Chapter 4 Solutions

1. a, b and c) The force is always

$$\begin{aligned} F_g &= m \cdot 9.8 \tfrac{N}{kg} \\ &= 100kg \cdot 9.8 \tfrac{N}{kg} \\ &= 980N \end{aligned}$$

2. *Forces Acting on the Object*

There are 2 forces acting on William.

1) The 705.6 N weight directed downwards.
2) A normal force (F_N) directed upwards exerted by the floor of the lift.

Sum of the Forces

The sum of the *y*-component of the forces is (with a *y*-axis directed upwards)

$$\sum F_y = -705.6N + F_N$$

Newton's Second Law

$$\sum F_y = ma_y \qquad \rightarrow \qquad -705.6N + F_N = 72kg \cdot a_y$$

Solving the Equations

a) If the speed is constant, the acceleration is zero and we have

$$\begin{aligned} -705.6N + F_N &= 0 \\ F_N &= 705.6N \end{aligned}$$

b) If the speed increases, the acceleration is in the same direction as the velocity. It is then directed upwards. Then

$$\begin{aligned} -705.6N + F_N &= ma_y \\ -705.6N + F_N &= 72kg \cdot 2 \tfrac{m}{s^2} \\ F_N &= 849.6N \end{aligned}$$

c) If the speed decreases, the acceleration is in the opposite direction to the velocity. It is then directed downwards. Then

$$-705.6N + F_N = ma_y$$
$$-705.6N + F_N = 72kg \cdot \left(-3\tfrac{m}{s^2}\right)$$
$$F_N = 489.6N$$

3. Let's look at one box at a time, beginning by the 5 kg box.

5 kg Box

Forces acting on the Object

There are 2 forces acting on the 5 kg box.

1) The 49 N weight directed downwards.
2) A normal force (F_{N1}) directed upwards exerted by the 10 kg block

Sum of the Forces

The sum of the *y*-component of the forces is (with a *y*-axis directed upwards)

$$\sum F_y = -49N + F_{N1}$$

Newton's Second Law

If the elevator slows down, then the acceleration is in the direction opposite to the velocity. It is then directed downwards.

$$\sum F_y = m_1 a_y$$
$$-49N + F_{N1} = 5kg \cdot \left(-1\tfrac{m}{s^2}\right)$$
$$-49N + F_{N1} = -5N$$

Solving the Equations

The normal force is therefore

$$-49N + F_{N1} = -5N$$
$$F_{N1} = 44N$$

Now, let's look at the force acting on the 10 kg box.

10 kg box

Forces Acting on the Object

There are 3 forces acting on the 10 kg box.

1) The 98 N weight directed downwards.
2) A normal force (F_{N1}) directed downwards exerted by the 5 kg block, which has the same magnitude has the normal force exerted by the 10 kg block on the 5 kg block.
3) A normal force (F_{N2}) directed upwards exerted by the floor.

Sum of the Forces

The sum of the *y*-component of the forces is (with a *y*-axis directed upwards)

$$\sum F_y = -49N - F_{N1} + F_{N2}$$

Newton's Second Law

If the elevator slows down, then the acceleration is in the opposite direction to the velocity. So, the acceleration id directed downwards. Thus,

$$\begin{gathered}\sum F_y = m_2 a_y \\ -49N - F_{N1} + F_{N2} = 10kg \cdot \left(-1\tfrac{m}{s^2}\right) \\ -49N - F_{N1} + F_{N2} = -10N\end{gathered}$$

Solving the Equations

As F_{N1} = 44 N, we have

$$\begin{gathered}-98N - 44N + F_{N2} = -10N \\ F_{N2} = 132N\end{gathered}$$

4. *Forces Acting on the Object*

There are 2 forces acting on the 300 kg part.

1) The 2940 N weight directed downwards.
2) The tension force (T) directed upwards.

Sum of the Forces

The sum of the y-component of the forces is (with a y-axis directed upwards)

$$\sum F_y = T - 2940N$$

Newton's Second Law

$$\sum F_y = ma_y \qquad \rightarrow \qquad T - 2940N = 300kg \cdot a_y$$

Solving the Equations

a) If there is no acceleration, then

$$\begin{gathered} T - 2940N = 300kg \cdot a_y \\ T - 2940N = 0 \\ T = 2940N \end{gathered}$$

b) If the acceleration is 3 m/s² upwards, then

$$\begin{gathered} T - 2940N = 300kg \cdot a_y \\ T - 2940N = 300kg \cdot 3\tfrac{m}{s^2} \\ T = 3840N \end{gathered}$$

c) If the acceleration is 2 m/s² downwards, then

$$\begin{gathered} T - 2940N = 300kg \cdot a_y \\ T - 2940N = 300kg \cdot \left(-2\tfrac{m}{s^2}\right) \\ T = 2340N \end{gathered}$$

5. Let's look at one box at a time, beginning by the 10 kg box.

10 kg Box

Forces acting on the Object

There are 2 forces acting on the 10 kg box.

