2} \cdot 4.051\tfrac{m}{s^2} \cdot (5s)^2 \\ &= 100.6m \end{aligned}$$

3. Let's start with the situation on the horizontal surface to find the coefficient of friction

Forces acting on the Object

There are 3 forces exerted on Manon.

1) The weight (mg) directed downwards.
2) A normal force (F_N) directed upwards.
3) A friction force ($F_f = \mu_k F_N$) directed towards the left.

Sum of the Forces

The sums of the forces are (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\begin{aligned} \sum F_x &= -\mu_k F_N \\ \sum F_y &= -mg + F_N \end{aligned}$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\begin{aligned} \sum F_x &= ma_x &&\rightarrow && -\mu_k F_N = ma \\ \sum F_y &= ma_y &&\rightarrow && -mg + F_N = 0 \end{aligned}$$

Solving the Equations

The coefficient can be obtained from the equation for the x-component of Newton's second law. However, the normal force and the acceleration must be known first.

The normal force can be found with the sum of the y-component of the forces.

$$-mg + F_N = 0$$
$$F_N = mg$$

As Manon stops over a distance of 40 m, the acceleration is

$$2a\left(x - x_0\right) = v_x^2 - v_{0x}^2$$
$$2 \cdot a \cdot \left(40m - 0m\right) = \left(0\tfrac{m}{s}\right)^2 - \left(10\tfrac{m}{s}\right)^2$$
$$a = -1.25\tfrac{m}{s^2}$$

Then

$$-\mu_k F_N = ma$$
$$-\mu_k mg = ma$$
$$-\mu_k g = a$$
$$-\mu_k \cdot 9.8\tfrac{N}{kg} = -1.25\tfrac{m}{s^2}$$
$$\mu_c = 0.12755$$

Let us now examine the situation on the slope.

Forces acting on the Object

There are still 3 forces exerted on Manon.

1) The weight (mg) directed downwards.
2) A normal force (F_N) perpendicular to the slope.
3) A friction force ($F_f = \mu_k F_N$) opposed to the motion so directed downhill.

Sum of the Forces

The table of force is (using an x-axis directed uphill)

Forces	*x*	*y*
Weight	mg cos (-120°)	mg sin (-120°)
Normal force	0	F_N
Friction force	$-\mu_k F_N$	0

The sums of the forces are then

$$\sum F_x = mg\cos\left(-120°\right) - \mu_c F_N$$
$$\sum F_y = mg\sin\left(-120°\right) + F_N$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis or in the direction opposed to the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad mg\cos\left(-120°\right) - \mu_k F_N = ma$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad mg\sin\left(-120°\right) + F_N = 0$$

Solving the Equations

The acceleration can be obtained from the equation for the *x*-component of Newton's second law. However, the normal force must be known first.

The normal force can be found with the sum of the *y*-component of the forces.

$$mg\sin\left(-120°\right) + F_N = 0$$
$$F_N = -mg\sin\left(-120°\right)$$
$$F_N = mg\sin\left(120°\right)$$

This value is then substituted in the sum of the *x*-component of the forces.

$$mg\cos\left(-120°\right) - \mu_k mg\sin\left(120°\right) = ma$$
$$g\cos\left(120°\right) - \mu_k g\sin\left(120°\right) = a$$
$$9.8\tfrac{N}{kg}\cdot\cos\left(120°\right) - 0.12755\cdot 9.8\tfrac{N}{kg}\cdot\sin\left(120°\right) = a$$
$$a = -5.9825\tfrac{m}{s^2}$$

The stopping distance is therefore

$$2a(x-x_0)=v_x^2-v_{0x}^2$$
$$2\cdot\left(-5.9825\tfrac{m}{s^2}\right)\cdot(x-0m)=\left(0\tfrac{m}{s}\right)^2-\left(10\tfrac{m}{s}\right)^2$$
$$x=8.358m$$

4. Let's find the equations for each block.

24 kg Block

Forces acting on the Object

There are 4 forces exerted on the 24 kg block.

1) The 235.2 N weight directed downwards.
2) A normal force (F_{N1}) directed upwards.
3) A tension force (T) directed towards the right.
4) A friction force ($F_{f1} = \mu_k F_{N1}$) directed towards the right.

Sum of the Forces

The sums of the of force are (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

$$\sum F_x = T+\mu_k F_{N1}$$
$$\sum F_y = -235.2N+F_{N1}$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis or in the direction opposed to the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \to \qquad T+0.4\cdot F_{N1}=24kg\cdot a$$
$$\sum F_y = ma_y \qquad \to \qquad -235.2N+F_{N1}=0$$

We will use the sum of the y-component of the forces right now to find F_{N1}

$$-235.2N + F_{N1} = 0$$
$$F_{N1} = 235.2N$$

and use this value in the sum of the x-component of the forces to obtain

$$T + 0.4 \cdot F_{N1} = 24kg \cdot a$$
$$T + 0.4 \cdot 235.2N = 24kg \cdot a$$
$$T + 94.08N = 24kg \cdot a$$

18 kg Block

Forces acting on the Object

There are 4 forces acting on the 18 kg block.

1) The 176.4 N weight directed downwards.
2) A normal force (F_{N2}) perpendicular to the surface
3) A tension force (T) directed uphill.
4) A friction force ($F_{f2} = \mu_k F_{N2}$) directed downhill.

Sum of the Forces

The table of force is (using an x-axis directed downhill)

Forces	x	y
Weight	176.4 N · cos (-30°)	176.4 N · sin (-30°)
Normal	0	F_{N2}
Friction	$\mu_k F_{N2}$	0
Tension	$-T$	0

The sums of the force are then

$$\sum F_x = 176.4N \cdot \cos(-30°) + \mu_c F_{N2} - T$$
$$\sum F_y = 176.4N \cdot \sin(-30°) + F_{N2}$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad 176.4N \cdot \cos(-30°) + 0.4 \cdot F_{N2} - T = 18kg \cdot a$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad 176.4N \cdot \sin(-30°) + F_{N2} = 0$$

We will use the sum of the *y*-component of the forces right now to find F_{N2}

$$176.4N \cdot \sin(-30°) + F_{N2} = 0$$
$$F_{N2} = -176.4N \cdot \sin(-30°)$$
$$F_{N2} = 88.2N$$

and use this value in the sum of the *x*-component of the forces to obtain

$$176.4N \cdot \cos(30°) + 0.4 \cdot F_{N2} - T = 18kg \cdot a$$
$$176.4N \cdot \cos(30°) + 0.4 \cdot 88.2N - T = 18kg \cdot a$$
$$188.047N - T = 18kg \cdot a$$

Solving the Equations

The two equations for the sum of the *x*-component of the forces are then

$$T + 94.08N = 24kg \cdot a$$
$$188.047N - T = 18kg \cdot a$$

This system can be solved by adding these equations.

$$(T + 94.08N) + (188.047N - T) = 24kg \cdot a + 18kg \cdot a$$
$$94.08N + 188.047N = (24kg + 18kg)a$$
$$282.127N = 42kg \cdot a$$
$$a = 6.7173 \tfrac{m}{s^2}$$

The tension can then be found

$$T + 94.08N = 24kg \cdot a$$
$$T + 94.08N = 24kg \cdot 6.7173 \tfrac{m}{s^2}$$
$$T = 67.135N$$

5. Since the friction coefficients are different for each box, it would be difficult to find the acceleration of the system by considering it as a single object. Instead, the forces on each box must be found.

12 kg Box

Forces acting on the Object

There are 5 forces exerted on the 12 kg box.

1) The 117.6 N weight directed downwards.
2) A normal force (F_{N1}) directed upwards.
3) A friction force ($F_{f1} = \mu_{k1} F_{N1}$) directed towards the right.
4) The 30 N force.
5) The tension force (T_1) of the rope between the 12 kg block and the 8 kg block directed towards the left.

Sum of the Forces

The sums of the force are then (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\sum F_x = 30N \cdot \cos 37° + \mu_{k1} F_{N1} - T_1$$
$$\sum F_y = -117.6N + F_{N1} + 30N \cdot \sin 37°$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \quad \rightarrow \quad 30N \cdot \cos 37° + 0.5 \cdot F_{N1} - T_1 = 12kg \cdot a$$
$$\sum F_y = ma_y \quad \rightarrow \quad -117.6N + F_{N1} + 30N \cdot \sin 37° = 0$$

We will use the sum of the y-component of the forces right now to find F_{N1}

$$-117.6N + F_{N1} + 30N \cdot \sin 37° = 0$$
$$F_{N1} = 117.6N - 30N \cdot \sin 37°$$
$$F_{N1} = 99.55N$$

and use this value in the sum of the x-component of the forces to obtain

$$30N \cdot \cos 37° + 0.5 \cdot F_{N1} - T_1 = 12kg \cdot a$$
$$30N \cdot \cos 37° + 0.5 \cdot 99.55N - T_1 = 12kg \cdot a$$
$$73.732N - T_1 = 12kg \cdot a$$

8 kg Box

There are 5 forces acting on the 8 kg box.

1) The 78.4 N weight directed downwards.
2) A normal force (F_{N2}) directed upwards.
3) A friction force ($F_{f2} = \mu_{k2} F_{N2}$) directed towards the right.
4) The tension force (T_1) of the rope between the 12 kg block and the 8 kg block directed towards the right.
5) The tension force (T_2) of the rope between the 8 kg block and the 5 kg block directed towards the left.

Sum of the Forces

The sums of the force are then (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\sum F_x = \mu_{k2} F_{N2} + T_1 - T_2$$
$$\sum F_y = -78.4N + F_{N2}$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad 0.4 \cdot F_{N2} + T_1 - T_2 = 8kg \cdot a$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -78.4N + F_{N2} = 0$$

We will use the sum of the y-component of the forces right now to find F_{N2}

$$-78.4N + F_{N2} = 0$$
$$F_{N2} = 78.4N$$

and use this value in the sum of the *x*-component of the forces to obtain

$$0.4 \cdot F_{N2} + T_1 - T_2 = 8kg \cdot a$$
$$0.4 \cdot 78.4N + T_1 - T_2 = 8kg \cdot a$$
$$31.36N + T_1 - T_2 = 8kg \cdot a$$

5 kg Box

Forces Acting on the Object

There are 5 forces acting on the 5 kg box.

