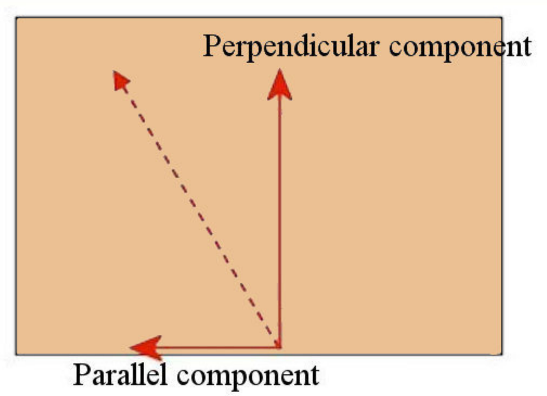
5 FORCES, PART 2

Julie is going to Saguenay in her car travelling at 126 km/h. Suddenly, she sees a moose on the road, and she must stop within a distance of 100 m. What must be the minimum value of the coefficient of static friction between the ground and the tires if she wants to stop before hitting the moose and if the wheels are not blocked?

wot.motortrend.com/animals-rejoice-volvo-developing-animal-detection-safety-technology-231543.html

Discover the answer to this question in this chapter.

5.1 FRICTION FORCE BETWEEN TWO SURFACES

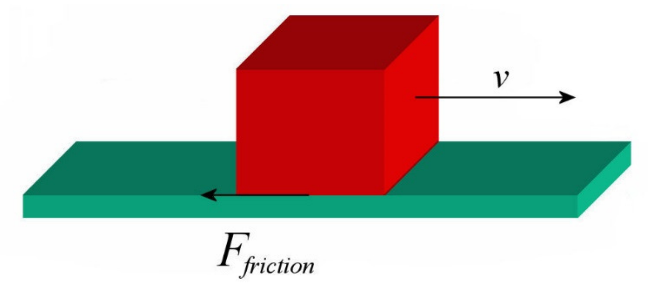


In the previous chapter, the contact force between two objects was resolved into two components: a perpendicular component (the normal force) and the parallel component. This component, which is the friction force, will now be explored.

There are two kinds of friction force: the static friction force and the kinetic friction force.

Kinetic Friction

The kinetic friction force acts on a body that slides on a surface. Often, this force in the opposite direction to the motion.



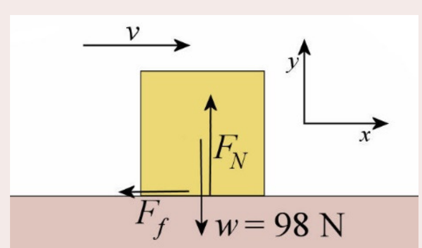
The frictional force has the value given by the formula

$$F_f = \mu_k F_N$$

where μ_k is a coefficient called the *coefficient of kinetic friction* and F_N is the normal between the two objects in contact. The value of μ_k depends only on the nature of the surfaces in contact. Typical values will be given a little further.

Example 5.1.1

A 10 kg box slides on the floor. The static coefficient of friction between the floor and the box is 0.4. What is the friction force acting on the box?



To find the force, the value of the normal force between the ground and the box must be found first. The forces acting on the box are shown in the diagram. The sum of the *y*-components is

$$\sum F_y = -98N + F_N$$

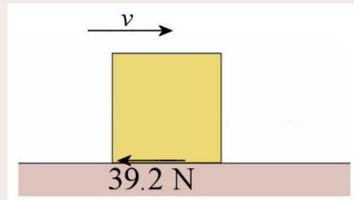
Since the y-component of the acceleration is zero, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -98N + F_N = 0$$

From this equation, we obtain $F_N = 98$ N.

The maximum of the static frictional force is, therefore.

$$\begin{aligned} F_f &= \mu_k F_N \\ &= 0.4 \cdot 98N \\ &= 39.2N \end{aligned}$$



The force is in the opposite direction to the motion.

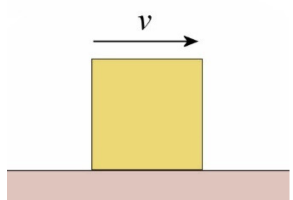
Do not assume that friction is always in the opposite direction to the motion. If a tablecloth is pulled and there are plates on the table, the plates, initially at rest, are set in motion as the tablecloth slides underneath them.
http://www.youtube.com/watch?v=T9KPwNeCdSg
What sets the plates in motion is the friction between the tablecloth and the plates. In this case, the friction force is not opposed to the motion of the plates. It causes the motion of the plates instead. If the tablecloth is removed quickly, the force of friction acts on the plate for a short amount of time, giving only a small speed to the plates. If there were no friction at all between the tablecloth and the plates, they would remain exactly at the same place.

If the tablecloth is not pulled fast enough, the friction force acts on the plates for a longer time. This gives them too much speed, and they fall to the ground.
https://www.youtube.com/watch?v=AcaSt5xD9Is

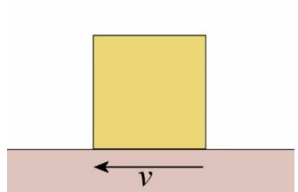
(Sometimes this trick fails, but for other reasons
https://www.youtube.com/watch?v=O3_51pXJ00M)

Here's the same trick, but on a much larger scale
http://www.youtube.com/watch?v=vfnt8Sdj7cs
(Here there is a trick used to reduce the friction between the tablecloth and the items on the table in this case. The friction must be reduced because the friction acts on the last plate on the table during too much time. View the Mythbuster video on this subject to find out.)
https://www.youtube.com/watch?v=lK1ci50DUgc

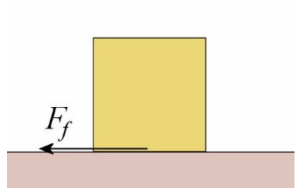
To correctly determine the direction of the friction force, take the point of view of the object being studied and try to determine in which direction the other surface moves. Let's start with the most common cases: an object sliding on a stationary floor.



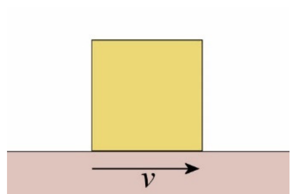
If the point of view of the block is taken, the block is stationary, and the floor is moving.



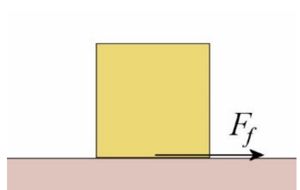
This is the direction of the relative motion of the other surface (the floor in this example). This direction is the direction of the friction force.



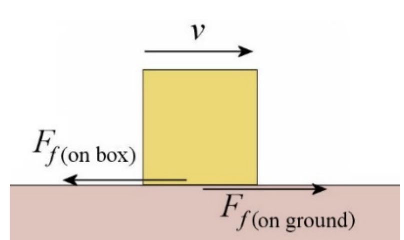
Let's take another example: a stationary object is at rest on a moving surface (such as the plates on a tablecloth that is withdrawn). This diagram illustrates this situation.



This is already the point of view of the block since it is stationary. So the direction of the relative motion of the other surface is towards the right, and the friction force is also towards the right.



Here's a small note on Newton's third law: if the floor exerts a frictional force on a block, then the block exerts a frictional force on the ground in the opposite direction.



Here is a summary of the characteristics of the kinetic friction force.

Kinetic Friction Force (F_f)

1) Magnitude of the force

$$F_f = \mu_k F_N$$

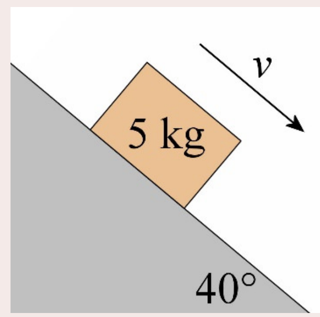
2) Direction of the force

Parallel to the contact surface,
In the direction of the relative motion of the other surface.

3) Application point of the force

Point of contact between the surfaces

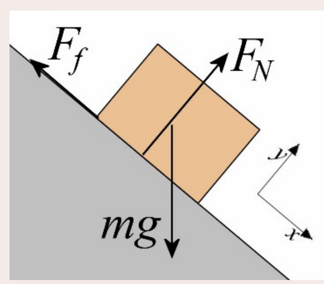
Example 5.1.2



A block slides down a 40° incline. The kinetic friction coefficient between the slope and the block is 0.4. What is the acceleration of the block?

The acceleration is found with Newton's second law by considering the forces acting on the block.

Forces Acting on the Object

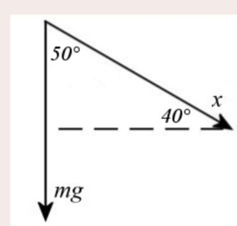


There are 3 forces acting on the block.

1) The weight (mg) directed downwards.
2) The normal force (F_N) made by the slope.
3) The friction force ($F_f = \mu_k F_N$) made by the slope directed uphill.

(mg was used for the weight rather than 49 N in order to be able to show something interesting at the end.)

Sum of the Forces



The normal force and friction force are directed in the direction of an axis, and it is not difficult to find their components. However, the situation is more complicated for the weight. The force and the positive x-axis are shown together in the diagram to the left. The angle between the x-axis and the weight is -50°.

The components of the force are then

Forces	x	y
Weight	mg cos(-50 °)	mg sin(-50 °)
Normal force	0	F_N
Friction force	$-\mu_k F_N$	0

The sums of the forces are

$$\sum F_x = mg\cos\left(-50°\right) - \mu_k F_N$$
$$\sum F_y = mg\sin\left(-50°\right) + F_N$$

Newton's Second Law

As the velocity is in the direction of the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus $a_x = a$ and $a_y = 0$.

Thus, Newton's second law gives

$$\sum F_x = ma_x \quad \rightarrow \quad mg\cos\left(-50°\right) - \mu_k F_N = ma$$
$$\sum F_y = ma_y \quad \rightarrow \quad mg\sin\left(-50°\right) + F_N = 0$$

Solving the equation

The acceleration can be found with the first equation, but the normal force is needed. However, the normal force can be found with the second equation.

$$mg\sin\left(-50°\right) + F_N = 0$$
$$F_N = -mg\sin\left(-50°\right)$$
$$F_N = mg\sin\left(50°\right)$$
$$F_N = 5kg \cdot 9.8\tfrac{N}{kg}\sin\left(50°\right)$$
$$F_N = 37.54N$$

The acceleration can now be found.

$$mg\cos\left(50°\right) - \mu_k F_N = ma$$
$$\cancel{m}g\cos\left(50°\right) - \mu_k \cancel{m}g\sin\left(50°\right) = \cancel{m}a$$
$$a = g\cos\left(50°\right) - \mu_k g\sin\left(50°\right)$$
$$a = 9.8\tfrac{m}{s^2}\cos\left(50°\right) - 0.4 \cdot 9.8\tfrac{m}{s^2}\sin\left(50°\right)$$
$$a = 3.296\tfrac{m}{s^2}$$

Here are two comments on this example.

1) Since the acceleration is positive, and thus in the same direction as the velocity, the object goes faster and faster. With a greater μ, the acceleration would be negative, which is in the opposite direction of the velocity. The block would have slowed down in this case.

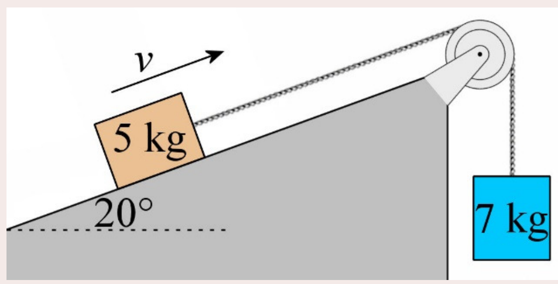
2) The normal force was substituted by $\mu mgsin(50\ °)$ in the second calculation, but 37.54 N could have been used too. This formula was used to illustrate something interesting. The following acceleration was obrained

$$a = g\cos(50°) - \mu_c g\sin(50°)$$

The acceleration of an object sliding on a slope does not depend on its mass. It depends only on μ, g and the angle of the slope. This shows that in a downhill sled race, all the sleds have the same acceleration, whatever their masses, even if the friction between the sled and the ground is taken into account. If they race on their buttocks, then there may be a winner depending on the value of the coefficient of friction between the snow and the snowsuit of the slider.

Example 5.1.3

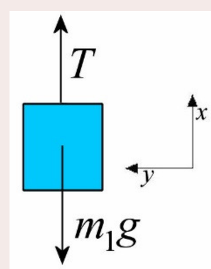
In the situation illustrated in the diagram, what is the acceleration of the blocks if the 5 kg block is moving uphill and the coefficient of kinetic friction between the 5 kg block and the slope is 0.7? (Neglect the mass of the pulley.)



The acceleration is found with Newton's second law by considering the forces acting on each block.

7 kg Block

Forces Acting on the Object



There are 2 forces acting on the block.

1) The weight (68.6 *N*) directed downwards.
2) The tension force (*T*) of the rope directed upwards.

Sum of the Forces

The sum of the forces is

$$\sum F_x = -68.6N + T$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus $a_x = a$ and $a_y = 0$.

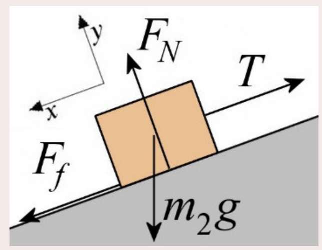
Thus, Newton's second law gives

$$\sum F_x = ma_x \quad \rightarrow \quad -68.6N + T = 7kg \cdot a$$

7 kg Block

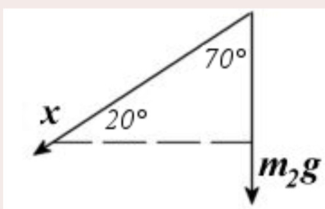
Forces Acting on the Object

There are 4 forces acting on the block.



1) The weight (49 *N*) directed downwards.
2) The tension force of the rope (*T*) directed uphill.
3) The normal force (F_N) made by the slope.
4) The friction force ($F_f = \mu_k F_N$) directed downhill.

Sum of the Forces



The angle between the weight and the *x*-axis being -70 °, the table of forces for this block is

Forces	*x*	*y*
Weight	49 N·cos(-70 °)	49 N·sin(-70 °)
Tension force	-*T*	0
Normal force	0	F_N
Friction force	$\mu_k F_N$	0

The sums of the forces are then

$$\sum F_x = 49N \cdot \cos(-70°) - T + \mu_k F_N$$
$$\sum F_y = 49N \cdot \sin(-70°) + F_N$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus $a_x = a$ and $a_y = 0$.