1) The 98 N weight directed downwards.
2) The tension force (T_2) directed upwards.

Sum of the Forces

The sum of the y-component of the forces is (with a y-axis directed upwards)

$$\sum F_y = -98N + T_2$$

Newton's Second Law

As the velocity is in the direction of the y-axis or in the direction opposed to the y-axis, the x-component of the acceleration is 0 and the y-component of the acceleration is a. Thus

$$a_x = 0 \qquad\qquad a_y = a$$

Therefore, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -98N + T_2 = 10kg \cdot a$$

6 kg box

Forces Acting on the Object

There are 3 forces acting on the 6 kg box.

1) The 58.8 N weight directed downwards.
2) The tension force (T_1) directed downwards.
3) The tension force (T_2) directed upwards.

Sum of the Forces

The sum of the y-component of the forces is (with a y-axis directed upwards)

$$\sum F_y = -58.8N + T_1 - T_2$$

Newton's Second Law

As the velocity is in the direction of the y-axis or in the direction opposed to the y-axis, the x-component of the acceleration is 0 and the y-component of the acceleration is a. Thus

$$a_x = 0 \qquad\qquad a_y = a$$

Therefore, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -58,8N + T_1 - T_2 = 6kg \cdot a$$

Solving the Equations

a) If the acceleration is 2.4 m/s² downwards, the two equations become

$$-98N + T_2 = 10kg \cdot \left(-2.4\tfrac{m}{s^2}\right)$$
$$-58.8N + T_1 - T_2 = 6kg \cdot \left(-2.4\tfrac{m}{s^2}\right)$$

The first equation gives T_2 = 74 N. By substituting this value in the second equation, we get T_1 = 118.4 N.

b) The peak acceleration is found by setting each of the strings at its maximum tension and find the accelerations. The lower of the two maximum accelerations will be the limit.

If T_1 is set at its maximal tension of 200 N, we have

$$-98N + T_2 = 10kg \cdot a_{\max}$$
$$-58.8N + 200N - T_2 = 6kg \cdot a_{\max}$$

Summing these equations, we get

$$\left(-98N + T_2\right) + \left(-58.8N + 200N - T_2\right) = 10kg \cdot a_{\max} + 6kg \cdot a_{\max}$$
$$43.2N = 16kg \cdot a_{\max}$$
$$a_{\max} = 2.7\tfrac{m}{s^2}$$

If T_2 is set at its maximal tension of 200 N, we have

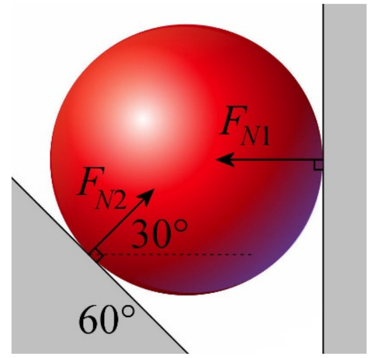
$$-98N + 200N = 10kg \cdot a_{\max}$$
$$-58.8N + T_1 - 200N = 6kg \cdot a_{\max}$$

With the first equation, we have

$$-98N + 200N = 10kg \cdot a_{\max}$$
$$a_{\max} = 10.2\tfrac{m}{s^2}$$

Therefore, rope 1 sets the limit to the acceleration at 2.7 m/s².

6. *Forces Acting on the Object*



There are 3 forces acting on the ball.

1) The 3.92 N weight directed downwards.
2) A normal force (F_{N1}) directed towards the left exerted by the wall.
3) A normal force (F_{N2}) directed at 30° exerted by the inclined.

Sum of the Forces

The table of force is (with an x-axis towards the right towards the right and a y-axis directed upwards)

Forces	x	y
Weight	0	-3.92N
Normal Force 1	$-F_{N1}$	0
Normal Force 2	F_{N2} cos 30°	F_{N2} sin 30°

Thus, the sums of the forces are

$$\sum F_x = -F_{N1} + F_{N2} \cos 30°$$
$$\sum F_y = -3.92N + F_{N2} \sin 30°$$

Newton's Second Law

Since the components of the acceleration are zero, we have

$$\sum F_x = ma_x \quad \rightarrow \quad -F_{N1} + F_{N2} \cos 30° = 0$$
$$\sum F_y = ma_y \quad \rightarrow \quad -3.92N + F_{N2} \sin 30° = 0$$

Solving the Equations

From the sum of the y-component of the forces, we have

$$-3.92N + F_{N2} \sin 30° = 0$$
$$F_{N2} = 7.84N$$

This answer is then substituted in the sum of the x-component of the forces to obtain

$$-F_{N1} + 7.84N \cdot \cos 30° = 0$$
$$F_{N1} = 6.79N$$

7. As the normal force acts on the 12 kg block, we will look at the forces acting on this block.

12 kg Block

Forces Acting on the Object

There are 3 forces acting on the 12 kg box.