1) The 49 N weight directed downwards.
2) A normal force (F_{N3}) directed upwards.
3) A friction force ($F_{f3} = \mu_{k3} F_{N3}$) directed towards the right.
4) The tension force (T_2) of the rope between the 8 kg block and the 5 kg block directed towards the right.

Sum of the Forces

The sums of the force are then (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

$$\sum F_x = \mu_{k3} F_{N3} + T_2$$
$$\sum F_y = -49N + F_{N3}$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis or in the direction opposed to the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad \mu_{k3} F_{N3} + T_2 = 5kg \cdot a$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -49N + F_{N3} = 0$$

We will use the sum of the *y*-component of the forces right now to find F_{N3}

$$-49N + F_{N3} = 0$$
$$F_{N3} = 49N$$

and use this value in the sum of the *x*-component of the forces to obtain

$$0.3 \cdot F_{N3} + T_2 = 5kg \cdot a$$
$$0.3 \cdot 49N + T_2 = 5kg \cdot a$$
$$14.7N + T_2 = 5kg \cdot a$$

Solving the Equations

The 3 equations obtained are

$$73.732N - T_1 = 12kg \cdot a$$
$$31.36N + T_1 - T_2 = 8kg \cdot a$$
$$14.7N + T_2 = 5kg \cdot a$$

This system can be solved by adding these three equations.

$$\left(73.732N - T_1\right) + \left(31.36N + T_1 - T_2\right) + \left(14.7N + T_2\right) = 12kg \cdot a + 8kg \cdot a + 5kg \cdot a$$
$$73.732N + 31.36N + 14.7N = \left(12kg + 8kg + 5kg\right) \cdot a$$
$$119.792N = 25kg \cdot a$$
$$a = 4.792 \tfrac{m}{s^2}$$

Then, the tensions can be found. T_1 is found with the sum of the *x*-component of the forces for the 12 kg block.

$$73.732N - T_1 = 12kg \cdot a$$
$$73.732N - T_1 = 12kg \cdot 4.792 \tfrac{m}{s^2}$$
$$T_1 = 16.232N$$

T_2 is found with the sum of the *x*-component of the forces for the 5 kg block.

$$14.7N + T_2 = 5kg \cdot a$$
$$14.7N + T_2 = 5kg \cdot 4.792 \tfrac{m}{s^2}$$
$$T_2 = 9.258N$$

6. *Forces Acting on the Object*

There are 4 forces exerted on the 100 kg block of ice.

1) The 980 N weight directed downwards.
2) A normal force (F_N) directed upwards.
3) A friction force ($F_f = \mu_k F_N$) directed towards the left.
4) The tension force (T).

Sum of the Forces

The sums of the force are then (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\sum F_x = -\mu_k F_N + T\cos 25°$$
$$\sum F_y = -980N + F_N + T\sin 25°$$

Newton's Second Law

As the velocity is constant, there is no acceleration. Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad -0.1\cdot F_N + T\cos 25° = 0$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -980N + F_N + T\sin 25° = 0$$

Solving the Equations

The normal force can be found with the sum of the y-component of the forces.

$$F_N = 980N - T\sin\left(25°\right)$$

This value is then substituted in the sum of the x-component of the forces.

$$-0.1\cdot F_N + T\cos 25° = 0$$
$$-0.1\cdot\left(980N - T\sin 25°\right) + T\cos 25° = 0$$
$$-0.1\cdot 980N + 0.1\cdot T\sin 25° + T\cos 25° = 0$$
$$T\left(0.1\cdot\sin 25° + \cos 25°\right) = 0.1\cdot 980N$$
$$T = \frac{0.1\cdot 980N}{0.1\cdot\sin 25° + \cos 25°}$$
$$T = 103.31N$$

7. *Forces Acting on the Object*

There are 4 forces exerted on the sled.

1) The 78.4 N weight directed downwards.
2) A normal force (F_N) perpendicular to the slope.
3) A friction force ($F_f = \mu_k F_N$) directed downhill.
4) The tension force of the rope (T).

Sum of the Forces

The table of force is (with an *x*-axis directed uphill)

Forces	***x***	***y***
Weight	78.4 N · cos (-100°)	78.4 N · sin (-100°)
Normal force	0	F_N
Friction force	$-\mu_k F_N$	0
Tension force	T cos 20°	T sin 20°

The sums of the force are then

$$\sum F_x = 78.4N \cdot \cos(-100°) - \mu_k F_N + T\cos 20°$$
$$\sum F_y = 78.4N \cdot \sin(-100°) + F_N + T\sin 20°$$

Newton's Second Law

As the velocity is constant, there is no acceleration. Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \to \qquad 78.4N \cdot \cos(-100°) - 0.12 \cdot F_N + T\cos 20° = 0$$
$$\sum F_y = ma_y \qquad \to \qquad 78.4N \cdot \sin(-100°) + F_N + T\sin 20° = 0$$

Solving the Equations

We have two equations and two unknowns.

$$-13,614N - 0,12 \cdot F_N + T\cos 20° = 0$$
$$-77,209N + F_N + T\sin 20° = 0$$

To resolve, we will solve for the normal force in the sum of the *y*-component of the forces.

$$F_N = 77.209N - T\sin 20°$$

This value is then substituted in the sum of the *x*-component of the forces.

$$\begin{aligned}
-13.614N - 0.12 \cdot F_N + T\cos 20° = 0 \\
-13.614N - 0.12 \cdot (77.209N - T\sin 20°) + T\cos 20° = 0 \\
-13.614N - 0.12 \cdot 77.209N + 0.12 \cdot T\sin 20° + T\cos 20° = 0 \\
-22.879N + 0.12 \cdot T\sin 20° + T\cos 20° = 0 \\
-22.879N = -0.12 \cdot T\sin 20° - T\cos 20° \\
-22.879N = T(-0.12 \cdot \sin 20° - \cos 20°) \\
-22.879N = T \cdot (-0.9807) \\
T = \frac{-22.879N}{-0.9807} \\
T = 23.33N
\end{aligned}$$

8. *Forces Acting on the Object*

There are 4 forces acting on the 50 kg box.

1) The 490 N weight directed downwards.
2) A normal force (F_N) directed upwards.
3) A friction force ($F_f = \mu_k F_N$) directed towards the left.
4) The 300 N force exerted by the rope.

Sum of the Forces

The table of force is (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

Forces	*x*	*y*
Weight	0	$-mg$
Normal force	0	F_N
Friction force	$-\mu_k F_N$	0
Tension force	$T\cos\theta$	$T\sin\theta$

The sums of the force are then

$$\sum F_x = -\mu_k F_N + T\cos\theta$$
$$\sum F_y = -mg + F_N + T\sin\theta$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad -\mu_k F_N + T\cos\theta = ma$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -mg + F_N + T\sin\theta = 0$$

Solving the Equations

With the second equation, the normal force is found.

$$F_N = mg - T\sin\theta$$

This value can then be substituted in the equation for the x-components of the forces.

$$-\mu_k\left(mg - T\sin\theta\right) + T\cos\theta = ma$$
$$a = \frac{-\mu_k\left(mg - T\sin\theta\right) + T\cos\theta}{m}$$
$$a = -\mu_k g + \frac{T}{m}\mu_k\sin\theta + \frac{T}{m}\cos\theta$$

At the maximum acceleration, we have $da/d\theta = 0$. Thus,

$$\frac{da}{d\theta} = 0 + \frac{T}{m}\mu_k\cos\theta - \frac{T}{m}\sin\theta = 0$$

This leads to

$$\frac{T}{m}\mu_k\cos\theta - \frac{T}{m}\sin\theta = 0$$
$$\mu_k\cos\theta - \sin\theta = 0$$
$$\mu_k\cos\theta = \sin\theta$$
$$\mu_k = \tan\theta$$

Therefore, the angle required to have the largest acceleration is

$$\begin{aligned}\theta &= \arctan 0.7 \\ &= 35°\end{aligned}$$

9. *Forces Acting on the Object*

There are 4 forces exerted on the crate.

1) The 980 N weight directed downwards.
2) A normal force (F_N) directed upwards.
3) A 600 N tension force directed towards the right.
4) A friction force (F_f) directed towards the left.

Sum of the Forces

The sums of the force are (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

$$\begin{aligned}\sum F_x &= 600N - F_f \\ \sum F_y &= -980N + F_N\end{aligned}$$

Newton's Second Law

If it is assumed that the crate is at rest, then

$$\begin{aligned}\sum F_x = ma_x \qquad &\to \qquad 600N - F_f = 0 \\ \sum F_y = ma_y \qquad &\to \qquad -980N + F_N = 0\end{aligned}$$

Solving the Equations

To solve problems with static friction, F_f and F_N must be found.

The sum of the *x*-component of the forces allows us to find the friction force.

$$\begin{aligned}600N - F_f &= 0 \\ F_f &= 600N\end{aligned}$$

A friction force of 600 N is required for the object to remain at rest.

The sum of the *y*-component of the forces allows us to find the normal force.

$$-980N + F_N = 0$$
$$F_N = 980N$$

Once F_f and F_N are obtained, F_f and $\mu_s F_N$ can be compared.

$$F_f = 600N \qquad \mu_s F_N = 0.6 \cdot 980N = 588N$$

As $F_f > \mu_s F_N$, the crate does slide.

With a maximum friction force of 588 N, we cannot have the 600 N of friction required to keep the object at rest. The crate will therefore move.

10. *Forces Acting on the Object*

There are 3 forces exerted on the box.