Thus, Newton's second law gives

$$\begin{aligned} \sum F_x = ma_x \quad &\to \quad 49N \cdot \cos(-70°) - T + \mu_c F_N = 5kg \cdot a \\ \sum F_y = ma_y \quad &\to \quad 49N \cdot \sin(-70°) + F_N = 0 \end{aligned}$$

Solving the Equations

We have a system of three equations to solve.

$$-68.6N + T = 7kg \cdot a$$
$$49N \cdot \cos(-70°) - T + \mu_k F_N = 5kg \cdot a$$
$$49N \cdot \sin(-70°) + F_N = 0$$

First, the normal force is found with the third equation.

$$49N \cdot \sin(-70°) + F_N = 0$$
$$F_N = -49N \cdot \sin(-70°)$$
$$F_N = 49N \cdot \sin(70°)$$
$$F_N = 46.04N$$

This value is then substituted into the second equation.

$$-68.6N + T = 7kg \cdot a$$
$$49N \cdot \cos(-70°) - T + \mu_k \cdot 46.04N = 5kg \cdot a$$

If the two equations are added, the tensions cancel each other, and we have

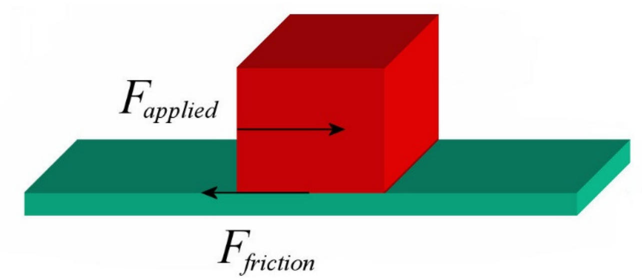
$$(-68.6N + T) + (49N \cdot \cos(70°) - T + \mu_k \cdot 46.04N) = 7kg \cdot a + 5kg \cdot a$$
$$-68.6N + 49N \cdot \cos(70°) + \mu_k \cdot 46.04N = (7kg + 5kg) \cdot a$$
$$-68.6N + 49N \cdot \cos(70°) + 0{,}7 \cdot 46.04N = 12kg \cdot a$$
$$a = -1.63 \tfrac{m}{s^2}$$

The acceleration is negative because it is directed uphill for the 5 kg block. As the direction of the acceleration is the same as the direction of the velocity, the speed of the blocks increases with an acceleration of 1.63 m/s².

Static Friction

When a very heavy crate at rest on the ground is pushed with a weak force, it does not move. If the force is gradually increased, then the crate still will not move until the force reaches a certain value. Only then will the crate begin to move. In this section, we will look at the situation where the crate does not move, i.e. the static situation.

If the crate remains in place, then it does not accelerate. This means that the sum of the forces acting on the crate is zero. That also means that the friction force must exactly cancel the applied force. This frictional force acting on an object that does not slide on a surface is called the *static friction force.*



Thus, if the applied force is 10 N and the crate does not move, then the friction force must be 10 N in the opposite direction. If the applied force is 20 N and the crate still does not move, then the friction force must be 20 N in the opposite direction. If the applied force is 30 N and the crate does not move, then the friction force must be 30 N in the opposite direction.

However, there is a maximum static frictional force. Studies, begun in 1699 by Guillaume Amontons, showed that the static frictional force cannot exceed the value given by the formula

$$F_{f\,\max} = \mu_s F_N$$

where μ_s is a coefficient called the *static coefficient of friction*, and F_N is the normal force acting on the two objects in contact. The value of μ_s depends only on the nature of the surfaces in contact.

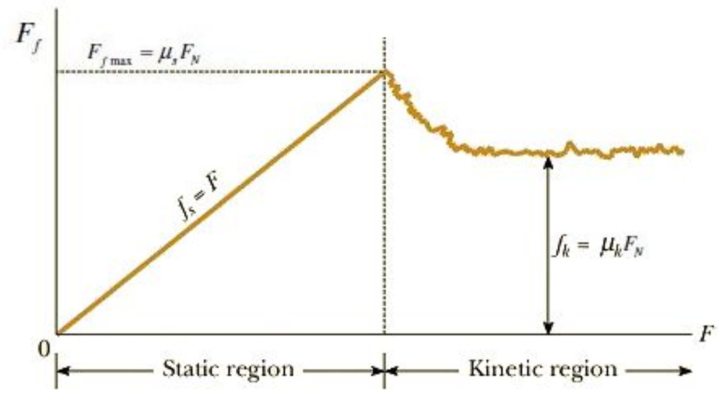
The values of μ_s and μ_c are not necessarily the same. Here are some values of the friction coefficient between various surfaces.

Surfaces	**μ (static)**	**μ (kinetic)**
Steel on steel	0.74	0.57
Glass on glass	0.94	0.40
Ice on Ice	0.10	0.03
Teflon on Teflon	0.04	0.04
Tires on dry asphalt	0.8 to 0.9	0.7 to 0.8
Tire on wet asphalt	0.5 to 0.7	0.4 to 0.5
Tire on snow	0.3	0.2
F1 tire on dry asphalt	1.7	

Obviously, these numbers are approximations, especially for last four because there are several types of tires. More friction coefficients can be found at:
http://www.roymech.co.uk/Useful_Tables/Tribology/co_of_frict.htm#coef

The static friction coefficients are almost always greater than the kinetic friction coefficients. Thus, there is a decrease of the friction force when the object begins to slide on a surface.

Suppose that a steadily increasing horizontal force is exerted on an object initially at rest on a horizontal surface. Here's a graph showing the friction force as a function of the force applied to the object.



www.kshitij-school.com/Study-Material/Class-11/Physics/Laws-of-motion/Forces-of-friction.aspx

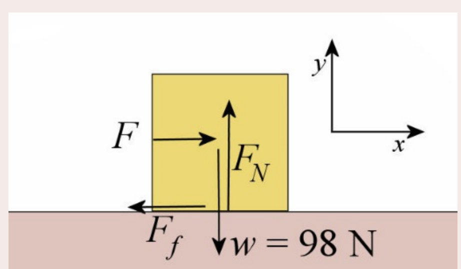
Initially, the friction force increases as the applied force increases. It continues to increase until the static friction reaches its maximum value ($\mu_s F_N$). Then the object starts to slip, and the friction force is now the kinetic friction force ($\mu_k F_N$). As $\mu_k < \mu_s$, there is a decrease of the friction force when the object begins to slide. This decrease of the friction force can be seen in the graph.

In this video, this decrease of the friction force can be seen when the object begins to slip. (It is not a huge decrease.)
http://www.youtube.com/watch?v=p7zMifirHLE

Example 5.1.4

A 10 kg box is on the floor. The static coefficient of friction between the floor and the box is 0.5, and the kinetic coefficient of friction is 0.4. What is the friction force acting on the box if a horizontal force acts on the box with a magnitude of a) 20 N b) 40 N c) 60 N?

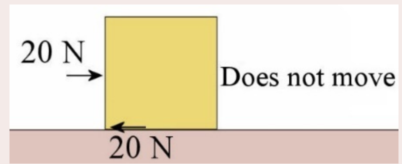
The maximum value of the static friction force will be calculated first. In order to do that, the value of the normal force between the ground and the box must be found first. The forces acting on the box are shown in the diagram. As the forces are the same as in example 5.1.1, we have the same normal $F_N = 98$ N.



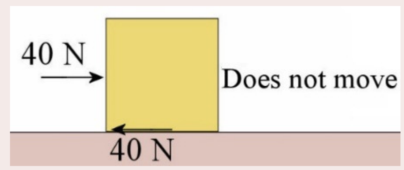
The maximum of the static frictional force is, therefore.

$$\begin{aligned} F_{f\max} &= \mu_s F_N \\ &= 0.5 \cdot 98N \\ &= 49N \end{aligned}$$

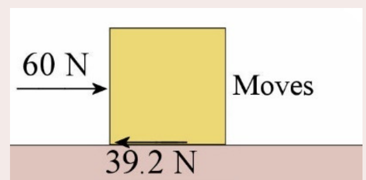
Thus, if the applied force (F) is 20 N, the friction force is also 20 N, and the object remains in place.



If the applied force (F) is 40 N, the friction force is also 40 N, and the object remains in place.



If the force is 60 N, then the object moves since the applied force exceeds the maximum static frictional force. We then have a kinetic friction force. The coefficient then drops to 0.4 and we obtain the same friction force as in the first example.



Here's a summary of what is known about the static frictional force.

Static Friction Force (F_f)

1) Magnitude of the force
   A magnitude which is exactly what it must be to prevent the surface from sliding on each other.
   Maximum value $F_{f\max} = \mu_s F_N$

2) Direction of the force
   Parallel to the contact surface,
   in the direction necessary to ensure that the surfaces do not slide on each other.

3) Point of application of the force
   Point of contact between the surfaces

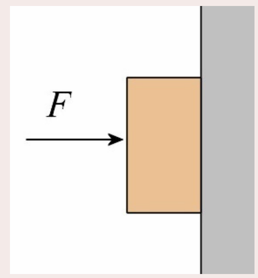
Problem-solving with static friction is a little more difficult because the value of the friction force is not known. The friction force cannot be substituted by $\mu_s F_N$ because this is not necessarily the magnitude of the frictional force. Its value can be anything between 0 and $\mu_s F_N$.

To solve static friction problems, the equations of the forces are written assuming that the object has a certain acceleration (often the one it must have so that it does not slip on the surface but not always as we'll see for the breaking car example). Then **F_f and F_N must be obtained** from theses equations. Finally, these results must be used in the following equation.

$$F_f \leq \mu_s F_N$$

(This equation indicates that the friction force must be less than the maximum value of the static friction force.) This equation is then solved to find the answer. This actually gives the condition for the object to have the supposed acceleration.

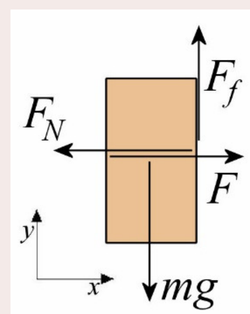
Example 5.1.5



A 1 kg block is held in place against a wall with a horizontal force. What minimum force should be applied on the block so that it does not slip if the coefficient of static friction between the block and the wall is 0.8?

The coefficient is found with Newton's second law by considering the forces acting on the block while assuming that it is not sliding.

Forces Acting on the Object



There are 4 forces acting on the block.

1) The weight (9.8 N) directed downwards.
2) A normal force (F_N) made by the wall directed towards the left.
3) A friction force (F_f) made by the wall directed upwards.
4) The force F directed towards the right.

A friction force must be present to keep the block in place as it is the only force that can oppose gravity.

Sum of the Forces

The sums of the forces are

$$\sum F_x = F - F_N$$
$$\sum F_y = -9.8N + F_f$$

Newton's Second Law

If the block is not slipping, there is no acceleration. (The condition for this block to remain at rest will then be obtained.)

Newton's second law then gives the following equations.

$$\sum F_x = ma_x \qquad \rightarrow \qquad F - F_N = 0$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -9.8N + F_f = 0$$

Solving the Equations

To solve problems with static friction, F_f and F_N must be found.

With the second equation F_f can be found.

$$-9.8N + F_f = 0$$
$$F_f = 9.8N$$

With the first equation F_N can be found.

$$F - F_N = 0$$
$$F_N = F$$

Using those values in $F_f \leq \mu_s F_N$, the result is

$$F_f \leq \mu_s F_N$$
$$9.8N \leq 0.8 \cdot F$$

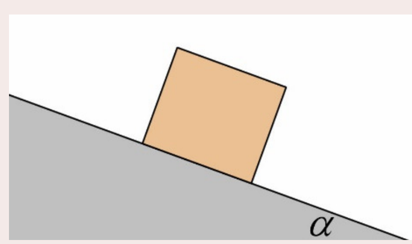
Then, this equation can be solved to obtain F.

$$9{,}8N \leq 0.8 \cdot F$$
$$\frac{9{,}8N}{0.8} \leq F$$
$$12.25N \leq F$$

The minimum force is, therefore, 12.25 N for this object to remain at rest.

Example 5.1.6

A block is on an incline, and the angle α is gradually increased. What is the angle when the block starts to slide if the coefficient of static friction between the block and the slope is 0.6?

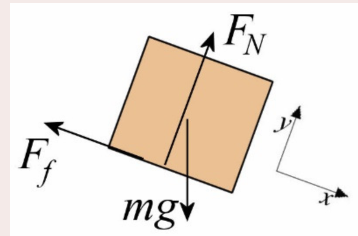


The angle is found with Newton's second law by considering the forces acting on the block while assuming that it is not sliding.

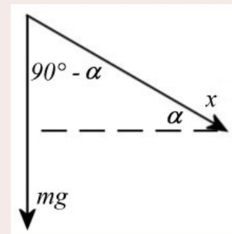
Forces Acting on the Object

There are 3 forces acting on the block.

1) The weight (mg) directed downwards.
2) A normal force (F_N) perpendicular to the slope.
3) A friction force (F_f) directed uphill.



Sum of the Forces



Since the angle between the positive x-axis and the weight is -(90° - α), the table of forces is

Forces	x	y
Weight	$mg\cos(-(90° - \alpha))$	$mg\sin(-(90° - \alpha))$
Normal force	0	F_N
Friction force	$-F_f$	0

The sums of the forces are then

$$\sum F_x = mg\cos(90° - \alpha) - F_f$$
$$\sum F_y = -mg\sin(90° - \alpha) + F_N$$

Newton's Second Law

If the block is not slipping, there is no acceleration. (The condition for this block to remain at rest will then be obtained.)

Newton's second law then gives the following equations.

$$\sum F_x = ma_x \quad \rightarrow \quad mg\cos(90° - \alpha) - F_f = 0$$
$$\sum F_y = ma_y \quad \rightarrow \quad -mg\sin(90° - \alpha) + F_N = 0$$

Solving the Equations

Using trigonometric identities, these equations are

$$mg\sin\alpha - F_f = 0$$
$$-mg\cos\alpha + F_N = 0$$

The first equation gives F_f.

$$F_f = mg \sin \alpha$$

The second equation gives F_N.

$$F_N = mg \cos \alpha$$

Using those values in $F_f \leq \mu_s F_N$, the result is

$$F_f \leq \mu_s F_N$$
$$mg \sin \alpha \leq \mu_s mg \cos \alpha$$

Now, this equation can be solved to find α.

$$mg \sin \alpha \leq \mu_s mg \cos \alpha$$
$$\sin \alpha \leq \mu_s \cos \alpha$$
$$\tan \alpha \leq \mu_s$$
$$\alpha \leq \arctan \mu_s$$

As the static friction coefficient is 0.6, the angle is

$$\alpha \leq \arctan 0.6$$
$$\alpha \leq 30.96°$$

This means that the angle must be less than 30.96° for the object to be at rest (since an acceleration of zero was used). Thus, if the angle is greater than 30.96 °, the object slips.

Note that this result gives us a simple way to know the static coefficient of friction between two surfaces. You just have to tilt the surface where the object is until it slips. The coefficient of static friction is then found with $\mu_s = \tan \alpha_{max}$. The coefficient depends only on the angle and on no other elements such as the mass. This is what is shown in this video.
http://www.youtube.com/watch?v=3miOIZKKYHs

Will an Object Slip on a Surface?

If you must determine whether an object is going to slide on a surface or not, the calculation is almost identical. The value of F_f and F_N must be calculated but it must be assumed that there is no slipping (it is assumed that the object does not slip in order to know the friction force needed to prevent the object from slipping).

Then the values of F_f and $\mu_s F_N$ must be compared.