1) The 117.6 N weight directed downwards.
2) The tension force (T) directed upwards.
3) A normal force (F_N) directed upwards exerted by the ground.

Sum of the Forces

The sum of the y-component of the forces is (with a y-axis directed upwards)

$$\sum F_y = T - 117.6N + F_N$$

Newton's Second Law

Since there is no acceleration, we have

$$\sum F_y = ma_y \qquad \rightarrow \qquad T - 117.6N + F_N = 0$$

To solve, the tension force is needed. It can be found by examining what is happening at the other end of the rope, so by examining the forces acting on the 4 kg block.

4 kg Block

Forces Acting on the Object

There are 2 forces acting on the 4 kg box.

1) The 39.2 N weight directed downwards.
2) The tension force (T) directed upwards.

Sum of the Forces

The sum of the y-component of the forces is (with a y-axis directed upwards)

$$\sum F_y = T - 39.2N$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad T - 39.2N = 0$$

Solving the equation

The tension force can be obtained with the 4 kg-block equation

$$T - 39.2N = 0$$
$$T = 39.2N$$

Then, this result can be used to calculated the normal force acting on the 12 kg block.

$$T - 117.6N + F_N = 0$$
$$39.2N - 117.6N + F_N = 0$$
$$F_N = 78.4N$$

8. To find the normal force acting on the 12 kg and 20 kg blocs, the sums of the force acting on each block must be calculated.

12 kg Block

Forces Acting on the Object

There are 3 forces acting on the 12 kg box.

1) The 117.6 N weight directed downwards.
2) The tension force (T) directed upwards.
3) A normal force (F_{N1}) directed upwards exerted by 20 kg box.

Sum of the Forces

The sum of the y-component of the forces is (with a y-axis directed upwards)

$$\sum F_y = T - 117.6N + F_{N1}$$

Newton's Second Law

Since there no acceleration, we obtain

$$\sum F_y = ma_y \qquad \rightarrow \qquad T - 117{,}6N + F_{N1} = 0$$

20 kg Block

Forces Acting on the Object

There are 3 forces acting on the 20 kg box.

1) The 196 N weight directed downwards.
2) A normal force (F_{N1}) directed downwards exerted by the 12 kg block, which has the same magnitude has the normal force exerted by the 20 kg block on the 12 kg block.
3) A normal force (F_{N2}) directed upwards exerted by the ground.

Sum of the Forces

The sum of the y-component of the forces is (with a y-axis directed upwards)

$$\sum F_y = F_{N2} - 196N - F_{N1}$$

Newton's Second Law

Since there no acceleration, we obtain

$$\sum F_y = ma_y \qquad \rightarrow \qquad F_{N2} - 196N - F_{N1} = 0$$

Equations Obtained

The equations obtained are

$$T - 117.6N + F_{N1} = 0$$
$$F_{N2} - 196N - F_{N1} = 0$$

We see that to solve, we need the tension. It can be found by examining what is happening at the other end of the rope, so by examining the forces on the 4 kg block.

4 kg Block

Forces Acting on the Object

There are 2 forces acting on the 4 kg box.

1) The 39.2 N weight directed downwards.

2) The tension force (T) directed upwards.

Sum of the Forces

The sum of the y-component of the forces is (with a y-axis directed upwards)

$$\sum F_y = T - 39.2N$$

Newton's Second Law

Since there no acceleration, we obtain

$$\sum F_y = ma_y \qquad \rightarrow \qquad T - 39,2N = 0$$

Solving the Equations

The tension can be found with the force equation on the 4 kg block.

$$T - 39.2N = 0$$
$$T = 39.2N$$

With the tension, the normal between the 12 kg and 20 kg blocks can be calculated.

$$T - 117.6N + F_{N1} = 0$$
$$39.2N - 117.6N + F_{N1} = 0$$
$$F_{N1} = 78.4N$$

With this normal force, normal exerted by the ground can be found.

$$F_{N2} - 196N - F_{N1} = 0$$
$$F_{N2} - 196N - 78.4N = 0$$
$$F_{N2} = 274.4N$$

9. *Forces acting on the Object*

There are 4 forces acting on the snowball.

1) The 392 N weight directed downwards.
2) A normal force (F_N) directed upwards exerted by the ground.
3) The 100 N force exerted by Gontran.
4) The 75 N force exerted by Philemon.