1) The weight (mg) directed downwards.
2) A normal force (F_N) directed upwards.
3) A horizontal friction force (F_f).

Sum of the Forces

The sums of the force are (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\sum F_x = F_f$$
$$\sum F_y = -mg + F_N$$

Newton's Second Law

Here, we assume that the box has the same acceleration as the truck. If the crate and truck have the same acceleration, the box does not slide on the floor of the truck. Assuming this acceleration, we will find the condition so that the box does not slip on the floor of the truck.

Thus

$$\sum F_x = ma_x \qquad \rightarrow \qquad F_f = ma$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -mg + F_N = 0$$

Solving the Equations

To solve problems with static friction, F_f and F_N must be found.

The sum of the *x*-component of the forces allows us to find the friction force.

$$F_f = ma$$

The sum of the *y*-component of the forces allows us to find the normal force.

$$-mg + F_N = 0$$
$$F_N = mg$$

Using those results in $F_f \leq \mu_s F_N$, the result is

$$F_f \leq \mu_s F_N$$
$$ma \leq \mu_s mg$$
$$a \leq \mu_s g$$
$$a \leq 0.65 \cdot 9.8 \tfrac{N}{kg}$$
$$a \leq 6.37 \tfrac{m}{s^2}$$

For the box not to slip (this was our condition), the acceleration must be smaller than 6.37 m/s². The maximum acceleration of the box is therefore of 6.37 m/s². If the truck has a greater acceleration than this, the box slips.

11. *Forces Acting on the Object*

There are 3 forces exerted on the block,

1) The weight (*mg*) directed downwards
2) A normal force (F_N) perpendicular to the slope
3) A friction force (F_f) directed uphill.

Sum of the Forces

The table of force is (with an *x*-axis direction downhill),

Forces	*x*	*y*
Weight	*mg* cos (-55°)	*mg* sin (-55°)
Normal Force	0	F_N

Friction Force	$-F_f$	0

The sums of the force are then

$$\sum F_x = mg\cos(-55°) - F_f$$
$$\sum F_y = mg\sin(-55°) + F_N$$

Newton's Second Law

If it is assumed that the block is at rest, then the acceleration is zero. We then have

$$\sum F_x = ma_x \qquad \to \qquad mg\cos(-55°) - F_f = 0$$
$$\sum F_y = ma_y \qquad \to \qquad mg\sin(-55°) + F_N = 0$$

Solving the Equations

The sum of the *x*-component of the forces allows us to find the friction force.

$$mg\cos(-55°) - F_f = 0$$
$$F_f = mg\cos(-55°)$$
$$F_f = mg\cos(55°)$$

The sum of the *y*-component of the forces allows us to find the normal force.

$$mg\sin(-55°) + F_N = 0$$
$$F_N = -mg\sin(-55°)$$
$$F_N = mg\sin(55°)$$

Once F_f and F_N are obtained, F_f and $\mu_s F_N$ can be compared.

$$F_f = mg\cos(55°) \qquad\qquad \mu_s F_N = 0.8 \cdot mg\sin(55°)$$

It's hard to tell if F_f required for the object to remain at rest is greater than $\mu_s F_N$. To find out, we will calculate the ratio of these forces.

$$\frac{\mu_s F_N}{F_f} = \frac{0.8 \cdot mg \sin(55°)}{mg \cos(55°)}$$

$$\frac{\mu_s F_N}{F_f} = \frac{0.8 \cdot \sin(55°)}{\cos(55°)}$$

$$\frac{\mu_s F_N}{F_f} = 0.8 \cdot \tan(55°)$$

$$\frac{\mu_s F_N}{F_f} = 1.1425$$

This means that $\mu_s F_N$ is 1.1425 times larger than F_f. Thus, the object remains at rest.

12. Here, there are two friction forces that prevent the block from sliding to the right. There is the friction force between box A and box B and the friction force between box B and the ground.

It will be assumed here that box B does not move (so it has zero acceleration). So we're going to find the condition for the box to stay in place.

Box B

Forces Acting on the Object

First, let's look at the forces on the 10 kg box (box B). There are 6 forces.

1) The 98 N weight directed downwards.
2) A normal force (F_{N1}) directed downwards exerted by box A.
3) A normal force (F_{N2}) directed upwards exerted by the ground.
4) A friction force (F_{f1}) directed towards the left exerted by box A.
5) A friction force (F_{f2}) directed towards the left exerted by the ground.
6) The force F directed towards the right.

Sum of the forces

The sum of forces is (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\sum F_x = -F_{f1} - F_{f2} + F$$
$$\sum F_y = -98N - F_{N1} + F_{N2}$$

Newton's Second Law

If it is assumed that the block is at rest, then the acceleration is zero. We then have

$$\begin{aligned} \sum F_x = ma_x \quad &\rightarrow \quad -F_{f1} - F_{f2} + F = 0 \\ \sum F_y = ma_y \quad &\rightarrow \quad -98N - F_{N1} + F_{N2} = 0 \end{aligned}$$

Solving the Equations

If the first equation is solved for the total friction force, the result is

$$\begin{gathered} -F_{f1} - F_{f2} + F = 0 \\ F_{f1} + F_{f2} = F \end{gathered}$$

The sum of the *y*-component of the forces is linked to the normal force.

$$-98N - F_{N1} + F_{N2} = 0$$

However, we do not have enough information to find the normal force. So we have to look at the forces on box A.

Box B

Forces Acting on the Object

There are 4 forces exerted on the 5 kg box (box A).

1) The 49 N weight directed downwards.
2) A normal force (F_{N1}) directed upwards exerted by box B.
3) A tension force (T_1) directed towards the left.
4) A friction force (F_{f1}) directed towards the right exerted by box B.

Sum of the forces

The sum of forces is (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

$$\begin{aligned} \sum F_x &= -T + F_{f1} \\ \sum F_y &= -49N + F_{N1} \end{aligned}$$

Newton's Second Law

If it is assumed that the block is at rest, then the acceleration is zero. We then have

$$\sum F_x = ma_x \qquad \rightarrow \qquad -T + F_{f1} = 0$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -49N + F_{N1} = 0$$

Solving the Equations

The normal force can be found with the sum of the *y*-component of the forces.

$$-49N + F_{N1} = 0$$
$$F_{N1} = 49N$$

Knowing this value, F_{N2} can now be found.

$$-98N - F_{N1} + F_{N2} = 0$$
$$-98N - 49N + F_{N2} = 0$$
$$F_{N2} = 147N$$

Since $F_f \leq \mu_s F_N$, we must have

$$F_{f1} \leq \mu_{s1} F_{N1} \qquad\qquad F_{f2} \leq \mu_{s2} F_{N2}$$

Thus

$$F = F_{f1} + F_{f2} \leq \mu_{s1} F_{N1} + \mu_{s2} F_{N2}$$
$$F \leq \mu_{s1} F_{N1} + \mu_{s2} F_{N2}$$
$$F \leq 0.8 \cdot 49N + 0.6 \cdot 147N$$
$$F \leq 127.4N$$

This is what is required for the box B to remain at rest. If we want this box to move, we must pull with a force larger than 127.4 N.

13. *Forces Acting on the Object*

There are 5 forces exerted on Donald.

1) The 490 N weight directed downwards.
2) A normal force (F_{N1}) directed towards the left exerted by the wall on the right.
3) A normal force (F_{N2}) directed towards the right exerted by the wall on the left.
4) A friction force (F_{f1}) directed upwards exerted by the wall on the right.

5) A friction force (F_{f2}) directed upwards exerted by the wall on the left.

Sum of the forces

The sum of forces is (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\sum F_x = -F_{N1} + F_{N2}$$
$$\sum F_y = -490N + F_{f1} + F_{f2}$$

Newton's Second Law

If it is assumed that Donald is at rest, then the acceleration is zero. We then have

$$\sum F_x = ma_x \qquad \rightarrow \qquad -T + F_{f1} = 0$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -49N + F_{N1} = 0$$

Solving the Equations

With the sum of the y-component of the forces, we find

$$-490N + F_{f1} + F_{f2} = 0$$
$$F_{f1} + F_{f2} = 490N$$

The normal forces cannot be found with the sum of the x-component of the forces. We can only know that the two normal forces are equal.

$$-F_{N1} + F_{N2} = 0$$
$$F_{N1} = F_{N2}$$

Since $F_f \leq \mu_s F_N$, we must have

$$F_{f1} \leq \mu_{s1} F_{N1} \qquad\qquad F_{f2} \leq \mu_{s2} F_{N2}$$

Thus

$$490N = F_{f1} + F_{f2} \leq \mu_s F_{N1} + \mu_s F_{N2}$$
$$490N \leq \mu_s F_{N1} + \mu_s F_{N2}$$
$$490N \leq \mu_s F_{N1} + \mu_s F_{N1}$$

(since the two normal forces are equal.)

$$490N \le \mu_s \left(F_{N1} + F_{N1}\right)$$
$$490N \le 2\mu_s F_{N1}$$
$$F_{N1} \ge \frac{490N}{2\mu_s}$$
$$F_{N1} \ge \frac{490N}{2 \cdot 1.4}$$
$$F_{N1} \ge 175N$$

Donald must, therefore, push on the walls so that the normal forces are at least 175 N.

14. *Forces Acting on the Object*

There are 4 forces exerted on the box.

1) The 980 N weight downwards.
2) A normal force (F_N) directed upwards.
3) A friction force (F_f) directed towards the left.
4) The force exerted by Boris.