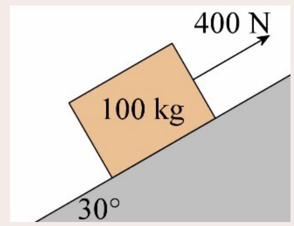
If $F_f \leq \mu_s F_N$, then the object is not slipping.

In this case, the static friction needed to prevent the object from slipping is below the maximum friction force possible, which means that there can be enough friction force to prevent the object from slipping.

$$\text{If } F_f > \mu_s F_N \text{, then the object is slipping.}$$

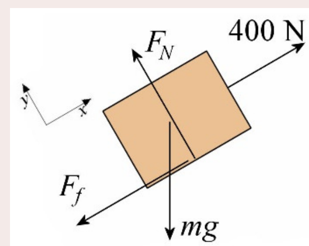
In this case, the static friction required to prevent the object from slipping is greater than the maximum friction force possible. This means that the friction force cannot be large enough to prevent the object from slipping.

Example 5.1.8



Will this block move if it is initially at rest? The coefficient of static friction between the block and the slope is 0.2.

Forces acting on the Object

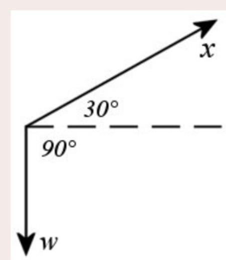


There are 4 forces acting on the 100 kg block.

1) The weight (980 N) directed downwards.
2) A normal force (F_N) perpendicular to the slope.
3) A friction force (F_f) directed downhill.
4) The 400 N force directed uphill.

Actually, we do not know whether the friction force is directed downhill or uphill. Here, one of the two directions was arbitrarily chosen. If the value of the force of friction obtained at the end is positive, then we will know that the right direction was chosen. If the value obtained is negative, then we will know that we chose the wrong direction, and that friction is rather directed uphill.

Sum of the Forces



The angle between the positive x-axis and the weight is -120°.

Then, the table of force is

Forces	x	y
Weight	980 N·cos (-120 °)	980 N·sin (-120 °)
Normal force	0	F_N
Friction force	$-F_f$	0
400 N	400 N	0

The sums of the forces are thus

$$\sum F_x = 980N \cdot \cos\left(-120°\right) - F_f + 400N$$
$$\sum F_y = 980N \cdot \sin\left(-120°\right) + F_N$$

Newton's Second Law

If the block is not slipping, there is no acceleration. (The condition for this block to remain at rest will then be obtained.)

Newton's second law then gives the following equations.

$$\sum F_x = ma_x \qquad \rightarrow \qquad 980N \cdot \cos\left(-120°\right) - F_f + 400N = 0$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad 980N \cdot \sin\left(-120°\right) + F_N = 0$$

Solving the Equations

F_f is found with the sum of the *x*-components of the forces.

$$980N \cdot \cos\left(-120°\right) - F_f + 400N = 0$$
$$F_f = 980N \cdot \cos\left(-120°\right) + 400N$$
$$F_f = -90N$$

As it is negative, the assumption made for the direction of the friction force was incorrect. The friction force is directed in the opposite direction from what is shown in the diagram.

Thus, a 90 N frictional force directed uphill is needed to keep the block in place.

Solving for F_N in the sum of the *y*-components of the forces gives

$$980N \cdot \sin\left(-120°\right) + F_N = 0$$
$$F_N = 980N \cdot \sin\left(120°\right)$$
$$F_N = 848.7N$$

Once F_f and F_N are obtained, F_f and $\mu_s F_N$ must be compared.

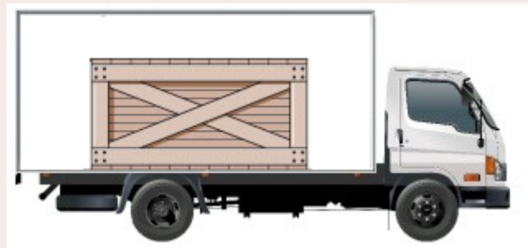
$$F_f = 90N \qquad\qquad \begin{aligned} \mu_s F_N &= 0.2 \cdot 848.7N \\ &= 169.7N \end{aligned}$$

As $F_f \leq \mu_s F_N$, the block does not slide.

With a maximum static friction force of 169.7 N, it is possible to obtain the 90 N required to prevent this bloc from moving. If the friction coefficient were 0.1, then the maximum friction would be only 84.9 N. As this value is smaller than 90 N, the static friction could not provide the 90 N required to keep the block in place, and the block would slide down the slope.

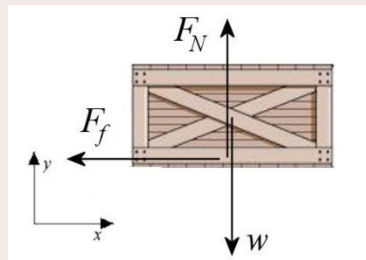
Example 5.1.8

This truck travelling at 90 km/h brakes over a distance of 50 m with a constant deceleration. Will the crate in the box of the truck slide if the coefficient of friction between the crate and the floor of the box is 0.4?



www.canstockphoto.com/delivery-cargo-truck-13682465.html

Forces Acting on the Object



There are 3 forces acting on the crate.

1) The weight (*mg*) directed downwards.
2) A normal force (F_N) directed upwards.
3) A friction force (F_f) directed towards the left.

When the truck brakes, the crate must also brake since it is assumed that it does not slide. The friction force being the only horizontal force, it must be the force that slows down the crate. The force must, therefore, be directed towards the left since the crate moves towards the right.

Sum of the Forces

The sums of the forces are

$$\sum F_x = -F_f$$
$$\sum F_y = -mg + F_N$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus $a_x = a$ and $a_y = 0$.

Not sliding on a surface does not necessarily mean that the acceleration must be zero. Rather, it means that both surfaces must have the same acceleration. As the truck decelerates, the crate must have the same deceleration so that it does not slide.

Newton's second law thus gives the following equations.

$$\sum F_x = ma_x \quad \rightarrow \quad -F_f = ma$$
$$\sum F_y = ma_y \quad \rightarrow \quad -mg + F_N = 0$$

<u>*Solving the Equations*</u>

Solving for F_f in the sum of the x-components of the forces gives

$$F_f = -ma$$

The acceleration of the crate, which is the same as the truck's acceleration, must be found. This acceleration is

$$2a(x - x_0) = v^2 - v_0^2$$
$$2 \cdot a \cdot (50m - 0m) = \left(0 \tfrac{m}{s}\right)^2 - \left(25 \tfrac{m}{s}\right)^2$$
$$a = -6.25 \tfrac{m}{s^2}$$

The mass of the crate is also needed, but it is not given. Therefore, the force of friction is

$$\begin{aligned} F_f &= -ma \\ &= -m\left(-6.25 \tfrac{m}{s^2}\right) \\ &= m \cdot 6.25 \tfrac{m}{s^2} \end{aligned}$$

Solving for F_N in the sum of the y-components of the forces gives

$$\begin{aligned} F_N &= mg \\ &= m \cdot 9.8 \tfrac{m}{s^2} \end{aligned}$$

Once F_f and F_N are obtained, F_f and $\mu_s F_N$ must be compared.

$$F_f = m \cdot 6.25 \tfrac{m}{s^2}$$

$$\begin{aligned} \mu_s F_N &= 0.4 \cdot m \cdot 9.8 \tfrac{m}{s^2} \\ &= m \cdot 3.92 \tfrac{m}{s^2} \end{aligned}$$

Even if the value of m is not known, it is easy to see that $F_f > \mu_s F_N$. This means that the crate will slide on the surface.

Here, the friction is not great enough to prevent the box from slipping. There will still be a friction force (kinetic friction) directed towards the left. This friction will slow down the box, but with a deceleration smaller than the truck's deceleration. If the box slows down more slowly than the truck, then this means that it slides forward relative to the truck.

This example also shows that the static frictional force is not necessarily a force acting on an object at rest. If the static coefficient of friction had been large enough so that the box would not slip when the truck brakes, the static frictional force would act on the box and the box would be accelerating. We would then have a static frictional force acting on a box that is accelerating.

Friction Force and Cars

The frictional force is not just a force that is detrimental to the movement of cars. In fact, the static friction force between the tires and the road is practically responsible for all the movements of your car: acceleration, braking, and change of direction in corners (gravity can also accelerate your car, but only in hills).

Increasing the Speed of a Car

Let's see how friction can bring a stopped car into motion. If the driver presses the accelerator hard, the tires will slide on the asphalt. In this case, the tire slid towards the back of the car on a road at rest. This means that the kinetic friction force on the tires will be directed towards the front of the car, causing it to accelerate.

science4fun.info/what-is-friction/

If the driver pushes gently on the accelerator at the start, then the wheels will not slide on the asphalt and the static friction force on the tires will speed the car up. This is a situation similar to someone trying to push a stationary block on a surface. When the person exerts a force to the left to move the block, then there is a friction force towards the right. When the driver presses the accelerator, the engine exerts a force that seeks to push the wheel backwards, just like the person pushing on the box. If the wheel does not slip, it is because, as with the box, the force of friction is opposed to this force made by the engine.

www.vectorstock.com/royalty-free-vector/car-wheel-vector-34385

This force is thus directed towards the front of the car. This frictional force is accelerating the forward. In this case, the friction force is not harmful at all. Without any friction, the car would not be able to move. Without friction, the wheels would rotate on the spot and slide on the ground when the driver would press the accelerator. That is what happens when you try to accelerate on ice. In this case, the friction force is very small, and the acceleration of the car is very small. Finally, notice that this friction force only acts on the wheels connected to your engine. For front-wheel drive, the force is on the front wheels, for a rear-wheel drive, the force is on the rear wheels and for a 4-wheel drive car, the force is distributed on the 4 wheels.

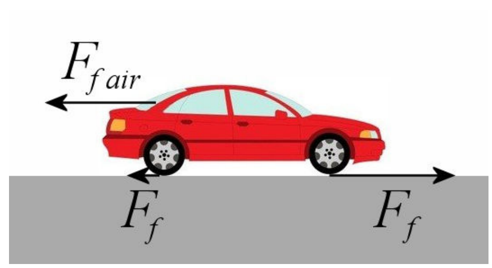
Decreasing the Speed of a Car

The frictional force is also the force that slows down your car when you brake. If you brake very hard, up to the point of locking the wheels, the tires will then slide on the asphalt as the car moves forward. In this case, the kinetic friction force on the tires will be directed towards the rear of the car, causing it to slow down.

Now let's see what happens if the wheels don't lock. If there were no friction at all, the wheels would slide as soon as the driver brake. If they do not slide, it is because the friction force made by the road on the wheel forces the wheel to continue spinning. This means that the friction force on the wheel is in the direction of the rotation of the wheel. This frictional force towards the back of the car causes the car to slow down. Without friction, it would be impossible to slow down the car. That's what you notice when you brake on ice. In this case, the friction force is very low, and the deceleration of the car is very small. The friction force that causes your car to slow down is acting on all 4 wheels of your car (not necessarily evenly distributed, the force being often greater on the front wheels).

Constant Speed Motion in a Straight Line

The frictional force is also the force that allows a car to move at a constant speed. For a front-wheel drive car (the engine turns the front wheels), there is a forward friction force acting on the front wheels. As the engine seeks to rotate the wheels faster than they turn, there is friction opposed to the rotation of the wheels, and this friction is directed towards the front of the car.



This force will be the force opposed to the frictional force made by the air and the small static frictional force that there is between the ground and the rear wheels. The sum of all these forces is zero, which means that the speed of the car is constant. It's still surprising to see that the frictional force is the force that allows the car to move at a constant speed! As mentioned earlier, friction is not always in the opposite direction to the motion.

When braking, the friction on the front wheels reverses, and all the frictional forces are towards the rear.

In the next chapter, it will be seen that the friction force is also responsible for the changing the direction of your car. Without friction, it's impossible to take a curve!

Example 5.1.9

Julie is going to Saguenay in her car travelling at 126 km/h. Suddenly, she sees a moose on the road, and she must stop within a distance of 100 m. What must be the minimum value of the coefficient of static friction between the ground and the tires if she wants to stop before hitting the moose and if the wheels are not blocked?

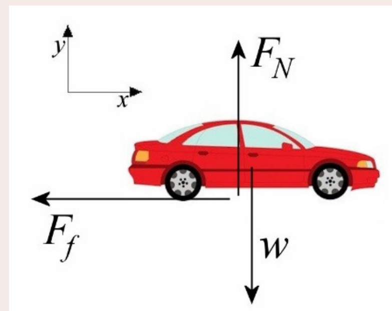
The coefficient is found with Newton's second law by considering the forces acting on the car while assuming that the tires are not sliding.

Forces Acting on the Object.

There are 3 forces acting on the car.

1) The weight (mg) directed downwards.
2) A normal force (F_N) directed upwards.
3) A friction force (F_f) directed towards the left.

(In the diagram, the starting point of the friction force is under the rear wheels but in reality, the friction acts on all 4 wheels.)



fr.depositphotos.com/2577683/stock-illustration-Car.html

Sum of the Forces

The sums of the forces are

$$\sum F_x = -F_f$$
$$\sum F_y = -mg + F_N$$

Newton's Second Law

In these equations, it is assumed that the acceleration is exactly the acceleration needed to stop the car just at the moose. Thus, the condition for the car to stop just before the moose will be obtained. (In this case, the acceleration is not 0 since the car is moving even if the wheels are not slipping.)

This friction force is a static friction force since there is no slippage between the wheels and the ground when the wheels rotate. If the wheels were blocked, there would be slippage and a kinetic friction force would be acting instead.

Newton's second law thus gives the following equations

$$\sum F_x = ma_x \qquad \to \qquad -F_f = ma$$
$$\sum F_y = ma_y \qquad \to \qquad -mg + F_N = 0$$

Solving the Equations

To solve problems with static friction, F_f and F_N must be found.

Solving for F_f in the sum of the x-components of the forces gives

$$-F_f = ma$$
$$F_f = -ma$$

Solving for F_N in the sum of the y-components of the forces gives

$$-mg + F_N = 0$$
$$F_N = mg$$

Using those values in $F_f \leq \mu_s F_N$, the result is

$$F_f \leq \mu_s F_N$$
$$-ma \leq \mu_s mg$$

Then μ_s can be found.

$$-\cancel{m}a \leq \mu_s \cancel{m}g$$
$$-a \leq \mu_s g$$
$$\frac{-a}{g} \leq \mu_s$$

The acceleration is needed to calculate the coefficient. It can be found with the laws of kinematics.

$$2a\left(x - x_0\right) = v^2 - v_0^2$$
$$2 \cdot a \cdot \left(100m - 0m\right) = 0 - \left(35\tfrac{m}{s}\right)^2$$
$$a = -6.125\tfrac{m}{s^2}$$

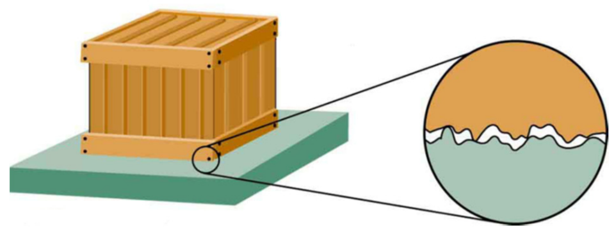
The coefficient can finally be calculated.

$$\mu_s \geq \frac{-a}{g}$$
$$\mu_s \geq \frac{-\left(-6.125\tfrac{m}{s^2}\right)}{9.8\tfrac{m}{s^2}}$$
$$\mu_s \geq 0.625$$

The minimum coefficient needed is 0.625 to stop just at the moose. If the friction coefficient is less than this value, there won't be enough friction to stop the vehicle in such a short distance.