Sum of the Forces

The table of force is (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

Forces	*x*	*y*
Weight	0	-392 N
Normal force	0	F_N
Gontran	100 N cos (-25°)	100 N sin (-25°)
Philemon	75 N cos 30°	75 N sin 30°

The sums of the forces are then

$$\sum F_x = 100N \cdot \cos(-25°) + 75N \cdot \cos 30°$$
$$\sum F_y = -392N + F_N + 100N \cdot \sin(-25°) + 75N \cdot \sin 30°$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis or in the direction opposed to the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \quad \rightarrow \quad 100N \cdot \cos(-25°) + 75N \cdot \cos 30° = 40kg \cdot a$$
$$\sum F_y = ma_y \quad \rightarrow \quad -392N + F_N + 100N \cdot \sin(-25°) + 75N \cdot \sin 30° = 0$$

Solving the Equations

The acceleration can be found with the sum of the *x*-component of the forces.

$$100N \cdot \cos(-25°) + 75N \cdot \cos 30° = 40kg \cdot a_x$$
$$a_x = 3.89 \tfrac{m}{s^2}$$

The normal force can be found with the sum of the *y*-component of the forces.

$$-392N + F_N + 100N \cdot \sin(-25°) + 75N \cdot \sin 30° = 0$$
$$F_N = 396.76N$$

10. *Forces acting on the Object*

There are 3 forces acting on Irina.

1) The 588 N weight directed downwards.
2) A normal force (F_N) exerted by the cliff.
3) The tension force (T).

Sum of the Forces

The table of force is (with an x-axis towards the right towards the right and a y-axis directed upwards)

Forces	x	y
Weight	0	-588 N
Normal force	F_N cos 15°	F_N sin 15°
Tension	T cos 121°	T sin 121°

The sums of forces are

$$\sum F_x = F_N \cos 15° + T \cos 121°$$
$$\sum F_y = -588N + F_N \sin 15° + T \sin 121°$$

Newton's Second Law

Since there is no acceleration, the equations of forces are

$$\sum F_x = ma_x$$
$$\rightarrow F_N \cos(15°) + T \cos(121°) = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -588N + F_N \sin(15°) + T \sin(121°) = 0$$

Solving the Equations

We have two equations and two unknowns. To solve, we will solve for the normal force in the first equation

$$F_N = \frac{-T \cos(121°)}{\cos(15°)}$$

And substitute this value in the second equation.

$$-588N + F_N \sin(15°) + T\sin(121°) = 0$$
$$-588N + \frac{-T\cos(121°)}{\cos(15°)}\sin(15°) + T\sin(121°) = 0$$
$$-588N + T\cdot\left(\frac{-\cos(121°)}{\cos(15°)}\sin(15°) + \sin(121°)\right) = 0$$
$$-588N + T\cdot(0.9952) = 0$$
$$T = 590.85N$$

With this tension force, the normal force can now be found.

$$\begin{aligned} F_N &= \frac{-T\cos(121°)}{\cos(15°)} \\ &= \frac{-590.85N\cdot\cos(121°)}{\cos(15°)} \\ &= 315.05N \end{aligned}$$

11. *Forces acting on the Object*

There are 3 forces acting on Indiana.

1) The 637 N weight directed downwards.
2) The tension force (*T*) directed at 5°.
3) The tension force (*T*) directed at 175°.

Sum of the Forces

The table of force is (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

Forces	***x***	***y***
Weight	0	-637 N
Tension 5°	$T\cos 5°$	$T\sin 5°$
Tension 175°	$T\cos 175°$	$T\sin 175°$

The sums of forces are

$$\sum F_x = T\cos 5° + T\cos 175°$$
$$\sum F_y = -637N + T\sin 5° + T\sin 175°$$

Newton's Second Law

Since there is no acceleration, the equations of forces are

$$\sum F_x = ma_x$$
$$\rightarrow T\cos 5° + T\cos 175° = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -637N + T\sin 5° + T\sin 175° = 0$$

Solving the Equations

The first equation gives no information. The sum is always zero, regardless of the value of *T*, since cos (5°) = - cos (175°). However, the tension can be found with the second equation.

$$-637N + T\sin(5°) + T\sin(175°) = 0$$
$$-637N + T\left(\sin(5°) + \sin(175°)\right) = 0$$
$$T = 3654N$$

12. *Forces acting on the Object*

There are 3 forces acting on the box.

1) The 392 N weight directed downwards.
2) The tension force (T_1) exerted by the string on the right.
3) The tension force (T_2) exerted by the string on the left.