Sum of the forces

The table of force is (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

Forces	*x*	*y*
Weight	0	-980 N
Normal force	0	F_N
Friction force	$-F_f$	0
Boris	F cos (-30°)	F sin (-30°)

The sum of forces is

$$\sum F_x = -F_f + F\cos\left(-30°\right)$$
$$\sum F_y = -980N + F_N + F\sin\left(-30°\right)$$

Newton's Second Law

If it is assumed that the box is at rest, then the acceleration is zero. We then have

$$\sum F_x = ma_x \qquad \rightarrow \qquad -F_f + F\cos(-30°) = 0$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -980N + F_N + F\sin(-30°) = 0$$

Solving the Equations

With the sum of the *x*-component of the forces, we have

$$-F_f + F\cos(30°) = 0$$
$$F_f = F\cos(30°)$$

With the sum of the *y*-component of the forces, we have

$$-980N + F_N + F\sin(-30°) = 0$$
$$F_N = 980N - F\sin(-30°)$$
$$F_N = 980N + F\sin(30°)$$

Using those results in $F_f \leq \mu_s F_N$, we have

$$F_f \leq \mu_s F_N$$
$$F\cos 30° \leq 0.5\cdot(980N + F\sin 30°)$$
$$F\cos 30° \leq 0.5\cdot 980N + 0.5\cdot F\sin 30°$$
$$F\cos 30° \leq 490N + 0.5\cdot F\sin 30°$$
$$F\cos 30° - 0.5\cdot F\sin 30° \leq 490N$$
$$F\cdot(\cos 30° - 0.5\cdot\sin 30°) \leq 490N$$
$$F\cdot(0.616) \leq 490N$$
$$F \leq 795.4N$$

The object remains at rest if the applied force is less than 795.4 N. Therefore, if the force exerted is greater than 795.4 N, the object will move.

15. *Forces Acting on the Object*

There are 4 forces exerted on the block.

1) The 9.8 N weight directed downwards.
2) A normal force (F_N) directed towards the left.
3) A friction force (F_f) directed upwards or downwards.
4) The 30 N force F.

The direction of the frictional force is not known because we do not know if the vertical component of the force *F* is larger or smaller than the weight of the block. If this component is smaller than the weight, there must be a friction directed upwards to help support the block. If this component is larger than the weight, there must be a friction force downwards to prevent the block from sliding upwards. We will, therefore, presume that friction force F_f is directed upwards and then found its magnitude. If we get a positive answer, the force is in the assumed direction (upwards). If we get a negative answer, the force is in the direction opposite to the direction assumed (downwards).

Sum of the Forces

The sum of forces is (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

$$\sum F_x = -F_N + 30N \cdot \cos 30°$$
$$\sum F_y = -9{,}8N + F_f + 30N \cdot \sin 30°$$

Newton's Second Law

If it is assumed that the box is at rest, then the acceleration is zero. We then have

$$\sum F_x = ma_x \qquad \rightarrow \qquad -F_N + 30N \cdot \cos 30° = 0$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -9{,}8N + F_f + 30N \cdot \sin 30° = 0$$

Solving the Equations

The sum of the *y*-component of the forces gives

$$-9.8N + F_f + 30N \cdot \sin(30°) = 0$$
$$F_f = 9.8N - 30N \cdot \sin(30°)$$
$$F_f = -5.2N$$

Since the answer is negative, the frictional force is 5.2 N downwards. This is the answer to a).

The sum of the *x*-component of the forces gives

$$-F_N + 30N \cdot \cos(30°) = 0$$
$$F_N = 30N \cdot \cos(30°)$$
$$F_N = 25.98N$$

Using those results in $F_f \leq \mu_s F_N$ (in this equation, F_f is the magnitude of the friction force so it is not negative), the result is

$$\begin{gathered} F_f \leq \mu_s F_N \\ 5.2N \leq \mu_s \cdot 25.98N \\ \mu_s \geq 0.2001 \end{gathered}$$

The coefficient must, therefore, be larger than 0.2001.

16. To know the force of friction, we need to know if we have to deal with kinetic friction or static friction. We will first explore the situation where the stone remains at rest to see if it’s the correct solution.

Stone

Forces Acting on the Object

There are 4 forces exerted on the stone

1) The 980 N weight directed downwards.
2) A normal force (F_N) directed upwards.
3) The tension force (T) directed towards the right.
4) A friction force (F_f) directed towards the left.

Sum of the Forces

The sum of forces is (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\begin{gathered} \sum F_x = T - F_f \\ \sum F_y = -980N + F_N \end{gathered}$$

Newton’s Second Law

If it is assumed that the stone is at rest, then the acceleration is zero. We then have

$$\begin{aligned} \sum F_x &= ma_x & &\rightarrow & T - F_f &= 0 \\ \sum F_y &= ma_y & &\rightarrow & -980N + F_N &= 0 \end{aligned}$$

The friction force is, therefore, equal to the tension of the rope. This tension is found by considering the forces acting on Thierry.

Thierry

Forces Acting on the Object

There are 2 forces exerted on Thierry.

1) The 686 N weight directed downwards.
2) The tension force (T) directed upwards.

Sum of the Forces

The sum of forces is (with a y-axis directed upwards)

$$\sum F_y = -686N + T$$

Newton's Second Law

Since Thierry has no acceleration, we have

$$\sum F_y = -686N + T = 0$$

Solving the Equations

According to the equation of the forces acting on Thierry, the tension force is 686 N. Therefore, the equations of forces on the stone become

$$686N - F_f = 0$$
$$-980N + F_N = 0$$

The sum of the x-component of the forces gives

$$686N - F_f = 0$$
$$F_f = 686N$$

The sum of the y-component of the forces gives

$$-980N + F_N = 0$$
$$F_N = 980N$$

Once F_f and F_N are obtained, F_f and $\mu_s F_N$ can be compared.

$$F_f = 686N \qquad \begin{aligned}\mu_s F_N &= 0.5\times 980N\\ &= 490N\end{aligned}$$

As $F_f > \mu_s F_N$, the stone does slide.

If the stone is slipping, we have kinetic friction, whose magnitude is

$$\begin{aligned}F_f &= \mu_k F_N\\ &= 0.45N\cdot 980N\\ &= 441N\end{aligned}$$

The force of friction is therefore 441 N towards the left.

17. If this system is in equilibrium, then the 30 kg mass does not slide. Thus, the force acting on the 30 kg mass will be examined.

30 kg Block

Forces Acting on the Object

There are 4 forces exerted on the block.

1) The 294 N weight directed downwards.
2) A normal force (F_N) directed upwards.
3) The tension force (T_1) directed towards the right.
4) A friction force (F_f) directed towards the left.

Sum of the Forces

The sum of forces is (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

$$\begin{aligned}\sum F_x &= T_1 - F_f\\ \sum F_y &= -294N + F_N\end{aligned}$$

Newton's Second Law

If it is assumed that the block is at rest, then the acceleration is zero. We then have

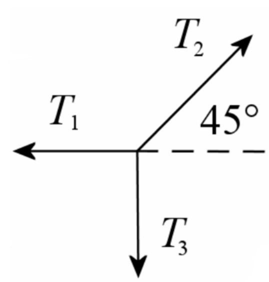
$$\sum F_x = ma_x \qquad \rightarrow \qquad T_1 - F_f = 0$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -294N + F_N = 0$$

The friction force is, therefore, equal to the tension of the rope. This tension is found by considering the forces acting on the object at the other end of the rope, i.e. on the knot linking the 3 ropes.

Knot

Forces Acting on the Object

There are 3 tensions acting on the knot.



Obviously, the tension T_3 is equal to the weight of the 20 kg block, thus T_3 = 196 N.

Sum of the Forces

The table of force is (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

Forces	*x*	*y*
Tension 1	$-T_1$	0
Tension 2	$T_2 \cos 45°$	$T_2 \sin 45°$
Tension 3	0	-196 N

The sums of forces are then

$$\sum F_x = -T_1 + T_2 \cos 45°$$
$$\sum F_y = T_2 \sin 45° - 196N$$

Newton's Second Law

Since the acceleration of the knot is zero, we have

$$\sum F_x = ma_x \qquad \to \qquad -T_1 + T_2 \cos 45° = 0$$
$$\sum F_y = ma_y \qquad \to \qquad T_2 \sin 45° - 196N = 0$$

Solving the Equations

The tension T_2 can be found with the sum of the y-component of the forces on the knot.

$$T_2 \sin 45° - 196N = 0$$
$$T_2 = 277.2N$$

Then, the tension T_1 can be found with the sum of the x-component of the forces.

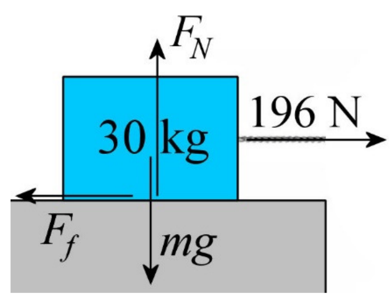
$$-T_1 + T_2 \cos 45° = 0$$
$$-T_1 + 277.2N \cdot \cos 45° = 0$$
$$T_1 = 196N$$

This is the tension of the rope pulling the 30 kg block. So, the equations of the forces acting on the block

$$T_1 - F_f = 0$$
$$-294N + F_N = 0$$

become

$$196N - F_f = 0$$
$$-294N + F_N = 0$$



The friction force can be found with the sum of the x-component of the forces.

$$196N - F_f = 0$$
$$F_f = 196N$$

The normal force can be found with the sum of the y-component of the forces.

$$-294N + F_N = 0$$
$$F_N = 294N$$

Using those results in $F_f \leq \mu_s F_N$, the result is

$$F_f \leq \mu_s F_N$$
$$196N \leq \mu_s \cdot 294N$$
$$\frac{196N}{294N} \leq \mu_s$$
$$\frac{2}{3} \leq \mu_s$$

In order for the block to remain in place, the friction coefficient must be greater than 2/3.

18. *Forces Acting on the Object*

There are 3 forces exerted on the Gastonne.

1) The weight (mg) directed downwards.
2) A normal force (F_N) perpendicular to the slope.
3) A friction force ($F_f = \mu_k F_N$) opposed to the motion so directed uphill.