Origin of the Frictional Force

The friction force comes from contacts between some of the surface molecules of the two objects in contact. Not all the molecules of the surface of one object interacts with the molecules of the other object because the surfaces are far from being smooth at the molecular level. By zooming a lot, it would be seen that the surfaces are rather irregular, as shown in the diagram.



cnx.org/content/m42139/latest/?collection=col11406/latest

Because of these irregularities, the two objects do not touch everywhere, and only a few molecules are close enough to each other to create molecular interactions between objects. If the area where the two objects actually touch each other is measured, the actual contact area is obtained. To give you an idea, the actual contact area of a copper cube with 22 cm long sides placed on a copper plate is only 1 mm², whereas the area of the bottom surface of the cube is 48,400 mm²! Obviously, the two surfaces do not actually touch each other very often.

If a surface must be moved relatively to the other, then all these molecular interactions must be broken. A minimal force is therefore required to move the object. These molecular interactions are not necessarily going to break exactly between the two surfaces. It is possible that the break tears a piece of one of the surfaces. This is what happens when a chalk is passed on a chalkboard or if sandpaper is rubbed on wood.

Even when the object slips, new interactions between molecules are continually re-formed so that the friction force remains ever-present.

Friction coefficient values for Teflon are low because Teflon molecules do not bond easily with other types of molecules.

Why Is μ_k Smaller Than μ_s?

There is a difference between the two types of friction because there is often a third substance between the two objects. This may simply be air, or a thin oxide layer found on the surface of many metals.

When the two objects are at rest relative to each other, there is sufficient time for the impurities between the two surfaces to move so that the surfaces can come into direct contact where they actually touch. However, when the surfaces slide over each other, the substance between the two surfaces has less time to make room for the asperities of the two objects. So, there is less direct contact between the two objects and thus fewer molecular contacts. The frictional force is, therefore, smaller than when the two objects were at rest relative to each other.

This means that if it were possible to eliminate this third substance between the two surfaces completely, there would be virtually no difference between the static friction and kinetic friction coefficient. Tests carried out in a vacuum with perfectly clean metals show that there is indeed no difference between the two types of friction in this case.

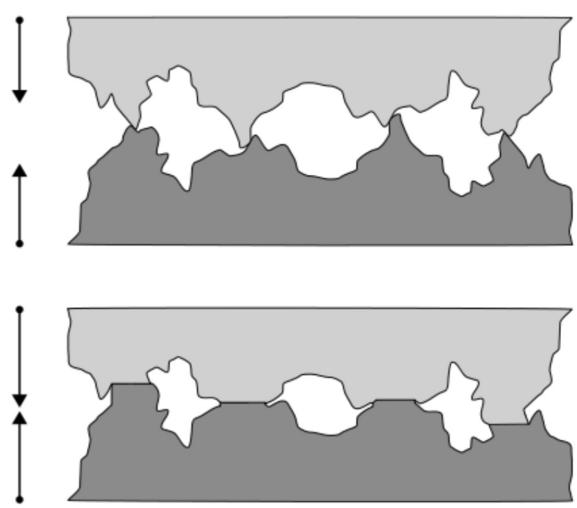
Two Common Mistakes

Two important elements are common sources of mistakes.

Common Mistake: Thinking That the Friction Force Depends on the Contact Area

Many people believe that the area of the contact surface is critical for the friction force, but this is wrong. The frictional force is independent of the contact surface. This is Amontons's second law.

This law may seem contrary to what is known about the friction force. If the contact area is larger, there should be more interactions between the molecules and it should take a greater force to break these interactions. The frictional force should, therefore, be greater. This argument is basically correct, but it must be remembered that the two contact surfaces do not touch each other everywhere and that they truly touch each other at only a few locations.



en.wikipedia.org/wiki/Asperity_(materials_science)

Let's see how the actual contact area between a brick and ground changes if the orientation of the brick is changed. If it is set initially on its largest surface (the underside of the brick), then it exerts pressure on the ground and touches the ground in some places, which form the actual contact area (top diagram on the left). If it is now set on its smallest surface (one of the ends of the brick), then the brick exerts more pressure on the ground since the weight is distributed over a smaller area. This means that the brick sinks deeper into the ground so that each of the actual contact area between the two objects becomes larger (second picture on the left). However, the total number of contacts is smaller, because the end surface of the brick is smaller than the underside surface of the brick. A wonderful thing then happens: these larger and less numerous contacts give exactly the same actual contact area between the two objects! Thus, even if the contact area seems smaller when the brick is standing on its end rather than flat on the ground, the real contact area remains the same! Now, it can be seen that the argument was correct: a larger actual contact area means that the friction force is greater. However, the actual contact area depends only on the normal force and not of the apparent contact area.

This destroys the argument usually given to explain why it is hard to separate two books whose pages are interleaved. (See this video)
http://www.youtube.com/watch?v=AX_lCOjLCTo
It is generally argued that it is hard to separate the books because there is a lot of friction and that this large friction force comes from the large contact surface. However, the friction force does not depend on the contact surface area! In fact, when a force is applied to separate the books, there is a component of the force that pushes the pages on each other, which increases the normal force between the pages, thereby increasing the friction force. As the force to separate the books increases, the normal force also increases and so does the friction force.

What would happen if there was no impurity between the surfaces and if the objects were so smooth that there was no bump on the surface so that all the molecules of the object would come into contact with the molecules of the other object? There would be many more contacts between the molecules compared to when there are irregularities on the surfaces. The friction force would then become much greater. This experiment was done, and it was observed that the friction force becomes so large that it is difficult to separate the two objects!

Wide Tires: Is It Better?

Why do cars have a better grip on the road with wider tires? When the width of the rear tires was reduced and grooves were added on all four tires in Formula 1 in 1998, all the drivers were complaining that their cars were sliding more than before.

www.goodwood.com/grr/f1/this-williams-f1-car-could-be-yours-for-150000/

These changes decreased the apparent contact area between the tires and the road, and everybody said that this lowered the friction between the tires and the track, thereby increasing the chances of sliding. Surely the cars were sliding more, but it was surely not because the contact surface had decreased since we know that the friction force is independent of the apparent contact area. What was happening then?

When the tires were changed, the tire composition had to be modified to make the tires stiffer. Narrow tires must have stiffer sides to support the weight of the car and to avoid a quick wear out. However, a more rigid rubber sticks less to the road, and that is why the cars were slipping more. Thus, the advantage of a wide tire is not to have a larger contact area; it is to enable the manufacturer to use a softer rubber compound which has a greater coefficient of friction with the asphalt. If two tires were made with the same type of rubber, but with different sizes, there would be no difference in the frictional force.

Common Mistake: Thinking That the Friction Force Depends on the Sliding Velocity

Many people think that the friction between two surfaces changes if the sliding speed increases. This is also false. Tests made by Augustin Coulomb and others that followed showed that the force does not change when the speed changes. It is Coulomb's law of friction. The confusion comes perhaps from the fact that the **air resistance** (which will be seen later) indeed increases with speed.

Some Exceptions

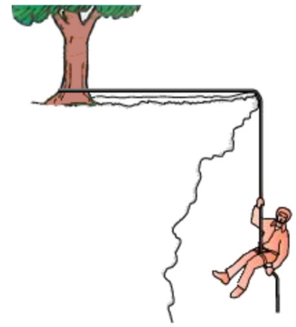
The formulas obtained previously are not applicable to every situation. First of all, the kinetic friction formula is an approximation. When an object slides, the friction force is not exactly constant. The kinetic friction formula gives only the average of the force.

The kinetic friction formula is not right when one of the objects in contact (or both objects in contact) deforms too much. When skiing with miniskis in soft snow, you are going more slowly than with long skis. However, the normal is the same, and the friction coefficient is the same (if both kinds of skis are made from the same material). The friction should be the same. With miniskis, the pressure on the ground is greater and, if the snow is too soft, you sink deeper in the snow than with long skis. When moving with short skis, you'll then create deep tracks by plowing through the surface of the snow. This destruction of the surface of the slope significantly increases the frictional force. Thus, when marks or tracks are formed on one of the surfaces, the friction may be much greater than predicted by the formula.

If the sliding velocity becomes too high, the heat generated by the friction can be so large that one of the surfaces begins to melt. A liquid substance between the two objects will now lubricate the sliding, and the friction will decrease. This is not what is happening with ice. It can sometimes be read that there is not much friction between the ice and a skate because the friction melts the ice, thereby forming a thin layer of water. The pressure exerted by the skates also is sometimes mentioned as a cause of the ice melting. Both statements are false. Actually, there is always a thin layer of water on the ice surface. The colder the ice is, the thinner this layer is. It only disappears at temperatures below -157 °C. (The layer is actually a mixture of water and small particles of ice, like slush.) It is this layer of water, always present on ice, which lubricates the sliding and makes ice so slippery.

Note on Rope Tensions

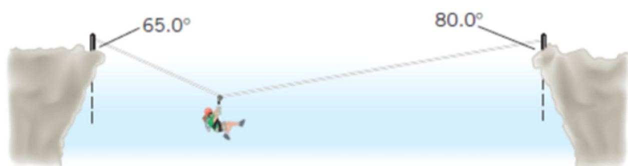
Friction can change the tension of a rope. This happens in the following situation.



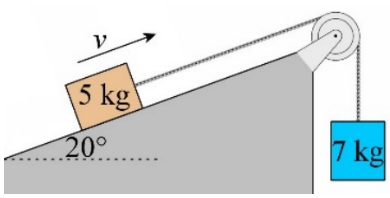
The rope tension will change at the point of contact with the ground at the edge of the cliff. Since the force of friction can exert a force in the direction of the rope, the tension of the rope may change. There is, therefore, a certain tension in the vertical part of the rope, and another tension in the horizontal part of the rope.

So, it is no longer true to say that the tension of a string or a rope is the same everywhere in the rope. Whenever the rope touches something and friction is taken into account, the tension can change.

For example, in the following situation, the tension of the rope is not the same in both sections of the rope. The force of friction made by the harness on the string changes the tension. (If there were no friction, the tension would be the same, but the person could not be in equilibrium at this position. He would slip to the middle of the rope). To solve this kind of problem, work as if there were two different ropes.



www.chegg.com/homework-help/questions-and-answers/mountain-climber-process-crossing-two-cliffs-rope-pauses-rest-weighs-565-n-drawing-shows-c-q23204469



However, the tension will not change if the rope passes over a pulley without mass, even if there is friction between the rope and pulley. (The pulley would not turn if there were no friction.)

Indeed, as the pulley has no mass, it takes no force to speed it up. This means that the force of friction between the rope and pulley is zero and that the tension of the rope does not change.

In Chapter 12, we will consider pulleys having masses. In this case, the tension of the string will not be the same before and after the pulley.

5.2 AIR RESISTANCE

Drag Force Formula

Measurements made on objects moving in fluids (gas and liquids) have shown that the friction force, called the drag force, is

Drag Force (F_d)

1) Magnitude of the force

$$F_d = \frac{1}{2} C_d A \rho v^2$$

2) Direction of the force

In the direction of the relative motion of the fluid with respect to the object.

3) Application point of the force

Surface of the object

ρ is the density of the fluid in which the object is travelling. In air, this density is around 1.29 kg/m³ at 0 °C and 101.3 kPa and around 1.20 kg/m³ at 20 °C and 101.3 kPa (it depends on the temperature and the pressure).

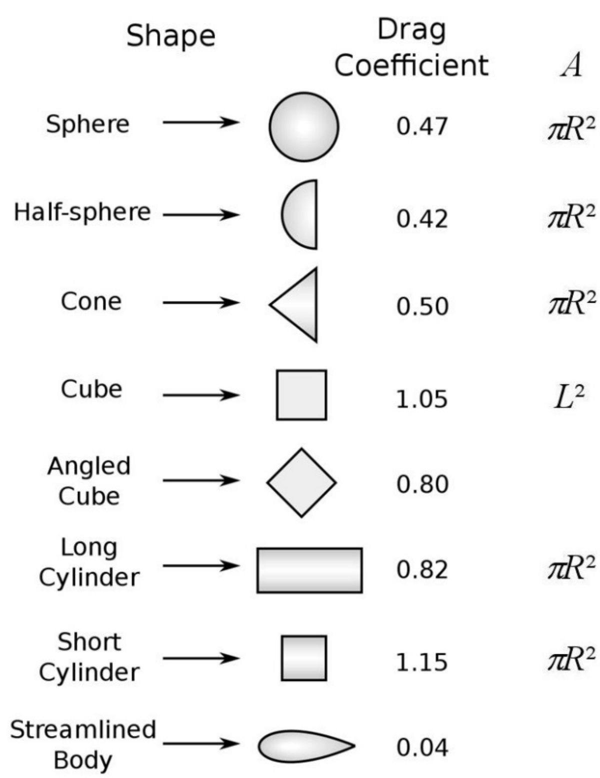
v is the velocity of the object in the fluid. Note that the drag increases rapidly with speed. Double the speed and drag force is four times greater. Christian Huygens discovered that the friction force made by fluids increases with v^2 in 1669.

C_d is the drag coefficient. Its value depends on the shape and orientation of the object relative to the fluid motion.

A is the reference area of the object whose definition varies according to the shape of the object. For simple shapes, it corresponds to the area of the section. This area is the area of the shadow if light comes from the same direction as the fluid. For example, with the sphere, a circular shadow having the same radius as the ball is obtained and the area is πR^2.

(The ½ is there and was not included in C_d because they wanted to have $\frac{1}{2}\rho v^2$, a quantity called *dynamic pressure* used in fluid mechanics.)

The following table provides the values of C_d and A for different forms.



en.wikipedia.org/wiki/Drag_coefficient

Note that the drag force does not come from the friction of the air molecules on the object or even from the collisions of the air molecules on the object. It actually comes from the friction between the layers of air going at different speeds near the object (this is the viscosity of the fluid) and the pressure differences between the front and the back of the object.

Drag is an important element in the design of cars as low drag means low fuel consumption. The value of C_dA must, therefore, be as small as possible. This table gives some values of C_dA for different car models.

Model	C_dA (m²)
2015 Audi A4 2.0 TDI	0.490
2015 BMW i8	0.548
2012 Tesla Model S P85	0.562
2014 Toyota Prius	0.576
2014 Chevrolet Volt	0.622
2016 Chrysler Pacifica	0.924
2019 RAM 1500	1.21
2003 Hummer H2	2.46

Several other values can be found on this site.
http://en.wikipedia.org/wiki/Automobile_drag_coefficient

Example 5.2.1

What is the drag force on a baseball (mass = 145 g and radius = 3.7 cm) travelling in air (density = 1.20 kg/m³) at 90 km/h?