Sum of the Forces

The table of force is (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

Forces	***x***	***y***
Weight	0	-392 N
Tension 1	T_1 cos 20°	T_1 sin 20°
Tension 2	T_2 cos 120°	T_2 sin 120°

The sums of forces are

$$\sum F_x = T_1\cos 20° + T_2\cos 120°$$
$$\sum F_y = -392N + T_1\sin 20° + T_2\sin 120°$$

Newton's Second Law

Since there is no acceleration, the equations of forces are

$$\begin{aligned}
&\sum F_x = ma_x \\
&\qquad \to T_1 \cos 20° + T_2 \cos 120° = 0 \\
&\sum F_y = ma_y \\
&\qquad \to -392N + T_1 \sin 20° + T_2 \sin 120° = 0
\end{aligned}$$

Solving the Equations

We have two equations and two unknowns. To obtain a solution, we will solve for T_2 in the first equation.

$$T_2 = -\frac{T_1 \cos 20°}{\cos 120°}$$

And substitute the value in the second equation.

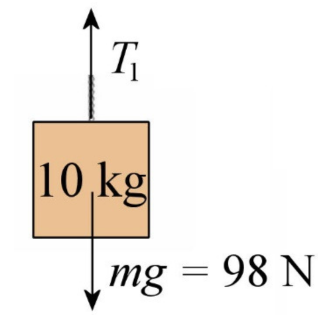
$$\begin{gathered}
-392N + T_1 \sin 20° + T_2 \sin 120° = 0 \\
-392N + T_1 \sin 20° + -\frac{T_1 \cos 20°}{\cos 120°} \cdot \sin 120° = 0 \\
-392N + T_1 \cdot \left(\sin 20° - \frac{\cos 20°}{\cos 120°} \cdot \sin 120° \right) = 0 \\
-392N + T_1 \cdot (1.9696) = 0 \\
T_1 = 199.02N
\end{gathered}$$

From this tension, the other tension can be found.

$$\begin{aligned}
T_2 &= -\frac{T_1 \cos 20°}{\cos 120°} \\
&= -\frac{199.02N \cdot \cos 20°}{\cos 120°} \\
&= 374.04N
\end{aligned}$$

13. Let's start by finding the tension of the string that supports the box.

Forces Acting on the Object



There are 2 forces acting on the 10 kg box.

1) The 98 N weight directed downwards.
2) The tension force (T_1) directed upwards.

Sum of the Forces

The sum of the *y*-component of the forces is (with a *y*-axis directed upwards)

$$\sum F_y = T_1 - 98N$$

Newton's Second Law

Since there is no acceleration, the equations of forces are

$$\sum F_y = ma_y \qquad \rightarrow \qquad T_1 - 98N = 0$$

Solving the Equations

A first tension can now be obtained.

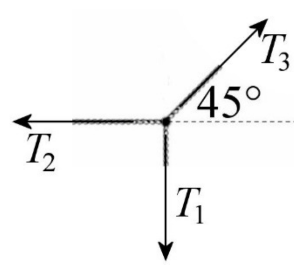
$$T_1 - 98N = 0$$
$$T_1 = 98N$$

Now let's look at the forces on the node that connects the three strings.

Forces Acting on the Object

There are 3 forces acting on the knot.

1) The 98 N tension force (T_1) directed downwards.
2) The tension force (T_2) directed towards the left exerted by the string on the left.
3) The tension force (T_3) directed towards the left at 45° exerted by the string on the right.



Sum of the Forces

The table of force is (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

Forces	*x*	*y*
Tension 1	0	-98 N
Tension 2	$-T_2$	0
Tension 3	$T_3 \cos 45°$	$T_3 \sin 45°$

The sums of forces are

$$\sum F_x = -T_2 + T_3 \cos 45°$$
$$\sum F_y = -98N + T_3 \sin 45°$$

Newton's Second Law

Since there is no acceleration, the equations of forces are

$$\sum F_x = ma_x$$
$$\rightarrow -T_2 + T_3 \cos 45° = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -98N + T_3 \sin 45° = 0$$

Solving the Equations

With the sum of the *y*-component of the forces, we find

$$-98N + T_3 \sin 45° = 0$$
$$T_3 = 138.59N$$

This value is then used in the sum of the *x*-component of the forces to find T_2.

$$-T_2 + T_3 \cos 45° = 0$$
$$-T_2 + 138.59N \cdot \cos 45° = 0$$
$$T_2 = 98N$$

14. *Forces acting on the Object*

There are 3 forces acting on the 10 kg block.

1) The 98 N weight directed downwards.
2) The tension force exerted by the string (T).
3) The force (F).