Sum of the Forces

The table of force is (using an x-axis directed downhill)

Forces	***x***	***y***
Weight	$mg \cos(-60°)$	$mg \sin(-60°)$
Normal force	0	F_N
Friction force	$-\mu_k F_N$	0

The sums of forces are then

$$\sum F_x = mg\cos(-60°) - \mu_k F_N$$
$$\sum F_y = mg\sin(-60°) + F_N$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad mg\cos(-60°) - \mu_k F_N = ma$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad mg\sin(-60°) + F_N = 0$$

Solving the Equations

To obtain the coefficient for the equation of the x-component of the forces, the normal and the acceleration must be known.

The normal force can be found with the sum of the y-component of the forces.

$$mg\sin(-60°) + F_N = 0$$
$$F_N = -mg\sin(-60°)$$
$$F_N = mg\sin(60°)$$

This acceleration can be found with the change of speed over 30 m. This acceleration is

$$2a_x(x - x_0) = v^2 - v_0^2$$
$$2 \cdot a \cdot (30m - 0m) = (16\tfrac{m}{s})^2 - (10\tfrac{m}{s})^2$$
$$a_x = 2.6\tfrac{m}{s^2}$$

The equation of the x-component of the forces now becomes

$$mg\cos(-60°) - \mu_k F_N = ma$$
$$mg\cos(-60°) - \mu_k mg\sin 60° = m \cdot 2.6\tfrac{m}{s^2}$$
$$g\cos(-60°) - \mu_c g\sin 60° = 2.6\tfrac{m}{s^2}$$

Then, the coefficient can be calculated.

$$g\cos(-60°) - \mu_c g\sin 60° = 2.6\tfrac{m}{s^2}$$
$$9.8\tfrac{m}{s^2} \cdot \cos(-60°) - \mu_c \cdot 9.8\tfrac{m}{s^2} \cdot \sin 60° = 2.6\tfrac{m}{s^2}$$
$$\mu_c = 0.271$$

19. The force is

$$\begin{aligned} F_d &= \frac{1}{2} C_d A \rho v^2 \\ &= \frac{1}{2} \cdot 0.47 \cdot \pi (0.11m)^2 \cdot 1.3 \tfrac{kg}{m^3} \cdot \left(20 \tfrac{m}{s}\right)^2 \\ &= 4.645N \end{aligned}$$

20. The force is

$$\begin{aligned} F_d &= \frac{1}{2} (C_d A) \rho v^2 \\ &= \frac{1}{2} \cdot 0.682m^2 \cdot 1.3 \tfrac{kg}{m^3} \cdot \left(33.33 \tfrac{m}{s}\right)^2 \\ &= 492.6N \end{aligned}$$

21. The terminal velocity is

$$\begin{aligned} v_t &= \sqrt{\frac{2mg}{C_d A \rho}} \\ &= \sqrt{\frac{2 \cdot 0.44kg \cdot 9.8 \tfrac{N}{kg}}{0.47 \cdot \pi (0.11m)^2 \cdot 1.3 \tfrac{kg}{m^3}}} \\ &= 19.27 \tfrac{m}{s} \end{aligned}$$

22. The terminal velocity is

$$v_t = \sqrt{\frac{2mg}{C_d A \rho}}$$

If the volume is 0.01 m³, then the edge of the cube has a length of

$$\begin{aligned} L^3 &= 0.01m^3 \\ L &= 0.21544m \end{aligned}$$

The mass of the cube is then

$$\begin{aligned} m &= \rho \cdot (volume) \\ &= 7320 \tfrac{kg}{m^3} \cdot 0.01 m^3 \\ &= 73.2 kg \end{aligned}$$

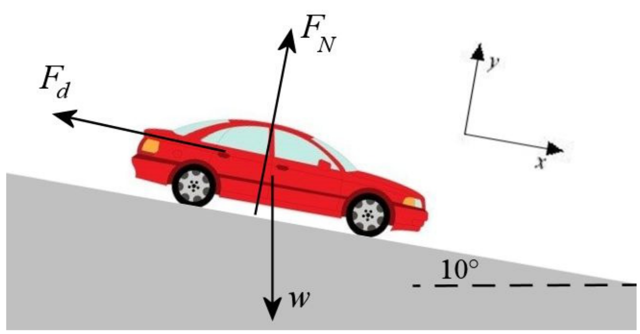
Since the value of C_x is 1.05, the terminal velocity is

$$\begin{aligned} v_t &= \sqrt{\frac{2mg}{C_d A \rho}} \\ &= \sqrt{\frac{2 \cdot 73.2kg \cdot 9.8 \frac{N}{kg}}{1.05 \cdot (0.21544m)^2 \cdot 1.3 \frac{kg}{m^3}}} \\ &= 150.5 \tfrac{m}{s} \end{aligned}$$

23. *Forces Acting on the Object*

There are 3 forces acting on the Honda Civic.

1) The weight (mg) directed downwards.
2) A normal force (F_N) perpendicular to the slope.
3) A drag force (F_d) directed uphill.



Sum of the forces

The table of force is

Forces	x	y
Weight	mg cos (-80°)	mg sin (-80°)
Normal Force	0	F_N
Drag Force	$-F_d$	0

The sums of the forces are therefore

$$\sum F_x = mg\cos(-80°) - F_d$$
$$\sum F_y = mg\sin(-80°) + F_N$$

Newton's Second Law

At the terminal velocity, the acceleration is zero. The equations of the forces are, therefore,

$$\sum F_x = ma_x \qquad \rightarrow \qquad mg\cos(-80°) - F_d = 0$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad mg\sin(-80°) + F_N = 0$$

Solving the Equations

The first of these equations gives

$$mg\cos(-80°) - F_d = 0$$
$$mg\cos 80° - \frac{1}{2}C_d A\rho v_t^2 = 0$$
$$v_t = \sqrt{\frac{2mg\cos 80°}{C_d A\rho}}$$

For a 2001 Honda Civic, C_dA = 0.682 m². With an 1142 kg mass, the terminal velocity is

$$\begin{aligned} v_t &= \sqrt{\frac{2\cdot 1142kg\cdot 9.8\frac{N}{kg}\cdot\cos 80°}{0.682m^2\cdot 1.3\frac{kg}{m^3}}} \\ &= 66.21\tfrac{m}{s} \\ &= 238.4\tfrac{km}{h} \end{aligned}$$

24. With a terminal velocity of 30 m/s, we find that

$$v_t = \sqrt{\frac{2mg}{C_d A\rho}}$$
$$30\tfrac{m}{s} = \sqrt{\frac{2\cdot 100kg\cdot 9.8\frac{N}{kg}}{C_d A\cdot 1.3\frac{kg}{m^3}}}$$
$$C_d A = 1.675m^2$$

The frictional force at half the terminal velocity is therefore

$$\begin{aligned}F_d &= \frac{1}{2}(C_d A)\rho v^2 \\ &= \frac{1}{2}\cdot 1.675m^2 \cdot 1.3\tfrac{kg}{m^3}\cdot\left(15\tfrac{m}{s}\right)^2 \\ &= 245N\end{aligned}$$

25. If the car moved, the drag force made by the wind must have exceeded the force of static friction between the tires and the asphalt. Therefore

$$F_D > F_f$$

$$\frac{1}{2}(C_d A)\rho v^2 > \mu_s mg$$

$$\frac{1}{2}\cdot 0.682\cdot 1.3\tfrac{kg}{m^3}\cdot v^2 > 0.8\cdot 1095kg\cdot 9.8\tfrac{N}{kg}$$

$$v > 139.2\tfrac{m}{s}$$

The minimum wind speed is thus 139.2 m/s.

26. *Forces Acting on the Object*

There are 3 forces exerted on the 2 kg carriage.

1) The 19.6 N weight directed downwards.
2) A normal force (F_N) directed upwards.
3) The spring force ($F_{sp} = kx$) directed towards the right.

Sum of the Forces

The sum of the x-component of the forces is then (using an x-axis in the direction of the spring)

$$\sum F_x = kx$$

Newton's Second Law

Since the acceleration is 4 m/s², we have

$$\sum F_x = ma_x \qquad \rightarrow \qquad kx = 2kg\cdot 4\tfrac{m}{s^2}$$

Solving the Equations

Therefore, the stretching of the spring is

$$kx = 2kg \cdot 4\tfrac{m}{s^2}$$
$$200\tfrac{N}{m} \cdot x = 2kg \cdot 4\tfrac{m}{s^2}$$
$$x = 0.04m = 4cm$$

27. a)

Let's assume that the box is at rest. This will allow us to know the condition that must be met for the box to be at rest.

Forces Acting on the Object

There are 4 forces exerted on the 100 kg box.

1) The 980 N weight directed downwards.
2) A normal force (F_N) directed upwards.
3) The spring force ($F_{sp} = kx$) directed towards the right.
4) A friction force (F_f) directed towards the left.

Sum of the Forces

The sum of forces is (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\sum F_x = kx - F_f$$
$$\sum F_y = -980N + F_N$$

Newton's Second Law

Since there is no acceleration, we have

$$\sum F_x = ma_x \qquad \rightarrow \qquad kx - F_f = 0$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -980N + F_N = 0$$

Solving the Equations

Solving for F_f in the first equation, we obtain

$$kx - F_f = 0$$
$$F_f = kx$$

Solving for F_N in the second equation, we obtain

$$-980N + F_N = 0$$
$$F_N = 980N$$

Using those results in $F_f \leq \mu_s F_N$, we have

$$F_f \leq \mu_s F_N$$
$$kx \leq \mu_s \cdot 980N$$
$$1000\tfrac{N}{m} \cdot x \leq 0.5 \cdot 980N$$
$$x \leq 0{,}49m$$

Thus, the box remains in place as long as the stretching of the spring is less than 49 cm. Therefore, if the stretching of the spring is larger than 49 cm, then the box moves.

b) *Forces Acting on the Object*

There are 4 forces exerted on the 100 kg box.