We'll start by assuming that the ball is a simple sphere. In this case, the force is

$$\begin{aligned} F_d &= \frac{1}{2} C_d A \rho v^2 \\ &= \frac{1}{2} \cdot 0.47 \cdot \pi \cdot (0.037m)^2 \cdot 1.20 \tfrac{kg}{m^3} \cdot \left(25 \tfrac{m}{s}\right)^2 \\ &= 0.758N \end{aligned}$$

However, a baseball is not exactly a smooth sphere. The presence of stitches changes the value of C_d so it is rather C_d = 0.31. Therefore, the real drag force is

$$\begin{aligned} F_d &= \frac{1}{2} C_d A \rho v^2 \\ &= \frac{1}{2} \cdot 0.31 \cdot \pi \cdot (0.037m)^2 \cdot 1.20 \tfrac{kg}{m^3} \cdot \left(25 \tfrac{m}{s}\right)^2 \\ &= 0.500N \end{aligned}$$

Example 5.2.2

What is the drag force on a Hummer H2 2003 travelling in air (density = 1.20 kg/m³) at 160 km/h?

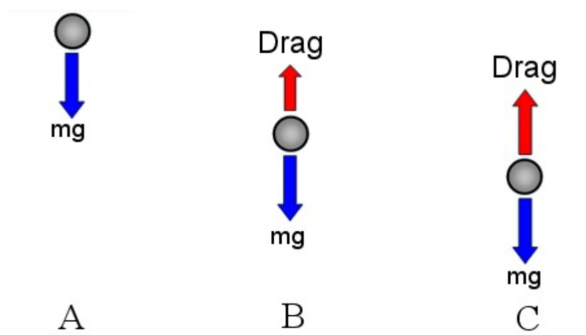
The drag force is

$$\begin{aligned} F_d &= \frac{1}{2}(C_d A)\rho v^2 \\ &= \frac{1}{2}\cdot 2.46m^2 \cdot 1.20\tfrac{kg}{m^3}\cdot\left(45\tfrac{m}{s}\right)^2 \\ &= 2989N \end{aligned}$$

This is the force required to lift 305 kg!

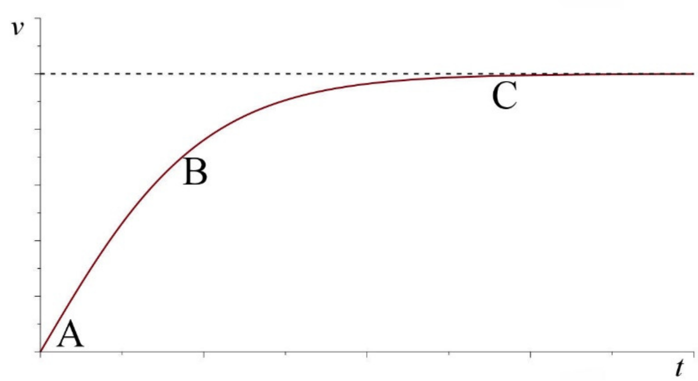
Incidentally, the value of C_d of a Formula 1 racecar is relatively large since it can be as high as 1.1 on some circuits. Knowing that $C_d = 0.57$ for a Hummer, this is indeed a very impressive value. This large value comes from the wings on the racecar. In the next chapter, it will be explained why it is necessary to have wings on a Formula 1 racecar.

Terminal Velocity



tap.iop.org/mechanics/drag/209/page_46353.html

When an object is free-falling from (starting from rest), it accelerates downwards because gravity exerts a downward force. Initially (A), gravity is the only force acting on the object, and its acceleration is 9.8 m/s². As the velocity of the object increase, the drag opposing its motion also increases. A little later (B) the drag force cancels half the weight and the acceleration is now reduced to 4.9 m/s². Later (C), the drag force has the same magnitude as the weight and the sum of the forces on the object is zero. The object is no longer accelerating and is moving at a constant speed. This speed is called the terminal velocity of the falling object.



The graph on the right shows the fall velocity versus time for an object that has a terminal velocity of 30 m/s.

Initially (point A), the slope is 9.8 m/s². The slope always diminishes thereafter, which means that the acceleration also decreases as time goes on. The slope (and the acceleration) gets closer and closer to 0 as the terminal velocity is approached. (For those who want to know, this function is a hyperbolic tangent $v = v_L \tanh(gt/v_L)$.)

As the terminal velocity is reached when the drag force is equal to the weight, this velocity is found with the equation

$$mg = \frac{1}{2} C_d A \rho v_t^2$$

The terminal velocity is found by solving this equation for the speed.

Terminal Velocity of a Free-Falling Object

$$v_t = \sqrt{\frac{2mg}{C_d A \rho}}$$

Example 5.2.3

What is the terminal velocity of a baseball (mass = 145 g and radius = 3.7 cm) in air (density = 1.20 kg/m³)?

The terminal velocity is

$$\begin{aligned} v_t &= \sqrt{\frac{2mg}{C_d A \rho}} \\ &= \sqrt{\frac{2 \cdot 0.145kg \cdot 9.8\frac{N}{kg}}{0.31 \cdot \pi (0.037m)^2 \cdot 1.20\frac{kg}{m^3}}} \\ &= 42.1\frac{m}{s} \end{aligned}$$

If an object is thrown downwards faster than the terminal velocity, then the drag force is greater than the weight and the object slows down until it reaches the terminal velocity.

An Object Falling in Air

Object falling down can now be reconsidered, this time taking into account air friction. In chapter 1, it was said that all objects, regardless of their mass, fall with the same acceleration. Taking into account the air friction, this is not true anymore.

To illustrate, suppose two spheres of the same size but with different masses (say a soccer ball and a metal ball of the same size) are dropped simultaneously from the same height. During the fall, the acceleration of the balls is (taking an axis pointing downwards)

$$ma = mg - \frac{1}{2} C_d A \rho v^2$$
$$a = g - \frac{C_d A \rho v^2}{2m}$$

Now, the acceleration depends on the mass of the object. The acceleration is smaller than g and the term that is subtracted from g is divided by the mass. As the subtracted term is smaller for the most massive object, the acceleration of the massive object is larger. This can be seen in this clip.
http://www.youtube.com/watch?v=8ytIANs7Nz8
Initially, the acceleration is virtually the same since the speed is small and the term subtracted from g is very small for both spheres. But as the speed increases, the friction term becomes more important and the term subtracted from g becomes more important for the soccer ball. As the acceleration of the soccer ball is smaller, the metal ball takes the lead. To see this difference, a pretty significant drop distance is needed to have speeds approaching the terminal velocity of the spheres.

However, the acceleration does not depend solely on the mass. It depends on several other factors like C_x and A. To see more clearly the effect of each, the acceleration can be written in the following form.

$$a = g - \frac{C_d A \rho v^2}{2m}$$
$$a = g - g\frac{C_d A \rho}{2mg} v^2$$
$$a = g - g\frac{v^2}{v_t^2}$$
$$a = g\left(1 - \frac{v^2}{v_t^2}\right)$$

The acceleration being always larger for the object having the higher terminal velocity, the object that has the highest terminal velocity will always reach the ground first if they are released at the same time from the same height. Thus, the shape, the size of the object and its mass are all factors that will determine which object will reach the ground first. If these factors are well adjusted to obtain the same terminal velocity, the two objects will reach the ground at the same time even if they have very different shapes and masses.
http://www.youtube.com/watch?v=Pgs3wv3VYy4
(In fact, in all the videos that show different objects arriving at the same time on the ground in which the fall distance is relatively large, they had to cheat a little and use objects having virtually identical terminal velocity. That's what I did when I dropped these pumpkins.
http://www.youtube.com/watch?v=gVAJcd4JXyE
As they are empty, the mass is almost proportional to A, so that the effect due to the mass and the effect due to the surface cancel each other to give the same terminal velocity to all pumpkins.)

It would even be possible to have a situation in which the object with the smallest mass reaches the ground first. If a baseball (m = 145 g, v_t = 42 m/s) and a basketball (m = 600 g, v_t = 20 m/s), are released simultaneously from the same height, the baseball will hit the ground first, even though its mass is smaller!
(Clip to come…)

Projectiles

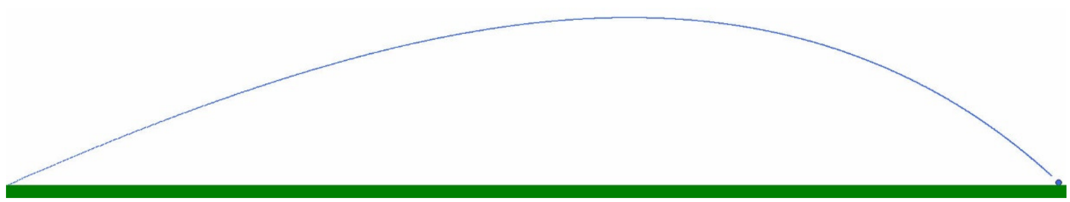
Drag can significantly change the conclusions reached in Chapter 2 concerning projectiles. However, it is tough to determine the exact form of the trajectory since there is no analytical solution (a formula) giving the position of the projectile as a function of time when there is drag. Still, it is possible to know what happens because computer simulations can be made. You can even download my own simulator.
http://www.college-merici.qc.ca/professeurs/luc-tremblay/Programmes.html

Here are some interesting conclusions:

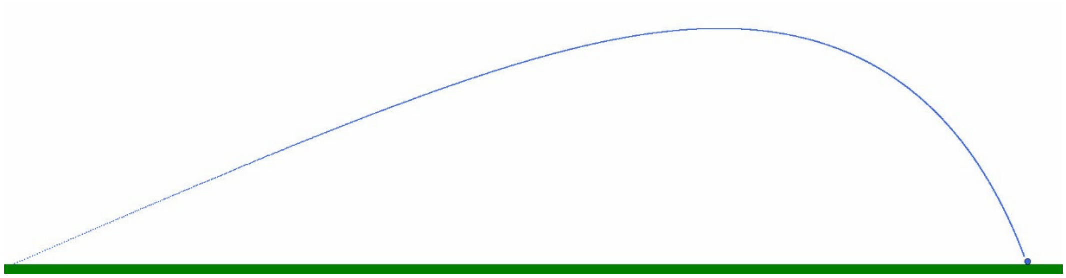
1- It is no longer true that the launch angle for the largest range is 45°. The angle becomes smaller in order to reduce the time during which the drag acts. For example, for an object having a terminal velocity of 42 m/s and launched at 70 m/s, the maximum range is obtained with a launch angle of approximately 37,6°.

2- The range decreases significantly, especially if the object is launched with a speed much higher than the terminal velocity. For example, for an object that has a terminal velocity of 42 m/s launched at 70 m/s with a launch angle of 40°, the range without drag is 492 m while it is only 186 m with drag.

3- The shape of the trajectory is not a parabola anymore. Here's the path of an object having a terminal velocity of 42 m/s launch with a speed of 70 m/s at 40°.

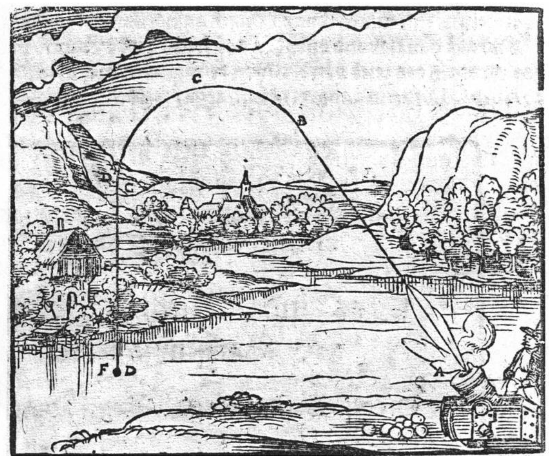


It can be noted that the end of the trajectory is more vertical than the beginning, which breaks the symmetry of the parabola. If the initial speed is increased, this effect is amplified. The following diagram shows the trajectory of the same projectile launched with a speed of 300 m/s with the same launch angle.



It now fell 434 m from the starting point (it would have been 9044 m without friction.)

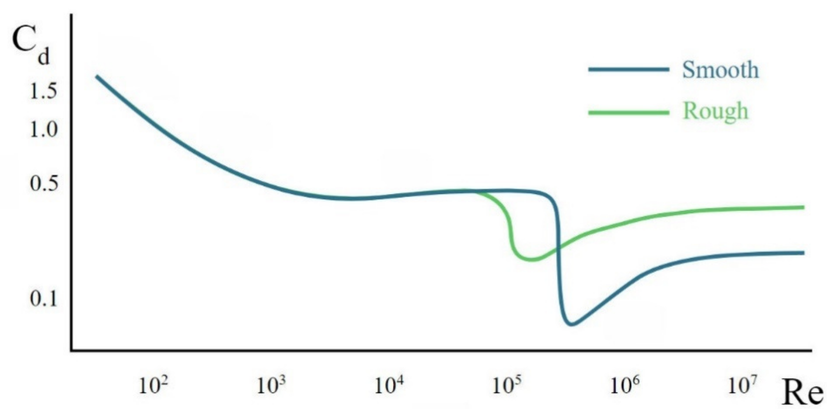
Note the striking similarity between this trajectory and the trajectory of the projectiles according to the physics before Galileo! Maybe they were not so crazy after all.



www.sciencedirect.com/science/article/pii/S0893608011000852

Real Drag Force

Actually, the calculation of the drag force is a little more complicated than what was shown. The force depends on the way the air flows around the object, which depends on the speed of the fluid relative to the object, the viscosity and the density of the fluid and the size of the object. For example, the following graph shows that the coefficient C_d is not always the same for a sphere.



commons.wikimedia.org/wiki/File:Drag_coefficient_on_a_sphere_vs._Reynolds_number_-_main_trends.svg

The value of C_d varies according to the Reynolds number that is given, for a sphere, by

$$\mathrm{Re} = \frac{\rho v D}{\mu}$$

where ρ is the density of the fluid, v is the speed of the sphere, D is the diameter of the sphere and μ is the viscosity of the fluid. It can be seen that the coefficient C_d of a sphere is near 0.5 for Reynolds numbers ranging from about 500 to 200,000. For a smooth sphere moving in air (whose viscosity is about 1.8 x 10^{-5} kg/ms), this, approximately, corresponds to speeds between 0.1 m/s and 40 m/s, a range of speeds that practically covers all applications made here (where C_d = 0.47 for the sphere was used). For a smooth sphere, the coefficient drops to 0.1 (when the Reynold number is 500,000). For a rough sphere, the coefficient drops to 0.15 (when the Reynold number is 100,000).

A simplified law, where C_d was considered constant, was, therefore, used here. However, this is justified since the coefficient is roughly constant for “normal” speeds.