Sum of the Forces

The table of force is (with an x-axis towards the right towards the right and a y-axis directed upwards)

Forces	$\boldsymbol{x}$	$\boldsymbol{y}$
Weight	0	-98 N
Tension	T cos 60°	T sin 60°
Force *F*	-F	0

The sums of the forces are

$$\sum F_x = T\cos 60° - F$$
$$\sum F_y = -98N + T\sin 60°$$

Newton's Second Law

Since there is no acceleration, the equations of forces are

$$\sum F_x = ma_x$$
$$\rightarrow T\cos 60° - F = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -98N + T\sin 60° = 0$$

Solving the Equation

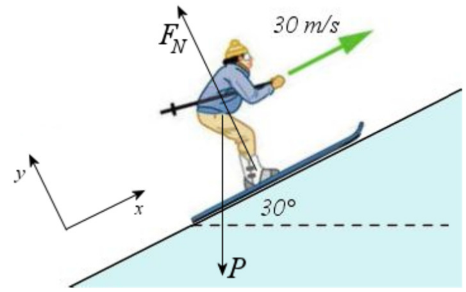
The tension can be found with the sum of the y-component of the forces.

$$-98N + T\sin 60° = 0$$
$$T = 113.16N$$

This value is then used in the sum of the x-component of the forces to find F.

$$T\cos 60° - F = 0$$
$$113.16N \cdot \cos 60° - F = 0$$
$$F = 56.58N$$

15. *Forces acting on the Object*



There are 2 forces acting on Yannick.

1) The weight (mg) directed downwards.
2) A normal force (F_N) perpendicular to the slope surface.

Sum of the Forces

With the axes shown in the figure, the table of force is

Forces	***x***	***y***
Weight	mg cos (-120°)	mg sin (-120°)
Normal force	0	F_N

The sums of the forces are

$$\sum F_x = mg\cos(-120°)$$
$$\sum F_y = mg\sin(-120°) + F_N$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x$$
$$\rightarrow mg\cos(-120°) = ma$$
$$\sum F_y = ma_y$$
$$\rightarrow mg\sin(-120°) + F_N = 0$$

Solving the Equations

The acceleration can be found with the sum of the x-component of the forces.

$$mg\cos(-120°) = ma$$
$$g\cos(-120°) = a$$
$$a = -4.9\tfrac{m}{s^2}$$

From there, the distance travelled can be found with

$$2a_x(x - x_0) = v^2 - v_0^2$$
$$2\cdot\left(-4.9\tfrac{m}{s^2}\right)\cdot(x - 0m) = \left(0\tfrac{m}{s}\right)^2 - \left(30\tfrac{m}{s}\right)^2$$
$$x = 91.84m$$

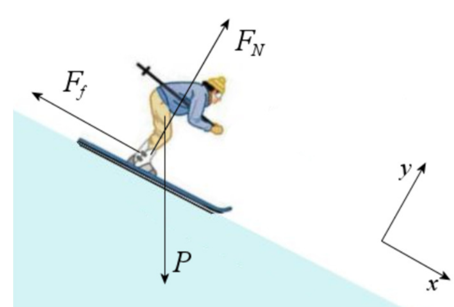
And the stopping time with

$$v_x = v_{0x} + a_x t$$
$$0\tfrac{m}{s} = 30\tfrac{m}{s} + \left(-4.9\tfrac{m}{s^2}\right)\cdot t$$
$$t = 6.122s$$

16. *Forces acting on the Object*

On the slope, there are 3 forces acting on Wolfgang.

1) The weight (mg) directed downwards.
2) A normal force (F_N) perpendicular to the slope surface.
3) A friction force (F_f) opposed to the velocity.



Sum of the Forces

With the axes shown in the figure, the table of force is

Forces	*x*	*y*
Weight	mg cos (-60°)	mg sin (-60°)
Normal force	0	F_N
Friction force	$-F_f$	0

The sums of the forces are

$$\sum F_x = mg\cos(-60°) - F_f$$
$$\sum F_y = mg\sin(-60°) + F_N$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\begin{aligned} &\sum F_x = ma_x \\ &\qquad \to mg\cos(-60°) - F_f = ma \\ &\sum F_y = ma_y \\ &\qquad \to mg\sin(-60°) + F_N = 0 \end{aligned}$$

Solving the Equations

In order to resolve, we will need the acceleration, which can be found with

$$\begin{aligned} 2a_x(x - x_0) &= v^2 - v_0^2 \\ 2 \cdot a \cdot (50m - 0m) &= \left(20\tfrac{m}{s}\right)^2 - \left(10\tfrac{m}{s}\right)^2 \\ a &= 3\tfrac{m}{s^2} \end{aligned}$$

The force of friction can then be found with the sum of the x-component of the forces.

$$\begin{aligned} mg\cos(-60°) - F_f &= ma \\ 70kg \cdot 9.8\tfrac{m}{s^2} \cdot \cos(-60°) - F_f &= 70kg \cdot 3\tfrac{m}{s^2} \\ F_f &= 133N \end{aligned}$$

17. *Forces acting on the Object*

There are 3 forces acting on the 30 kg block.