1) The 980 N weight directed downwards.
2) A normal force (F_N) directed upwards.
3) The spring force ($F_{sp} = kx$) directed towards the right.
4) A friction force ($F_f = \mu_k F_N$) directed towards the left.

Sum of the Forces

The sum of forces is (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

$$\sum F_x = kx - \mu_k F_N$$
$$\sum F_y = -980N + F_N$$

Newton's Second Law

Since there is no acceleration (constant speed), we have

$$\sum F_x = ma_x \qquad \rightarrow \qquad kx - \mu_k F_N = 0$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -980N + F_N = 0$$

Solving the Equations

The first equation gives

$$kx = \mu_k F_N$$
$$1000\tfrac{N}{m} \cdot x = 0.4 \cdot 980N$$
$$x = 0.392m$$

(The normal comes from the sum of the y-component of the forces.)

The spring is therefore stretched 39.2 cm.

28. *Forces Acting on the Object*

There are 4 forces exerted on the 5 kg box

1) The 49 N weight directed downwards
2) A normal force (F_N) directed upwards
3) The spring force ($F_{sp} = kx$) directed towards the left
4) A friction force ($F_f = \mu_k F_N$) directed towards the left

Sum of the Forces

The sum of forces is (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\sum F_x = -kx - \mu_k F_N$$
$$\sum F_y = -49N + F_N$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad -kx - \mu_c F_N = 5kg \cdot a$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -49N + F_N = 0$$

Solving the Equations

The sum of the x-component of the forces gives us

$$-kx - \mu_k F_N = ma$$
$$-5000 \tfrac{N}{m} \cdot 0.1m - 0.6 \cdot 49N = 5kg \cdot a$$
$$a = -105.88 \tfrac{m}{s^2}$$

(The normal comes from the sum of the y-component of the forces.)

29. *Forces Acting on the Object*

There are 3 forces exerted on the 5 kg block.

1) The 49 N weight directed downwards.
2) The tension force (T) directed upwards.
3) The spring force (F_{sp}) directed upwards or downwards.

As the direction of the force exerted by the spring is not known (because we do not know if the spring is stretched or compressed), we assume that the force is positive, i.e. directed upwards. When we have the solution, we'll know that this direction is correct if the force is positive. If it is negative, the force is in the direction opposite to the direction assumed, i.e. downwards.

Sum of the Forces

The sum of forces is (with a y-axis directed upwards)

$$\sum F_y = -49N + T + F_{sp}$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -49N + T + F_{sp} = 0$$

Solving the Equations

Since the rope also supports a 2 kg block in equilibrium, the tension of the rope is 19.6 N. We then have

$$-49N + 19.6N + F_{sp} = 0$$
$$F_{sp} = 29.4N$$

This positive answer tells us that the force exerted by the spring is directed upwards. This also means that the spring is compressed. The compression is

$$F_{sp} = 29.4N$$
$$kx = 29.4N$$
$$250\tfrac{N}{m} \cdot x = 29.4N$$
$$x = 0.1176m$$

The spring is therefore compressed 11.76 cm.

30. *Forces Acting on the Object*

There are 2 forces exerted on the 100 g object.

1) The 0.98 N weight directed downwards.
2) The spring force ($F_{sp} = kx$) directed upwards.

Sum of the Forces

The sum of forces is (with a *y*-axis directed upwards)

$$\sum F_y = -0.98N + kx$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_x \qquad \rightarrow \qquad -0.98N + kx = 0$$

Solving the Equations

As the stretching of the spring is 4 cm then, we have

$$-0.98N + kx = 0$$
$$-0.98N + k \cdot 0.04m = 0$$
$$k = 24.5\tfrac{N}{m}$$

Now, let's see what will happen if 300 g are added.

Forces Acting on the Object

There are 2 forces exerted on the 400 g object.

1) The 3.92 N weight directed downwards.
2) The spring force ($F_{sp} = kx$) directed upwards.

Sum of the Forces

The sum of forces is (with a *y*-axis directed upwards)

$$\sum F_y = -3.92N + kx$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_x \qquad \rightarrow \qquad -3.92N + kx = 0$$

Solving the Equations

The spring, therefore, exerts a 3.92 N force. We thus have

$$-3.92N + kx = 0$$
$$-3.92N + 24.5\tfrac{N}{m} \cdot x = 0$$
$$x = 0.16m$$

The spring is thus stretched 16 cm.

31. The force exerted on one side of the box is

$$\begin{aligned} F_P &= PA \\ &= 300{,}000Pa \cdot (0.2m)^2 \\ &= 12{,}000N \end{aligned}$$

32. *Forces Acting on the Object*

There are 2 forces exerted on the cover.

1) A pressure force (F_p = PA) directed downwards.
2) The spring force (F_{sp} = kx) directed upwards.

Sum of the Forces

The sum of forces is (with a y-axis directed upwards)

$$\sum F_y = -PA + kx$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -PA + kx = 0$$

Solving the Equations

We then have

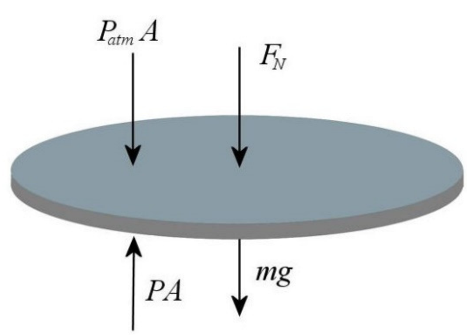
$$-PA + kx = 0$$
$$-102{,}000Pa \cdot \pi (0.1m)^2 + 10{,}000\tfrac{N}{m} \cdot x = 0$$
$$x = 0.3204m$$

The spring is therefore compressed 32.04 cm.

33. *Forces Acting on the Object*

There are 4 forces acting on the cover.

1) The 3.92 N weight of the cover (mg) directed downwards.
2) A 29.4 N normal force (F_N) directed downwards made by the 3 kg mass.
3) The pressure force ($P_{atm}A$) directed downwards made by the atmosphere.
4) The pressure force (PA) directed upwards made by the gas inside the piston.



Sum of the forces

Using a *y*-axis directed upwards, the sum of the forces is

$$\sum F_y = -3.92N - 29.4N - P_{atm}A + PA$$

Newton's Second Law

Since there is no acceleration at the equilibrium position, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -3.92N - 29.4N - P_{atm}A + PA = 0$$

Solving the Equation

Since the area of the cover is $A = \pi r^2$, the equation becomes

$$\begin{gathered} -3.92N - 29.4N - P_{atm}A + PA = 0 \\ -3.92N - 29.4N - 101{,}000Pa \cdot \pi \cdot (0.05m)^2 + P \cdot \pi \cdot (0.05m)^2 = 0 \\ P = 105{,}242Pa \\ P = 105.24kPa \end{gathered}$$

34. *Forces Acting on the Object*

There are 3 forces exerted on the block of cedar.

1) The 1.96 N weight directed downwards.
2) The buoyant force ($F_B = \rho g V_f$) directed upwards.
3) The tension force (T) directed downwards.

Sum of the Forces

The sum of forces is (with a *y*-axis directed upwards)

$$\sum F_y = -1.96N + \rho g V_f - T$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -1.96N + \rho g V_f - T = 0$$

Solving the Equations

To find the tension, the volume of the block must be found. This volume is found with the density.

$$m = \rho \cdot (volume)$$
$$0.2kg = 490 \tfrac{kg}{m^3} \cdot (volume)$$
$$volume = 0.0004082m^3$$

Therefore, the equation becomes

$$-1.96N + \rho g V_f - T = 0$$
$$-1.96N + 1000 \tfrac{kg}{m^3} \cdot 9.8 \tfrac{N}{kg} \cdot 0.0004081m^3 - T = 0$$
$$-1.96N + 4N - T = 0$$
$$T = 2.04N$$

35. As the stretching of the spring in water is sought, let's begin bay looking at the forces acting on the block in water.

Block in Water

Forces Acting on the Object

In water, the are 3 forces exerted on the piece of aluminum.

1) The weight (*mg*) directed downwards.
2) The spring force ($F_{sp} = kx$) directed upwards.
3) The buoyant force ($F_B = \rho g V_f$) directed upwards.

Sum of the Forces

The sum of forces is (with a *y*-axis directed upwards)

$$\sum F_y = -mg + kx + \rho g V_f$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -mg + kx + \rho g V_f = 0$$

To obtain *x*, the mass and the volume of the bloc are needed. These can be found by looking at the forces acting on the block in air.

Block in Air

In air, there are 2 forces exerted on the piece of aluminum.

1) The weight (*mg*) directed downwards.
2) The spring force ($F_{sp} = kx'$) directed upwards.

Sum of the Forces

The sum of forces is (with a *y*-axis directed upwards)

$$\sum F_y = -mg + kx'$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -mg + kx' = 0$$

Solving the Equations

Remember that *x* must be obtain in this equation

$$-mg + kx + \rho g V_f = 0$$

and that the mass and the volume were missing.

The mass of the aluminum piece can then be found with the equation of the forces in air.

$$\begin{gathered} -mg + kx' = 0 \\ -m \cdot 9.8 \tfrac{N}{kg} + 200 \tfrac{N}{m} \cdot 0.1m = 0 \\ m = 2.0408kg \end{gathered}$$

The volume is found with the density and the mass.

$$m = \rho \cdot (volume)$$
$$2.0408kg = 2700\tfrac{kg}{m^3} \cdot (volume)$$
$$volume = 0.0007559m^3$$

Now, the equation can be solved for *x*.

$$-mg + kx + \rho g V_f = 0$$
$$-2.0408kg \cdot 9.8\tfrac{N}{m} + 200\tfrac{N}{m} \cdot x + 1000\tfrac{kg}{m^3} \cdot 9.8\tfrac{N}{kg} \cdot 0.0007559m^3 = 0$$
$$20N + 200\tfrac{N}{m} \cdot x + 7.4074N = 0$$
$$x = 0.062963m$$

Therefore, the stretching of the spring is 6.2963 cm.