5.3 FORCE MADE BY A SPRING

The force exerted by a spring is not constant; it depends on the compression or stretching of the spring. The law for the force exerted by a spring, discovered by Robert Hooke in 1660, is

Force Made by a Spring (F_{sp})

1) Magnitude of the force

$$F_{sp} = kx$$

where k is the spring constant
and x is the compression or stretching of the spring

2) Direction of the force

Towards the equilibrium position of the spring

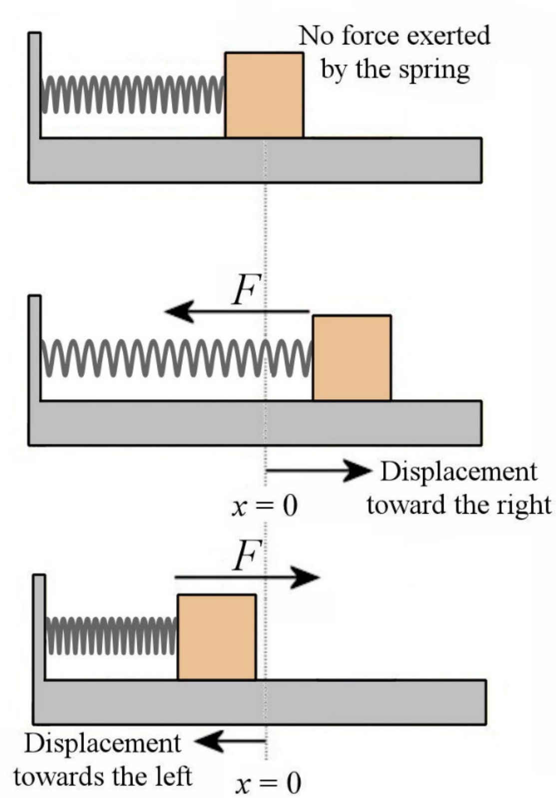
3) Application point of the force

At the point of attachment of the spring on the object

Each spring has an equilibrium length. At the equilibrium length, the spring is neither stretch nor compressed and does not exert any force. By definition, $x = 0$ is the position of an object when it is attached to a spring that does not exert any force. This position is the equilibrium position.

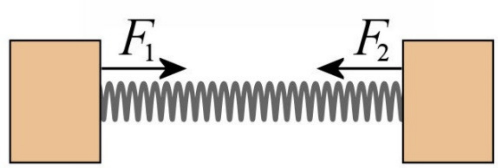
x is the value of the compression or stretching of the spring. This distance is always measured from the equilibrium position, i.e. the position of the object when the spring does not exert any force.

The spring constant k is in N/m, and it depends on the stiffness of the spring. If the spring is rather easy to compress or stretch, the constant is small, and if the spring is difficult to compress or stretch, the spring constant is large. If the spring constant is 1000 N/m, then a 1000 N force is needed to compress the spring 1 m or stretch it 1 m. 2000 N is needed to compress or stretch it 2 m.

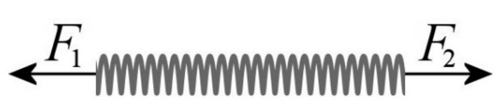


To understand the direction of the force, let's look at the diagram to the right where the object is moved away from the equilibrium position. If the object is moved to the right of the equilibrium position, the force made by the stretched spring is directed towards the left, thus towards the equilibrium position. If the object is moved to the left, the force made by the compressed spring is directed towards the right, thus towards the equilibrium position. It can be seen that the displacement and the force are always in the opposite direction. This is why the formula for the force made by a spring is often written as $F_{sp} = -kx$.

Note that the force exerted by a spring is the same on each side of the spring if the mass of the spring is neglected. To demonstrate this, suppose that a stretched spring exerts forces on each end.



If the spring exerts forces on each mass, then, according to Newton's 3rd law, the masses exert the same forces on the spring, but in opposite directions. Thus, the following forces act on the spring.



The sum of the forces on the spring gives

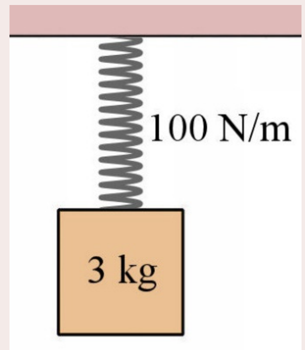
$$F_2 - F_1 = m_{ressort} a$$

If the mass of spring is neglected (by saying that $m_{spring} = 0$), the equation becomes

$$F_2 - F_1 = 0$$
$$F_2 = F_1$$

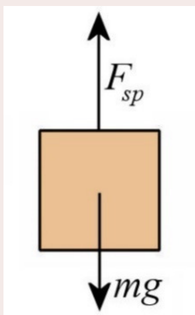
The force exerted by the spring is, therefore, the same at each end of the spring (but only the mass of the spring is neglected or if the acceleration of the spring is zero).

Example 5.3.1



A 3 kg mass is suspended from the ceiling using a spring having a 100 N/m constant. What is the stretching of the spring?

Forces acting on the Object



There are two forces acting on the mass.

1) The weight (29.4 N) directed downwards
2) The force made by the spring (F_{sp}) directed upwards.

It is rather obvious that the spring stretches in this situation and that the force exerted by the spring is directed upwards.

Sum of the Forces

With a y-axis directed upwards, the sum of the forces is

$$\sum F_y = -29.4N + F_{sp}$$

Newton's Second Law

Since there is no acceleration at the equilibrium point, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -29.4N + F_{sp} = 0$$

Solving the equation

Since $F_{sp} = kx$, the solution of this equation is

$$-29.4N + kx = 0$$
$$x = \frac{29.4N}{k}$$
$$x = \frac{29.4N}{100\frac{N}{m}}$$
$$x = 0.294m$$

5.4 PRESSURE FORCE

Pressure

When a fluid (liquid or gas) applies pressure against a wall, the force is

Pressure Force (F_p)

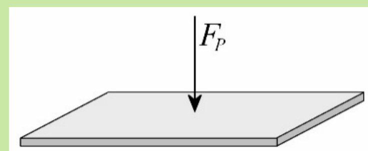
1) Magnitude of the force

$$F_p = PA$$

where P is the pressure in pascal (which are N/m²) and A is the area of the wall.

2) Direction of the force

Force acting on the wall, perpendicular to the surface

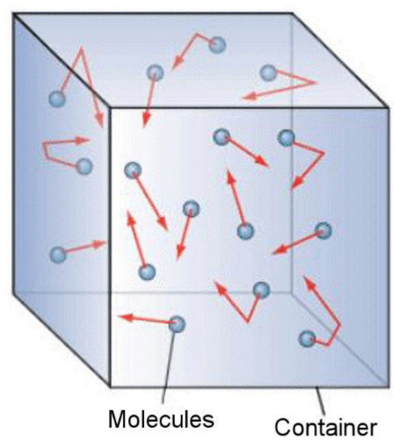


3) Application point of the force

Evenly distributed all over the surface of the object
(which is the same as to set the point of application at the centre of the surface).

This pressure force is produced by the multitude of collisions that the fluid molecules make with the walls of the container in which the fluid is located.

When the pressure is made by the air near the ground, there are billions and billions of collisions every second (about 10^{24}) per cm², and the molecules have a velocity of about 380 m/s.



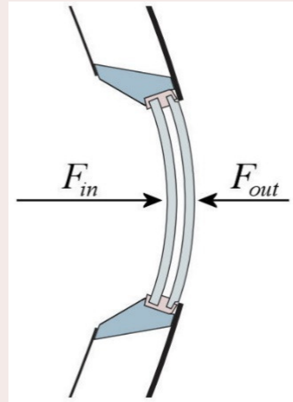
vacaero.com/information-resources/vac-aero-training/170466-the-fundamentals-of-vacuum-theory.html

Example 5.4.1

An airplane is flying at an altitude of 11,000 m. At this altitude, the atmospheric pressure is 22.7 kPa. The aircraft cabin is pressurized at 80 kPa. What is the net pressure force exerted on an aircraft window if the window is circular and has a diameter of 30 cm?

The air inside the cabin exerts an outwards force and the air outside the cabin exerts an inwards force. Net force is the difference between the two.

Pressure Force Made by the Air Outside the Aircraft



thepointsguy.com/news/what-are-airplane-windows-made-of/

The pressure force exerted by the outside air is

$$F_{out} = PA$$

To calculate this force, the area of the window is needed. Since it is a circle, the area is

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \cdot (0.15m)^2 \\ &= 0.070686m^2 \end{aligned}$$

Therefore, the force is

$$\begin{aligned} F_{out} &= 22\,700 Pa \cdot 0.070686m^2 \\ &= 1605N \end{aligned}$$

Pressure Force Made by the Air Inside the Aircraft

The pressure force exerted by the inside air is

$$\begin{aligned} F_{in} &= PA \\ &= 80\,000 Pa \cdot 0.07686m^2 \\ &= 5655N \end{aligned}$$

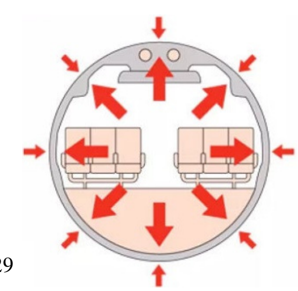
Net Force on the Window

We therefore have the outside air pushing on the window towards the interior of the aircraft with a force of 1605 N and the inside air pushing on the window towards the outside of the aircraft with a force of 5655 N. Therefore, the net force is directed outwards, and has a magnitude of

$$\begin{aligned} F_{net} &= 5655N - 1605N \\ &= 4050N \end{aligned}$$

This is a force equivalent to the weight of a 413 kg object! That's a lot for a small window with a 30 cm diameter.

This force is not only exerted on the windows. It actually acts everywhere on the surface of the cabin. With an external pressure of 22.7 kPa and an internal pressure of 75 kPa (the pressure is usually between 75 and 83 kPa), there is a net outward force of almost 5 N on every square centimetre of the cabin. The aircraft must be solidly built.

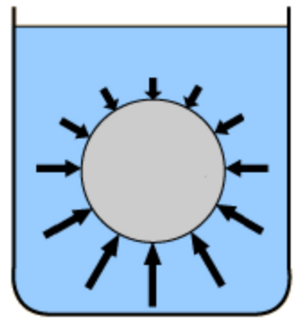

www.newsweek.com/boeings-737-airplane-prone-problems-63629

This force also makes it impossible to open the door of a pressurized aircraft in mid-flight at high altitude. The net force pushes on the door against its frame with an outwards force of about 100,000 N! (The door opens outwards, but there are extensions that unfold when closing so that the door sticks to the frame with pressure.) Thus, the 100,000 N force is a force that seeks to keep the door closed, not a force that seeks to open the door. But this also means that if you want to open the door, you're going to have to exert a force of at least 100,000 N to unstick the door to its frame. It's like trying to lift a 10-ton mass.

Buoyancy

When an object is in a fluid (liquid or gas), pressure forces are exerted on the object. However, the pressure is not the same everywhere as it decreases with altitude. This causes the pressure force to be greater on the bottom of the object than on the top of the object.

When all these forces are summed, a net force directed upwards is obtained. This is the buoyant force.



hyperphysics.phy-astr.gsu.edu/hbase/pbuoy.html

Buoyant Force (F_B)

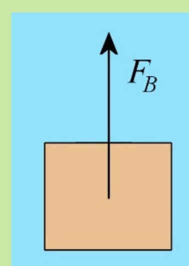
1) Magnitude of the force

$$F_B = \rho g V_f$$

where ρ is the density of the fluid
g is the gravitational acceleration
and V_f is the volume occupied by the object in the fluid.

2) Direction of the force

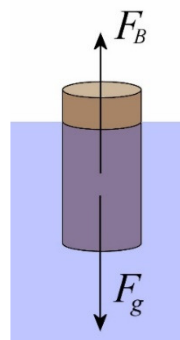
Upwards



3) Application point of the force

Distributed all over the surface of the part of the object that is in the fluid.
(Here, the point of application can be set at the centre of the part of the object that is in the fluid.)

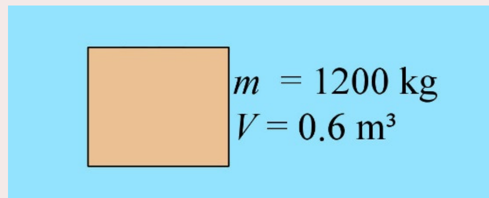
If the buoyant force is greater than the weight of the object, the net force on the object is directed upwards, and the object accelerates upwards until it reaches the surface. The object then floats. Once at the surface (diagram), a portion of the object comes out of the water and the buoyant force decreases. The submerged part of the object will, therefore, decrease until the buoyant force is equal to the force of gravity and the object is now in equilibrium at the surface.



If the gravitational force is greater than the buoyant force, the net force is directed downwards, and the object sinks to the bottom.

Example 5.4.2

A 1200 kg object is in water, whose density is 1000 kg/m³. The volume of the object is 0.6 m³.



a) What is the buoyant force exerted on this object?

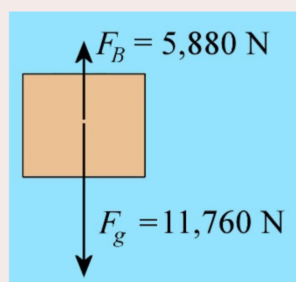
The force is

$$\begin{aligned} F_B &= \rho g V_f \\ &= 1000\tfrac{kg}{m^3}\cdot 9.8\tfrac{N}{kg}\cdot 0.6m^3 \\ &= 5880N \end{aligned}$$

b) Will this object float or sink?

The weight of this object is

$$\begin{aligned} F_g &= mg \\ &= 1200kg\cdot 9.8\tfrac{N}{kg} \\ &= 11,760N \end{aligned}$$



As the weight is greater than the buoyant force, this object sinks.

The buoyant force is the force that allows balloons or the Zeppelins to fly. The buoyant force is adjusted so that it is equal to the weight of the balloon. The balloon then remains in equilibrium in the air.

www.daysoftheyear.com/days/hot-air-balloon-day/

To achieve this, a gas lighter than air must be used. This gas can be simply hot air, which is less dense than the air in the atmosphere. Hydrogen or helium can also be used.

Example 5.4.3

The Hindenburg, a German zeppelin used in 1936 and 1937, had a mass of m = 235,000 kg (without hydrogen). How much hydrogen was to be used to fly it knowing that the air density is 1.3 kg/m³ and the hydrogen density is 0.09 kg/m³?

fr.wikipedia.org/wiki/LZ_129_Hindenburg

There are 2 forces on the zeppelin.

1) The weight ($(m + m_H)g$) directed downwards. (With hydrogen, the mass of the zeppelin and the mass of hydrogen must be added.)

2) The buoyant force (F_B) directed upwards.