1) The 294 N weight directed downwards.
2) The tension force (T) directed towards the right.
3) A normal force (F_N)

Sum of the Forces

The table of force is (with an *x*-axis towards the right towards the right and a *y*-axis directed upwards)

Forces	*x*	*y*
Weight	0	-294 N
Tension	T	0
Normal force	$F_N \cos 115°$	$F_N \sin 115°$

The sums of the forces are

$$\sum F_x = T + F_N \cos 115°$$
$$\sum F_y = -294N + F_N \sin 115°$$

Newton's Second Law

Since there is no acceleration, the equations of forces are

$$\sum F_x = ma_x$$
$$\rightarrow T + F_N \cos 115° = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -294N + F_N \sin 115° = 0$$

Solving the Equations

The normal force can be found with the sum of the *y*-component of the forces.

$$-294N + F_N \sin 115° = 0$$
$$F_N = 324.39N$$

This value is then substituted into the sum of the *x*-component of the forces to find *T*.

$$T + F_N \cos 115° = 0$$
$$T + 324.39N \cdot \cos 115° = 0$$
$$T = 137.09N$$

18. *Forces acting on the Object*

There are 3 forces acting on the 180 kg crate.

1) The 784 N weight directed downwards.
2) A normal force (F_N).
3) The 800 N force (F) directed towards the right.

Sum of the Forces

The table of force is (with axes tilted so that the *x*-axis is directed uphill)

Forces	*x*	*y*
Weight	784 N · cos (-130°)	784 N · sin (-130°)
Normal force	0	F_N
Force *F*	800 N · cos (-40°)	800 N · sin (-40°)

The sums of forces are

$$\sum F_x = 784N \cdot \cos\left(-130°\right) + 800N \cdot \cos\left(-40°\right)$$
$$\sum F_y = 784N \cdot \sin\left(-130°\right) + F_N + 800N \cdot \sin\left(-40°\right)$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis or in the direction opposed to the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x$$
$$\rightarrow 784N \cdot \cos\left(-130°\right) + 800N \cdot \cos\left(-40°\right) = ma$$
$$\sum F_y = ma_y$$
$$\rightarrow 784N \cdot \sin\left(-130°\right) + F_N + 800N \cdot \sin\left(-40°\right) = 0$$

Solving the Equations

The acceleration can be found with the sum of the *x*-component of the forces.

$$784N \cdot \cos\left(-130°\right) + 800N \cdot \cos\left(-40°\right) = ma$$
$$-503.95N + 612.84N = 80kg \cdot a$$
$$a_x = 1.361\tfrac{m}{s^2}$$

Then F_N can be found with the sum of the y-component of the forces.

$$784N \cdot \sin(-130°) + F_N + 800N \cdot \sin(-40°) = 0$$
$$F_N = 1114.81N$$

19. a)

Let's find the acceleration by considering both boxes as a single object whose mass is 5 kg.

Forces Acting on the Object

There are 4 forces acting on this 5 kg object.

1) The 49 N weight directed downwards.
2) A normal force (F_N) directed upwards, exerted by the ground.
3) The 50 N force (F) directed towards the right.
4) An 18 N friction force (F_f) directed towards the left.

Sum of the Forces

The sum of the x-components of the forces is (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\begin{aligned}\sum F_x &= 50N - 18N \\ &= 32N\end{aligned}$$

(The sum of the y-component is useless here.)

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad 32N = 5kg \cdot a$$

Solving the Equations

The acceleration is

$$32N = 5kg \cdot a$$
$$a = 6.4 \tfrac{m}{s^2}$$

b) To find the normal force between the boxes, the forces on one of the two boxes must be looked at. Let's take the 2 kg box.

Forces Acting on the Object

There are 4 forces on this box.

1) The 19.6 N weight directed downwards.
2) A normal force (F_{N1}) directed upwards.
3) A normal force (F_{N2}) directed towards the right, exerted by the 3 kg box.
4) An 8 N friction force directed towards the left.

Sum of the Forces

The sum of the x-components of the forces is (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\sum F_x = F_{N2} - 8N$$

Newton's Second Law

Since the x-component of the acceleration is 6.4 m/s², we have

$$\sum F_x = ma_x$$
$$F_{N2} - 8N = 2kg \cdot 6.4 \tfrac{m}{s^2}$$
$$F_{N2} - 8N = 12.8 \tfrac{m}{s^2}$$

Solving the Equation

The normal force is

$$F_{N2} - 8N = 12.8N$$
$$F_{N2} = 20.8N$$

20. There are 2 objects here.

12 kg Block

Forces Acting on the Object

There are 2 forces acting on the 12 kg block.

1) The 117.6 N weight directed downwards.
2) The tension force (T) directed upwards exerted by the rope.