36. *Forces Acting on the Object*

There are 3 forces acting on the object.

1) The weight (mg) directed downwards.
2) The buoyant force of the denser fluid ($F_{B1} = \rho_1 g V_1$) directed upwards.
3) The buoyant force of the less dense fluid ($F_{B2} = \rho_2 g V_2$) directed upwards.

Sum of the Forces

The sum of forces is (with a *y*-axis directed upwards)

$$\sum F_y = -mg + \rho_1 g V_{f1} + \rho_2 g V_{f2}$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -mg + \rho_1 g V_{f1} + \rho_2 g V_{f2} = 0$$

Solving the Equation

To obtain the density, the mass and the immersed volumes are needed.

The mass of the object is

$$\begin{aligned} m &= \rho_{object} \cdot Volume \\ &= \rho_{object} \cdot Ah \end{aligned}$$

where A is the area of the base of the object.

The immersed volumes are

$$\begin{aligned} V_{f1} &= A \cdot \tfrac{3}{4} h \\ V_{f2} &= A \cdot \tfrac{1}{4} h \end{aligned}$$

Using these formulas, the equation of forces then becomes

$$\begin{gathered} -mg + \rho_1 g V_{f1} + \rho_2 g V_{f2} = 0 \\ -\rho_{objet} \cdot Ahg + \rho_1 g A \tfrac{3}{4} h + \rho_2 g A \tfrac{1}{4} h = 0 \\ -\rho_{objet} + \rho_1 \tfrac{3}{4} + \rho_2 \tfrac{1}{4} = 0 \end{gathered}$$

The density of the object can now be found.

$$\begin{gathered} \rho_{object} = \tfrac{3}{4}\rho_1 + \tfrac{3}{4}\rho_2 \\ \rho_{object} = \tfrac{3}{4} \cdot 1000 \tfrac{kg}{m^3} + \tfrac{1}{4} \cdot 800 \tfrac{kg}{m^3} \\ \rho_{object} = 950 \tfrac{kg}{m^3} \end{gathered}$$

37. Sylvain in the Boat

Forces acting on the Object

When Sylvain is in the boat, there are 3 forces acting on the boat.

1) The weight (mg) directed downwards.
2) The buoyant force of the fluid ($F_B = \rho g V_{f1}$) directed upwards.
3) The normal force made by Sylvain (equal to its weight $m_s g$) directed downwards.

Sum of the Forces

The sum of forces is (with a y-axis directed upwards)

$$\sum F_y = -mg + \rho g V_{f1} - m_s g$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_y \qquad \to \qquad -mg + \rho g V_{f1} - m_s g = 0$$

To obtain Sylvain's mass, the immersed volume must be known. To solve this problem, we have to look at the forces on the boat when Sylvain is not in the boat.

Empty Boat

Forces Acting of the Object

There are 2 forces on the boat

1) The weight (mg) directed downwards.
2) The buoyant force of the fluid ($F_B = \rho g V_{f2}$) directed upwards.

Sum of the Forces

The sum of forces is (with a y-axis directed upwards)

$$\sum F_y = -mg + \rho g V_{f2}$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_y \qquad \to \qquad -mg + \rho g V_{f2} = 0$$

Solving the Equations

We have the following equations.

$$-mg + \rho g V_{f1} - m_s g = 0$$
$$-mg + \rho g V_{f2} = 0$$

The immersed volumes are not known, but the change of volume is known. This change is

$$\begin{aligned} V_{f1} - V_{f2} &= 4m^2 \cdot 0.05m \\ &= 0.2m^3 \end{aligned}$$

If we solve each force equation for the volume, we have

$$\rho g V_{f1} = mg + m_s g$$
$$V_{f1} = \frac{m + m_s}{\rho}$$

$$\rho g V_{f2} = mg$$
$$V_{f2} = \frac{m}{\rho}$$

Therefore

$$V_{f1} - V_{f2} = 0.2m^3$$
$$\frac{m + m_s}{\rho} - \frac{m}{\rho} = 0.2m^3$$
$$\frac{m}{\rho} + \frac{m_s}{\rho} - \frac{m}{\rho} = 0.2m^3$$
$$\frac{m_s}{\rho} = 0.2m^3$$

Thus, Sylvain's mass is

$$\begin{aligned} m_s &= 0,2m^3 \cdot \rho \\ &= 0.2m^3 \cdot 1000 \tfrac{kg}{m^3} \\ &= 200kg \end{aligned}$$

38. The terminal velocity is found with the drag force. To find this force, let's look at the forces acting on the 10 kg block. There is the tension of the rope upwards, the weight downwards, the buoyancy force pushing upwards and the drag force downwards (since the block rises) made by the fluid. Using an axis directed upwards, we have (knowing that at the terminal velocity, there is no more acceleration)

$$\sum F_y = ma_y$$
$$T - 98N + \rho g V_f - F_d = 0$$

To find the drag force, the tension of the rope is needed. It can be found by examining the forces acting on the 20 kg mass. For this mass, we have (using an axis directed downwards)

$$\sum F_y = ma_y$$
$$196N - T = 0$$
$$T = 196N$$

The equation of the forces on the 10 kg block then becomes

$$T - 98N + \rho g V_f - F_d = 0$$
$$196N - 98N + 1000\tfrac{kg}{m^3} \cdot 9.8\tfrac{N}{kg} \cdot (0.1m)^3 - F_d = 0$$
$$196N - 98N + 9.8N - F_d = 0$$
$$F_d = 107.8N$$

We now know that

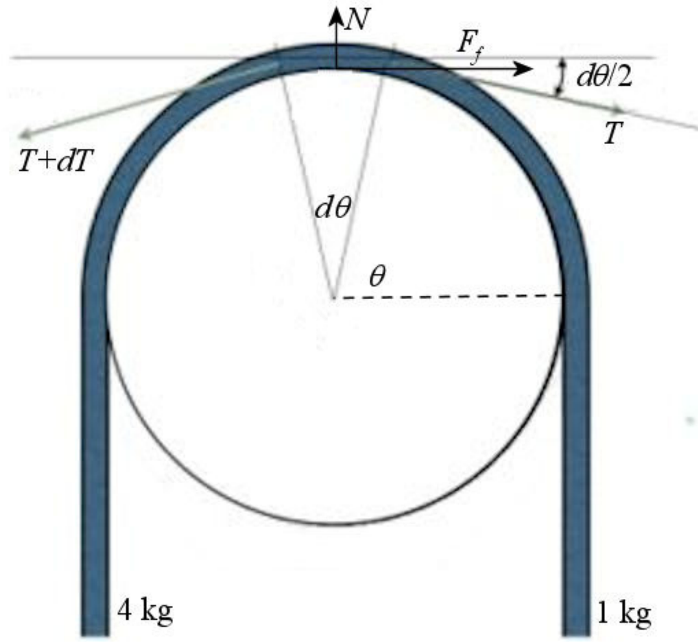
$$F_d = \frac{1}{2} C_d A \rho v^2$$
$$107.8N = \frac{1}{2} C_d A \rho v^2$$

For a cube, we have $C_d = 1.05$ and $A = L^2 = (0.1\text{m})^2$. Therefore, the velocity is

$$107.8N = \frac{1}{2} \cdot 1.05 \cdot (0.1m)^2 \cdot 1000\tfrac{kg}{m^3} \cdot v^2$$
$$v = 4.531\tfrac{m}{s}$$

39. We know that tension is 39.2 N on one side of the rope and 9.8 N the other side of the rope. To see how the friction changes the tension of the rope, we will consider a small piece of rope.

Forces Acting on the Object



wiki.imga.org.il/index.php?title=%3FHow_do_friction_devices_work

Sum of the Forces

The sum of the forces acting on the small piece of rope is (the mass of the rope is neglected) (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

$$\sum F_x = T\cos\left(\tfrac{d\theta}{2}\right) - \left(T + dT\right)\cos\left(\tfrac{d\theta}{2}\right) + F_f$$
$$\sum F_y = -T\sin\left(\tfrac{d\theta}{2}\right) - \left(T + dT\right)\sin\left(\tfrac{d\theta}{2}\right) + N$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_x = ma_x \quad \rightarrow \quad T\cos\left(\tfrac{d\theta}{2}\right) - \left(T + dT\right)\cos\left(\tfrac{d\theta}{2}\right) + F_f = 0$$
$$\sum F_y = ma_y \quad \rightarrow \quad -T\sin\left(\tfrac{d\theta}{2}\right) - \left(T + dT\right)\sin\left(\tfrac{d\theta}{2}\right) + N = 0$$

Solving the Equations

Since the angle is small, we can use cos x = 1 and sin x = x. then, the equation becomes

$$\sum F_x = T - \left(T + dT\right) + F_f = 0$$
$$\sum F_y = -T\frac{d\theta}{2} - \left(T + dT\right)\frac{d\theta}{2} + N = 0$$

The second equation gives

$$\begin{aligned}
-T\frac{d\theta}{2}-\left(T+dT\right)\frac{d\theta}{2}+N=0 \\
N=T\frac{d\theta}{2}+\left(T+dT\right)\frac{d\theta}{2} \\
N=T\frac{d\theta}{2}+T\frac{d\theta}{2}+dT\frac{d\theta}{2}
\end{aligned}$$

The third term is very small and can be neglected to obtain

$$\begin{aligned}
N&=T\frac{d\theta}{2}+T\frac{d\theta}{2} \\
&=Td\theta
\end{aligned}$$

Since the friction force is at its maximum, the sum of the x-component of the forces becomes

$$\begin{aligned}
\sum F_x=T-\left(T+dT\right)+\mu N=0 \\
dT=\mu N
\end{aligned}$$

With the value of the normal force, this equation is

$$dT=\mu Td\theta$$

The solution of this equation is

$$\begin{aligned}
\frac{dT}{T}=\mu d\theta \\
\int\frac{dT}{T}=\int\mu d\theta \\
\ln T=\mu\theta+cst
\end{aligned}$$

at $\theta = 0$, the tension is 9.8 N. Therefore,

$$\begin{aligned}
\ln 9.8N=0+cst \\
cst=\ln 9.8N
\end{aligned}$$

This, the tension is

$$\ln T = \mu\theta + \ln 9.8N$$
$$\ln T - \ln 9.8N = \mu\theta$$
$$\ln \tfrac{T}{9.8\text{N}} = \mu\theta$$
$$\frac{T}{9.8N} = e^{\mu\theta}$$
$$T = 9.8N \cdot e^{\mu\theta}$$

At 180°, the tension is 39,2 N. Thus,

$$39.2N = 9.8N \cdot e^{\mu\pi}$$

(We must work in radians because we used the approximation of small angles, which is true only for angles in radians.)