Since there is equilibrium, these two forces must be equal. So, we have

$$F_B = (m + m_H)\, g$$

As $F_B = \rho g V_f$, we have

$$\rho g V_f = (m + m_H)\, g$$
$$\rho V_f = m + m_H$$

The volume of the zeppelin is equal to the mass of the hydrogen divided by its density (here, the volume of the parts of the Zeppelin that are not part of the "balloon" are neglected).

$$V_f = \frac{m_H}{\rho_H}$$

Therefore

$$\rho \frac{m_H}{\rho_H} = m + m_H$$

This leads to

$$\rho \frac{m_H}{\rho_H} - m_H = m$$
$$\left(\frac{\rho}{\rho_H} - 1 \right) m_H = m$$
$$m_H = \frac{m}{\frac{\rho}{\rho_H} - 1}$$

Therefore, the mass is

$$\begin{aligned} m_H &= \frac{235,000kg}{\frac{1.3}{0.09}-1} \\ &= 17,479kg \end{aligned}$$

This represents a volume of 194,214 m³ of hydrogen.

In the 30s, the United States, the only producer of helium in the world, refused to sell it to several countries, including Germany. The Germans, great zeppelin enthusiasts, were therefore forced to use hydrogen, a gas 4 times lighter than helium that can easily be obtained from water.

Obviously, the use of hydrogen is not without danger since this gas is very flammable. The Germans believed they had succeeded in using hydrogen safely since there had not been a major incident in nearly 3 decades. Until May 6, 1937...
https://www.youtube.com/watch?v=5Mcg0mynVXE

5.5 THE 4 FUNDAMENTAL FORCES

Although there is a multitude of types of force in nature, it was realized that all these forces can be explained from only four fundamental forces.

The Gravitational Force

The gravitational attraction between masses is one of the fundamental forces of nature. The characteristics of this force are already known.

The Electromagnetic Force

Objects with an electrical charge (positive or negative) can attract or repel each other depending on the type of charge. There is a repulsive force between objects having charges of the same sign and an attractive force between objects with charges of opposite signs. The electromagnetic force also includes the force made by the magnets. This force will be explored in the course “Electricity and Magnetism”.

The Strong Nuclear Force

With the discovery of the atomic nucleus, scientists had to find the force that is keeping the nucleus together. Since all the protons in the nucleus have a positive charge, there is an electrical repulsion between them that should make them fly apart from each other.

Neutrons, being uncharged, cannot attract them with an electric force to compensate. The gravitational force between the nucleons (a family of particles comprising protons and neutrons) is way too small to cancel the electrical repulsion (10^{36} times too small!). So it was assumed that there is an attractive force between nucleons which has been called the *nuclear force*. This attraction between nucleons, which is greater than the electrical repulsion in the nucleus, prevents the protons from flying away. Experiments have subsequently confirmed this existence of this force.

The Weak Nuclear Force

The study of radioactivity led to the discovery an effect that no force could explain. So it was assumed that there was a second force acting in the atomic nucleus, which they called the *weak nuclear force* (from this moment on, the force of attraction between the nucleons took the name of *"strong" nuclear force*). Once again, experiments have confirmed the existence of this force. Although this force does not form any structure, it is involved in the transformation of protons into neutrons or of protons into neutrons that may occur in a nucleus.

Practically All the Forces of Everyday Life Come from the Electric Force.

As the nuclear forces only act within the nucleus, they have virtually no impact on the forces acting around us. Gravity is the force that keeps us on Earth but makes no other force around us. The electric force is thus ultimately responsible for all the other forces that can be observed around us: normal forces, tension forces, friction, pressure forces, muscle strength and many others. It is, therefore, not surprising that the study of the electric force is so important.

SUMMARY OF EQUATIONS

Static Friction Force (F_f)

1) Magnitude of the force

A magnitude which is exactly what it must be to prevent the surface from sliding on each other.

Maximum value $F_{f\max} = \mu_s F_N$

2) Direction of the force

Parallel to the contact surface,
in the direction necessary to ensure that the surfaces do not slide on each other.

3) Point of application of the force

Point of contact between the surfaces

Kinetic Friction Force (F_f)

1) Magnitude of the force

$$F_f = \mu_c F_N$$

2) Direction of the force

Parallel to the contact surface,
in the direction of the relative motion of the other surface.

3) Application point of the force

Point of contact between the surfaces

Drag Force (F_d)

1) Magnitude of the force

$$F_d = \frac{1}{2} C_d A \rho v^2$$

2) Direction of the force

In the direction of the relative motion of the fluid in respect to the object.

3) Application point of the force

Surface of the object

Terminal Velocity of a Free-Falling Object

$$v_t = \sqrt{\frac{2mg}{C_x A \rho}}$$

Force Made by a Spring (F_{sp})

1) Magnitude of the force

$$F_{sp} = kx$$

2) Direction of the force

Towards the equilibrium position of the spring

3) Application point of the force

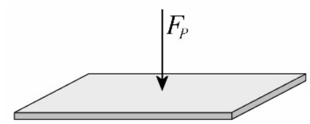
At the point of attachment of the spring on the object

Pressure Force (F_p)

1) Magnitude of the force

$$F_p = PA$$

2) Direction of the force
Force acting on the wall, perpendicular to the surface

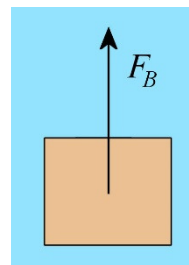


3) Application point of the force
Evenly distributed all over the surface of the object
(which is the same as to set the point of application at the centre of the surface).

Buoyant Force (F_B)

1) Magnitude of the force

$$F_B = \rho g V_f$$



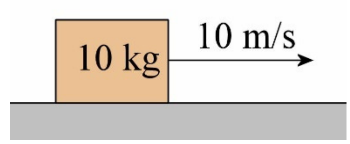
2) Direction of the force
Upwards

3) Application point of the force
Distributed all over the surface of the part of the object that is in the fluid.
(Here, the point of application can be set at the centre of the part of the object that is in the fluid.)

EXERCISES

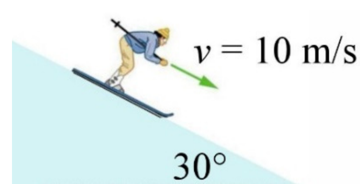
5.1 Friction Force between Two Surfaces

1. A 10 kg box is sliding at 10 m/s on a surface. If this box stops in 4 seconds, what is the kinetic coefficient of friction between the box and the ground?



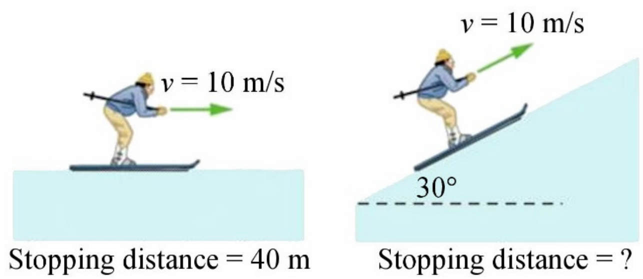
2. Guy skis downhill. The coefficient of kinetic friction between the skis and the snow is 0.1.

   a) How much time will it take for Guy to reach a speed of 50 m/s?
   b) How far will Guy travel in 5 seconds?



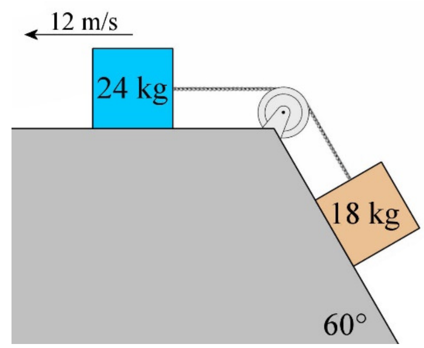
vhcc2.vhcc.edu/ph1fall9/frames_pages/openstax_problems.htm

3. When Manon is skiing on a flat surface, her stopping distance is 40 m if she has an initial speed of 10 m/s. What will the stopping distance be if Manon has an initial velocity of 10 m/s and is skiing uphill on a 30° slope?



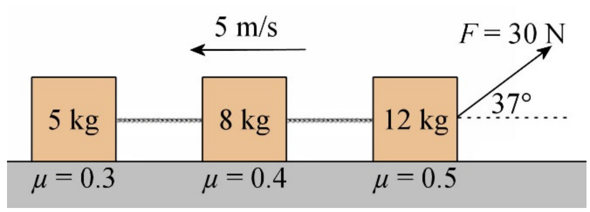
4. In the situation shown in the diagram, the kinetic friction coefficient between the blocks and the surface is 0.4.

   a) What is the acceleration of the blocks?
   b) What is the tension of the rope?



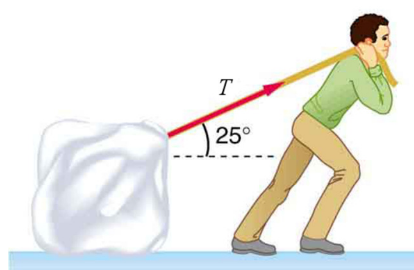
5. In the situation shown in the diagram (the coefficients shown in the diagram are the coefficients of kinetic friction),…

   a) what is the acceleration of this system?
   b) what are the tensions of the two ropes linking the blocks?



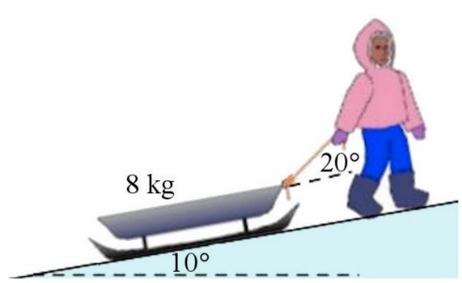
6. Ghislain is dragging a 100 kg block of ice with a rope, as shown in the diagram. The coefficient of kinetic friction between the block of ice and the ground is 0.1. What is the tension of the rope if the block moves at a constant speed?

cnx.org/content/m42139/latest/?collection=col11406/latest

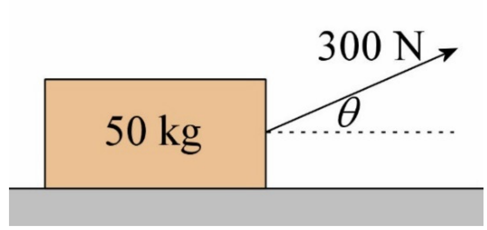


7. Little Catherine drags her 8 kg sled up a 10° slope at a constant speed. The coefficient of kinetic friction between the sled and slope is 0.12. What is the tension of the rope if the angle between the rope and the slope is 20 °?

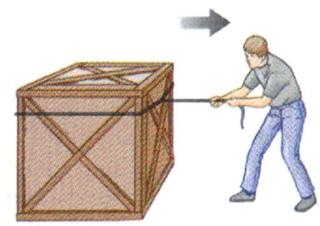
share.ehs.uen.org/node/8081



8. A 50 kg box is on the ground. The coefficient of friction between the ground and the box is 0.7. The crate is moved with a 300 N force making an angle θ with the horizontal. What should be the value of θ to obtain the largest possible acceleration? (Hint: find the value of acceleration assuming that angle is θ. Then, use the fact that at the maximum value of the acceleration, $da/d\theta = 0$.)

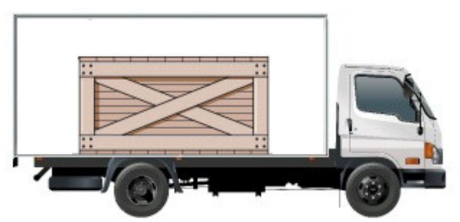


9. Hubert wants to move a 100 kg crate by pulling it with a rope. Will the crate be set in motion if he pulls on the rope with a 600 N force and the coefficient of static friction is 0.6?



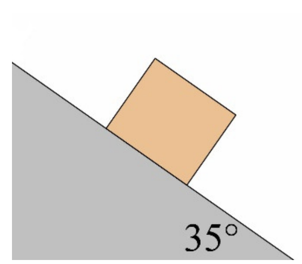
www.ux1.eiu.edu/~cfadd/1150/04Nwtn/frict.html

10. A box is in a truck. The coefficient of static friction between the box and the floor of the truck is 0.65. What is the maximum acceleration that the truck can have so that the box does not slip on the floor of the truck?

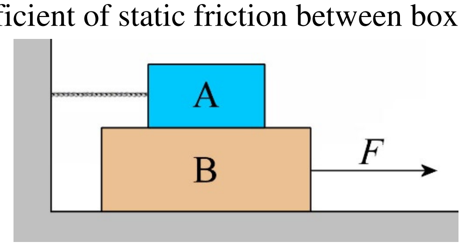


www.canstockphoto.com/delivery-cargo-truck-13682465.html

11. Will this block initially at rest slide down the slope if the coefficient of kinetic friction is 0.6 and the coefficient of static friction is 0.8?

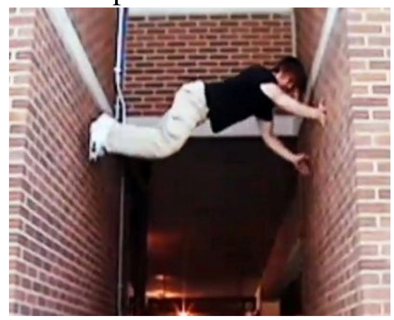


12. A 5 kg box (A) is placed on top of a 10 kg box (B). Box A is attached to the wall with a rope as shown in the diagram. The coefficient of static friction between box A and box B is 0.8 and the coefficient of static friction between the box B and the floor is 0.6. With what minimum force should the box B be pulled so that it moves?



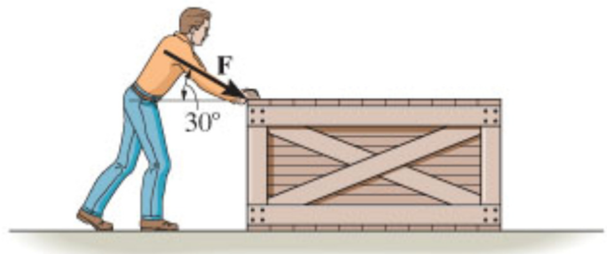
13. Donald, whose mass is 50 kg, remains in equilibrium in the position shown in the image. What should be the minimum normal forces exerted by Donald on each wall to keep him in this position if the coefficient of static friction between Donald and walls is 1.4?

(It is assumed here that frictional forces are the same on the hands and feet, which is not necessarily true in real life.)



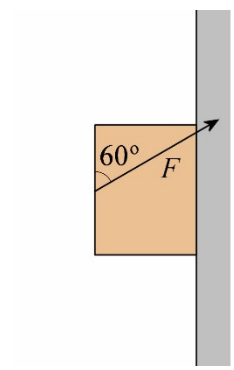
www.wired.com/wiredscience/2012/07/can-you-run-up-a-wall/

14. Boris pushes on a 100 kg box initially at rest with a force in the direction shown in the diagram. With what minimum force must he push on the box so that it begins to move if the coefficient of static friction is 0.5?



www.chegg.com/homework-help/questions-and-answers/man-pushes-the60-crate-force-force-alwaysdirected-30-horizontal-shown-magnitude-increased--q763391

15. A 1 kg block is held in place on a wall when a 30 N force is exerted on it in the direction shown in the diagram.



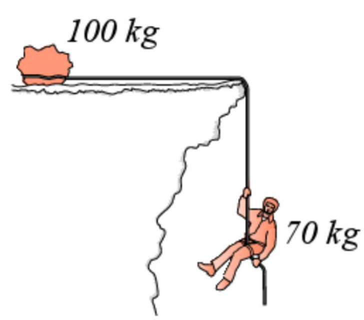
a) What is the friction force (magnitude and direction) acting on the block?

b) What is the minimum value that the coefficient of static friction must have for the block to remain in place?