It only remains to solve for μ.

$$4 = e^{\mu\pi}$$
$$\ln 4 = \mu\pi$$
$$\mu = \frac{\ln 4}{\pi}$$
$$\mu = 0.4413$$

40. *Force Acting on the Object*

The only force acting on the ball is the drag.

Sum of the Forces

With an *x*-axis in the direction of the velocity, the sum of the *x*-component of the forces is

$$\sum F_x = -F_d$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis or in the direction opposed to the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma$$
$$-F_d = ma$$
$$-\frac{1}{2}C\rho A v^2 = ma$$

Solving the Equations

To find the speed of the ball depending on the time, we use

$$a = \frac{dv}{dt}$$

Therefore

$$-\frac{1}{2}C\rho A v^2 = m\frac{dv}{dt}$$

This equation can then be solved to obtain

$$\frac{dv}{dt} = -\frac{C\rho A v^2}{2m}$$
$$\frac{dv}{v^2} = -\frac{C\rho A}{2m}dt$$
$$\int \frac{dv}{v^2} = -\int \frac{C\rho A}{2m}dt$$
$$-\frac{1}{v} = -\frac{C\rho A t}{2m} + Cst$$

Since the speed is v_0 at $t = 0$, the value of the constant can be found.

$$-\frac{1}{v_0} = 0 + Cst$$
$$Cst = -\frac{1}{v_0}$$

The formula of velocity as a function of time is thus

$$-\frac{1}{v} = -\frac{C\rho A t}{2m} - \frac{1}{v_0}$$

Solving for *v*, it becomes

$$\frac{1}{v} = \frac{C\rho At}{2m} + \frac{1}{v_0}$$
$$\frac{1}{v} = \frac{C\rho Atv_0}{2mv_0} + \frac{2m}{2mv_0}$$
$$\frac{1}{v} = \frac{C\rho Atv_0 + 2m}{2mv_0}$$
$$v = \frac{2mv_0}{C\rho Atv_0 + 2m}$$

The speed cannot yet be calculated because we don't know how long it will take for the ball to reach its target. To find out, the formula of the position as a function of time is needed. This position is found with

$$x = \int v dt$$

Thus

$$\begin{aligned} x &= \int \frac{2mv_0}{C\rho Atv_0 + 2m} dt \\ &= \frac{2m}{C\rho A} \int \frac{1}{\left(t + \frac{2m}{C\rho Atv_0} \right)} dt \\ &= \frac{2m}{C\rho A} \ln\left(t + \frac{2m}{C\rho Atv_0} \right) + Cst \end{aligned}$$

Since the position is 0 at $t = 0$, the value of the constant can be found.

$$0 = \frac{2m}{C\rho A} \ln\left(0 + \frac{2m}{C\rho Atv_0} \right) + Cst$$
$$Cst = -\frac{2m}{C\rho A} \ln\left(\frac{2m}{C\rho Atv_0} \right)$$

Therefore

$$\begin{aligned}
x &= \frac{2m}{C\rho A}\ln\left(t+\frac{2m}{C\rho A t v_0}\right)-\frac{2m}{C\rho A}\ln\left(\frac{2m}{C\rho A t v_0}\right) \\
&= \frac{2m}{C\rho A}\left(\ln\left(t+\frac{2m}{C\rho A t v_0}\right)-\ln\left(\frac{2m}{C\rho A t v_0}\right)\right) \\
&= \frac{2m}{C\rho A}\ln\left(\frac{t+\frac{2m}{C\rho A t v_0}}{\frac{2m}{C\rho A t v_0}}\right) \\
&= \frac{2m}{C\rho A}\ln\left(\frac{C\rho A t v_0 t+2m}{2m}\right) \\
&= \frac{2m}{C\rho A}\ln\left(\frac{C\rho A v_0}{2m}t+1\right)
\end{aligned}$$

The time needed to reach the target can now be found. For the baseball, we have

$$\begin{aligned}
\frac{2m}{C\rho A} &= \frac{2\cdot 0.145kg}{0.47\cdot 1.3\frac{kg}{m^3}\cdot\pi(0.037m)^2} \\
&= 110.36m
\end{aligned}$$

Therefore, the position is

$$x = 110.36m\cdot\ln\left(\frac{30\frac{m}{s}}{110.36m\cdot}\cdot t+1\right)$$
$$x = 110.36m\cdot\ln\left(0.27184\tfrac{1}{s}\cdot t+1\right)$$

The time to reach x = 100 m is thus

$$\begin{gathered}
100m = 110.36m\cdot\ln\left(0.27184\tfrac{1}{s}\cdot t+1\right) \\
0.90614 = \ln\left(0.27184\tfrac{1}{s}\cdot t+1\right) \\
e^{0.90614} = 0.27184\tfrac{1}{s}\cdot t+1 \\
2.4748 = 0.27184\tfrac{1}{s}\cdot t+1 \\
1.4748 = 0.27184\tfrac{1}{s}\cdot t \\
t = 5.425s
\end{gathered}$$

The speed is, therefore,

$$\begin{aligned} v &= \frac{2mv_0}{C\rho A t v_0 + 2m} \\ &= \frac{v_0}{\frac{C\rho A v_0}{2m}t + 1} \\ &= \frac{30\frac{m}{s}}{\frac{30\frac{m}{s}}{110.36m}5.425s + 1} \\ &= 12.12\tfrac{m}{s} \end{aligned}$$

41. *Force Acting on the Object*

On the mass, 3 forces are exerted: gravity and 2 springs.

Sum of the Forces

The sums of the forces on the mass are

$$\begin{aligned} \sum F_x &= -F_{sp}\cos\theta + F_{sp}\cos\theta \\ \sum F_y &= F_{sp}\sin\theta + F_{sp}\sin\theta - mg \end{aligned}$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

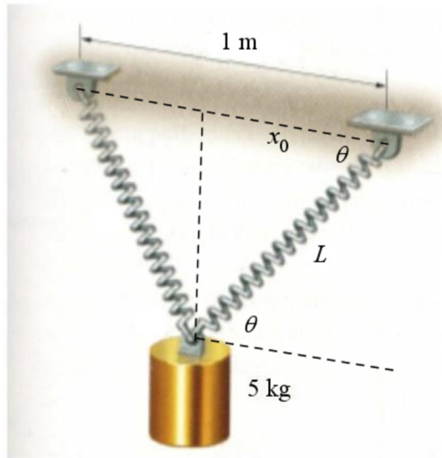
$$\begin{aligned} \sum F_x = ma_x \qquad &\to \qquad -F_R\cos\theta + F_R\cos\theta = 0 \\ \sum F_y = ma_y \qquad &\to \qquad F_R\sin\theta + F_R\sin\theta - mg = 0 \end{aligned}$$

Solving the Equations

The second equation gives

$$F_{sp} = \frac{mg}{2\sin\theta}$$

The force made by the spring and the angle both depend on the stretching of the spring.



The length of the spring is *L*. Thus, the force exerted by the spring is

$$F_{sp} = k\left(L - x_0\right)$$

where x_0 is the length of the unstretched spring (50 cm).

The sine of the angle is

$$\sin\theta = \frac{y}{L} = \frac{\sqrt{L^2 - x_0^2}}{L}$$

Therefore,

$$F_{sp} = \frac{mg}{2\sin\theta}$$

$$k\left(L - x_0\right) = \frac{mgL}{2\sqrt{L^2 - x_0^2}}$$

It only remains to solve this equation for *L*.

$$\left(L-x_0\right)\sqrt{L^2-x_0^2}=\frac{mgL}{2k}$$

$$\left(L-x_0\right)^2\left(L^2-x_0^2\right)=\left(\frac{mg}{2k}\right)^2 L^2$$

$$\left(L-x_0\right)^2\left(L-x_0\right)\left(L+x_0\right)=\left(\frac{mg}{2k}\right)^2 L^2$$

$$\left(L-x_0\right)^3\left(L+x_0\right)=\left(\frac{mg}{2k}\right)^2 L^2$$

$$\left(L^3-3x_0L^2+3x_0^2L-x_0^3\right)\left(L+x_0\right)=\left(\frac{mg}{2k}\right)^2 L^2$$

$$L^4-3x_0L^3+3x_0^2L^2-x_0^3L+x_0L^3-3x_0^2L^2+3x_0^3L-x_0^4=\left(\frac{mg}{2k}\right)^2 L^2$$

$$L^4-2x_0L^3+2x_0^3L-x_0^4=\left(\frac{mg}{2k}\right)^2 L^2$$

$$L^4-2x_0L^3-\left(\frac{mg}{2k}\right)^2 L^2+2x_0^3L-x_0^4=0$$

This is a 4$^{\text{th}}$-degree equation. With the values, it is

$$L^4-(1m)L^3-\left(\frac{1}{16}m^2\right)L^2+\left(\frac{1}{4}m^3\right)L-\left(\frac{1}{16}m^4\right)=0$$

$$16L^4-16m\cdot L^3-1m^2\cdot L^2+4m^3\cdot L-1m^4=0$$

The only positive solution of this equation is

$$L=0.81623\text{ m}$$