16. Thierry, whose mass is 70 kg, finds himself in the situation shown in the diagram.

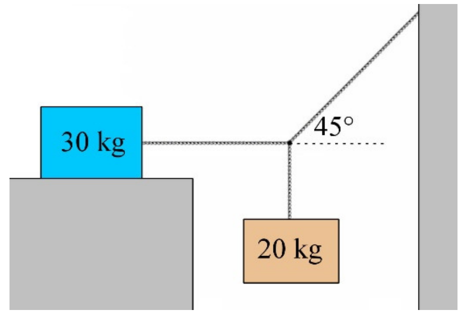
The coefficient of static friction between the stone and the ground is 0.5 while the coefficient of kinetic friction between the stone and the ground is 0.45. What is the force of friction between the stone and the ground if the stone is initially at rest?

(There is no friction between the ground and the rope on the edge of the cliff.)

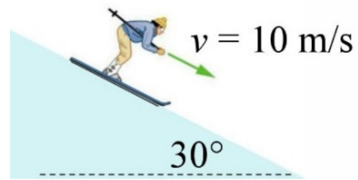


www.chegg.com/homework-help/questions-and-answers/75-climber-finds-dangling-edgeof-ice-cliff-shown-figure--fortunately-sroped-880-rock-locat-q780848

17. What is the minimum friction coefficient between the table surface and the 30 kg box if this system stays in equilibrium?



18. Gastonne skis downhill. Initially, her speed is 10 m/s. After sliding for a distance of 30 m, her speed is now 16 m/s. What is the coefficient of kinetic friction between her skis and the slope?



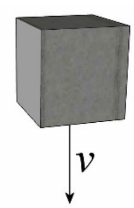
5.2 Drag Force

19. A soccer ball has a diameter of 22 cm. What is the drag force on the soccer ball if it travels at 20 m/s in air, whose density is 1.3 kg/m³?

20. What is the drag force on a Honda Civic moving at 120 km/h in air, whose density is 1.3 kg/m³? (The value $C_d A$ is 0.682 m² for this car.)

21. A soccer ball has a diameter of 22 cm and a weight of 440 g. What is the terminal velocity of the soccer ball in air, whose density is 1.3 kg/m³?

22. What is the terminal velocity of an iron cube falling in air with the orientation shown in the diagram? The air density is 1.3 kg/m³ and the cube has a volume of 0.01 m³ and a density of 7.32 g/cm³.



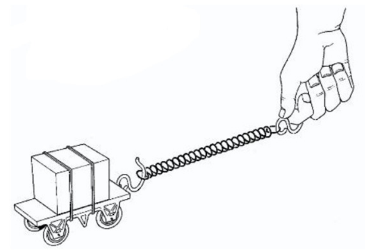
23. What is the terminal velocity (in km/h) of an 1142 kg Honda Civic 2001 rolling down a 10° slope if the drag force is the only force opposed to the motion of the car? (Use 1.3 kg/m³ for the density of air and 0.682 m² as $C_d A$ for the 2001 Honda Civic.)

24. Felix, whose mass is 100 kg, jumped from a balloon at an altitude of 2000 m. During the fall, Felix's terminal velocity is 30 m/s. What is the drag force acting on Felix when its speed is half the terminal velocity?

25. A Honda Civic having a mass of 1095 kg is parked. During the passage of a tornado, the wind (which was blowing towards the front of the car) has managed to move the car in the parking lot. Knowing that the coefficient of friction between the tires and the asphalt is 0.8, what was the minimum wind speed? (Use 1.3 kg/m³ for the density of air and 0.682 m² for $C_d A$ of the Honda Civic.)

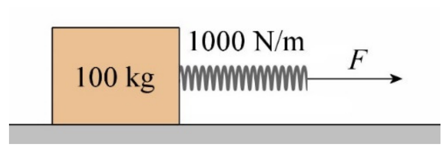
5.3 Force Made by Springs

26. A 2 kg cart is pulled by a spring having a constant of 200 N/m. What is the stretching of the spring if the acceleration of the cart is 4 m/s²?
(There is no friction.)



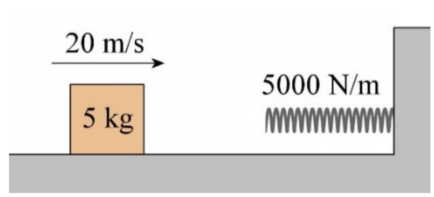
www.nuffieldfoundation.org/practical-physics/constant-and-varying-forces-between-trucks

27. A 100 kg box is pulled with a spring whose constant is 1000 N/m. The static coefficient of friction between the box and the floor is 0.5 and the kinetic coefficient of friction between the box and the floor is 0.4.

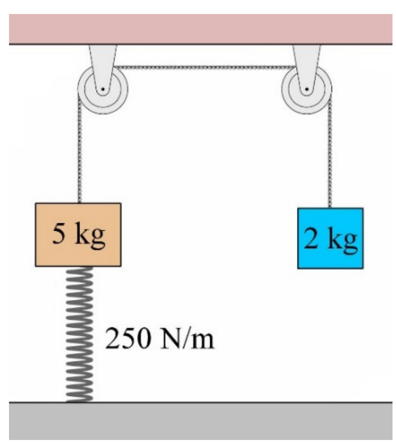


a) In this first case, the box is initially at rest. The force is then slowly increased until the box moves. What is the stretching of the spring when the box starts to move?
b) In this second case, the block is moved at a constant speed. What is the stretching of the spring?

28. A 5 kg mass arrives with a speed of 20 m/s on a spring as shown in this diagram. The coefficient of kinetic friction between the ground and the block is 0.6. What is the acceleration of the block when the spring is compressed 10 cm?

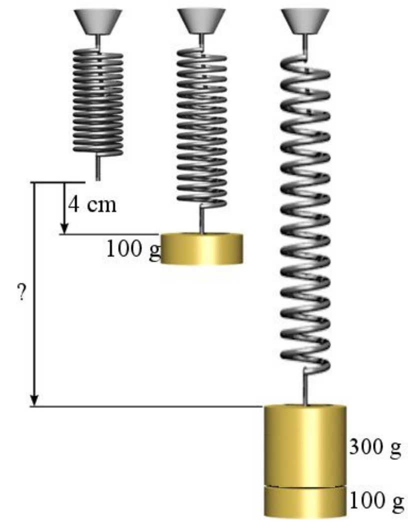


29. What is the compression or stretching of the spring at equilibrium in this situation? (There is no friction between the 5 kg block and the walls.)



30. When a 100 g mass is added at the end of a spring, it stretches 4 cm. What will the stretching of the spring be if an additional 300 g is added?

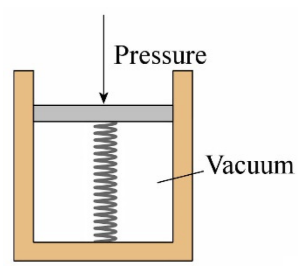
en.wikipedia.org/wiki/Hooke's_law



5.4 Pressure Force

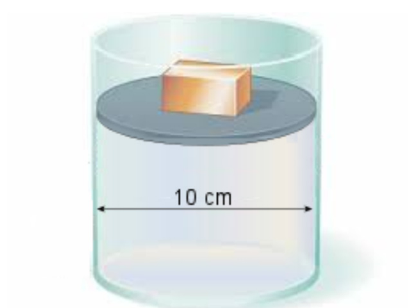
31. A gas is placed inside a sealed cubic metallic box whose edges have a length of 20 cm. What force is exerted on each side of the box by the gas if the pressure inside the box is 300 kPa?

32. A spring with a constant of 10,000 N/m is attached to the cover of a piston as shown in the diagram. The cover is a disk with a radius of 10 cm and a negligible mass. What will the compression of the spring be if there is a vacuum inside the cylinder and if the atmospheric pressure is 102 kPa?

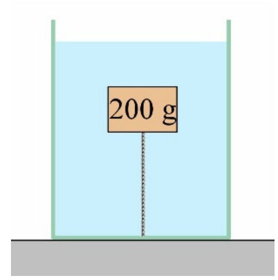


33. A 3 kg mass is placed on the cover of a piston. The mass of the cover is 400 g. The mass and the cover are held in position by the air pressure inside the piston. What is the pressure inside the piston if the atmospheric pressure is 101 kPa?

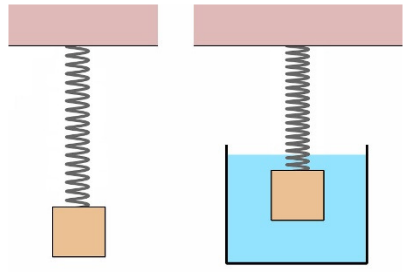
physics.unl.edu/~lnicholl/lectures/notes/lec33-notes.pdf



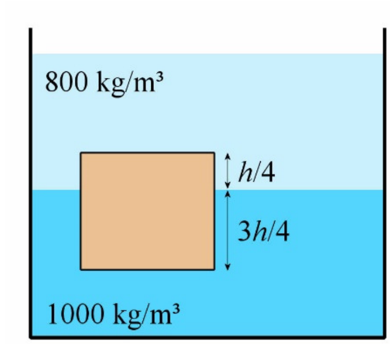
34. A 200 g block of cedar, whose density is 0.49 g/cm³, is attached at the bottom of a container filled with water as shown in the diagram. What is the tension of the rope if the density of water is 1 g/cm³?



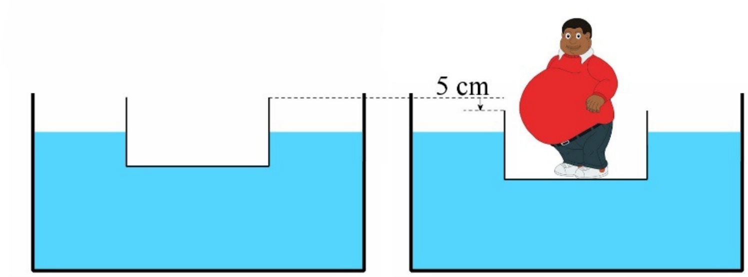
35. When a piece of aluminum (density = 2700 kg/m³) is fixed at the end of a spring with a constant of 200 N/m, the spring stretches 10 cm. What will the stretching of the spring be if the piece of aluminum is plunged into water (density = 1000 kg/m³)?



36. An object is in equilibrium between two fluids, as shown in the diagram. What is the density of the object?

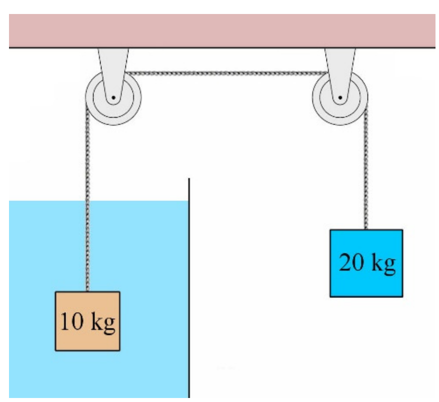


37. When Sylvain climbs aboard this "boat", it sinks 5 cm. What is Sylvain's mass knowing that the bottom of the boat has an area of 4 m²?



dk.pinterest.com/pin/507851295452117348/

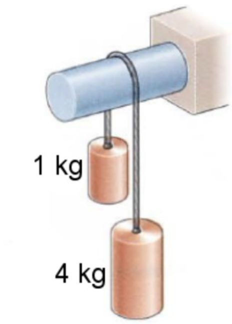
38. The 10 kg block is a cubic block with an edge of 10 cm. This block moves in the water (density = 1000 kg/m³). The 10 kg block accelerates upwards to reach a terminal velocity. What is this terminal velocity? (There is a terminal velocity because there is friction made by water on the 10 kg mass.)



Challenges

(Questions more difficult than the exam questions.)

39. In the situation shown in the diagram, the masses begin to slide on the pole if a small mass is added on the 4 kg mass. (In other words, the static friction between the rope and the post is at its maximum in the position shown in the diagram.) What is the static coefficient of friction between the rope and the post?

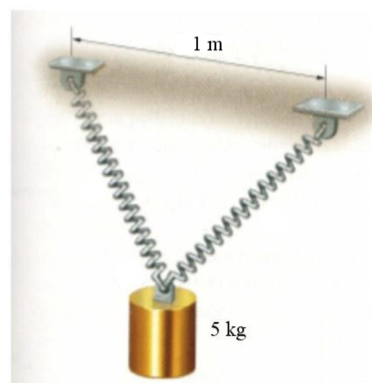


www.chegg.com/homework-help/questions-and-answers/-minimum-coefficient-friction-mu-rope-fixed-shaft-prevent-unbalanced-cylinders-moving-b--o-q4406714

40. In a spaceship far into space, Alexander throws a baseball (m = 145 g, r = 3.7 cm) at 30 m/s to hit a target 100 m away. As there is air in the spaceship, air friction will slow the ball down. The density of air in the spaceship is 1.3 kg/m³. How long will it take for the ball to reach the target and what will the speed of the ball be when it reaches the target?

41. Two identical springs are 50 cm long when they are neither stretched nor compressed. The spring constant is 98 N/m. What is the length of the springs when a 5 kg mass is fixed to the springs as shown in the diagram?

(Normally, a 4th-degree equation (quartic equation) is obtained. The solution of a quartic equation can be found with the site: http://www.wolframalpha.com/widgets/view.jsp?id=dcc8007e03af36a0bd3635b09e4cd5a2.)



www.chegg.com/homework-help/questions-and-answers/two-springs-identical-e-spring-constant-unstretched-length-04-m-50-kg-mass-suspended-b-len-q1487379

ANSWERS

5.1 Friction Force between Two Surfaces

1. 0.2551
2. a) 9.873 s b) 100.6 m
3. 8.358 m
4. a) 6.717 m/s² b) 67.135 N
5. a) 4.792 m/s² b) 16.232 N (between 12 kg and 8 kg) and 9.258 N (between 8 kg and 5 kg)
6. 103.3 N
7. 23.33 N

8. 35°
9. It moves
10. 6.37 m/s²
11. Stays at rest
12. 127.4 N
13. Both normal forces are 175 N
14. 795.4 N
15. a) 5.2 N downwards b) 0.2001
16. 441 N
17. 0.6667
18. 0.271

5.2 Drag Force

19. 4.645 N
20. 492.6 N
21. 19.27 m/s
22. 150.5 m/s
23. 238.4 km/h
24. 245 N
25. 139.2 m/s

5.3 Force Made by Springs

26. 4 cm
27. a) 49 cm b) 39.2 cm
28. 105.88 m/s²
29. 11.76 cm
30. 16 cm

5.4 Pressure Force

31. 12,000 N
32. 32.04 cm
33. 105.24 kPa
34. 2.04 N
35. 6.296 cm
36. 950 kg/m³
37. 200 kg
38. 4.531 m/s

Challenges

39. 0.4413
40. t = 5.425 s v = 12.12 m/s
41. 81.63 cm