Sum of the Forces

The sum of the x-component of the forces is (with an x-axis directed downwards)

$$\sum F_x = 117.6N - T$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_1 a_x \qquad \to \qquad 117,6N - T = 12kg \cdot a$$

10 kg Block

Forces Acting on the Object

There are 2 forces acting on the 10 kg block.

1) The 98 N weight directed downwards.
2) The tension force (T) directed upwards.

Sum of the Forces

The sum of the x-component of the forces is (with an x-axis directed upwards)

$$\sum F_x = -98N + T$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_1 a_x \qquad \rightarrow \qquad -98N + T = 10kg \cdot a$$

Solving the Equations

The two equations are

$$117.6N - T = 12kg \cdot a$$
$$-98N + T = 10kg \cdot a$$

This system can be solved by adding these two equations.

$$(117.6N - T) + (-98N + T) = 12kg \cdot a + 10kg \cdot a$$
$$117.6N - 98N = 22kg \cdot a$$
$$a = 0.891\tfrac{m}{s^2}$$

Therefore, the tension is

$$117.6N - T = 12kg \cdot a$$
$$117.6N - T = 12kg \cdot 0.891\tfrac{m}{s^2}$$
$$T = 106.91N$$

21. There are 2 objects here.

24 kg Block

Forces Acting on the Object

The forces acting on the 24 kg block.

1) The 235.2 N weight directed downwards.
2) A normal force (F_{N1}) directed upwards.
3) The tension force (T) directed towards the right.
4) The 300 N force (F).

Sum of the Forces

The table of force is (with an x-axis towards the right towards the right and a y-axis directed upwards)

Forces	x	y
Weight	0	-235,2 N
Normal force 1	0	F_{N1}
Tension force	T	0
Force *F*	300 N · cos (160°)	300 N · sin (160°)

The sums of the forces are

$$\sum F_x = T + 300N \cdot \cos 160°$$
$$\sum F_y = -235,2N + F_{N1} + 300N \cdot \sin 160°$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_1 a_x$$
$$\to T + 300N \cdot \cos 160° = 24kg \cdot a$$
$$\sum F_y = m_1 a_y$$
$$\to -235.2N + F_{N1} + 300N \cdot \sin 160° = 0$$

18 kg Block

Forces Acting on the Object

There are 3 forces acting on the 18 kg box.

1) The 176.4 N weight directed downwards.
2) A normal force (F_{N2}) perpendicular to the slope surface.
3) The tension force (T) directed uphill.

Sum of the Forces

The table of force is (with an x-axis directed downhill)

Forces	x	y
Weight	176.4 N · cos (-30°)	176.4 N · sin (-30°)
Normal force 2	0	F_{N2}
Tension force	$-T$	0

The sums of the forces are

$$\sum F_x = 176.4N \cdot \cos\left(-30°\right) - T$$
$$\sum F_y = 176.4N \cdot \sin\left(-30°\right) + F_{N2}$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_2 a_x$$
$$\rightarrow 176.4N \cdot \cos\left(-30°\right) - T = 18kg \cdot a$$
$$\sum F_y = m_2 a_y$$
$$\rightarrow 176.4N \cdot \sin\left(-30°\right) + F_{N2} = 0$$

Solving the Equations

a and b) The two equations for the sum of the x-component of the forces are

$$T + 300N \cdot \cos 160° = 24kg \cdot a$$
$$176.4N \cdot \cos\left(-30°\right) - T = 18kg \cdot a$$

The solution to this system of equations can be found by adding these equations

$$\left(T + 300N \cdot \cos 160°\right) + \left(176.4N \cdot \cos\left(-30°\right) - T\right) = 24kg \cdot a + 18kg \cdot a$$
$$300N \cdot \cos 160° + 176.4N \cdot \cos\left(-30°\right) = \left(24kg + 18kg\right) \cdot a$$
$$-281.91N + 152.77N = 42kg \cdot a$$
$$a = -3,075 \tfrac{m}{s^2}$$

The tension is therefore

$$T + 300N \cdot \cos 160° = 24kg \cdot a$$
$$T + 300N \cdot \cos 160° = 24kg \cdot \left(-3.075\tfrac{m}{s^2}\right)$$
$$T = 208.11N$$

c) The normal force in the 24 kg block is found with the sum of the y-component of the forces on this block.

$$-235.2N + F_{N1} + 300N \cdot \sin 160° = 0$$
$$F_{N1} = 132.59N$$

The normal force in the 18 kg block is found with the sum of the y-component of the forces on this block.

$$176.4N \sin\left(-30°\right) + F_{N2} = 0$$
$$F_{N2} = 88.2N$$

22. There are 2 objects here.

2 kg Block

Forces Acting on the Object

There are forces acting on the 2 kg block are

1) The 19.6 N weight directed downwards.
2) The tension force (T) directed upwards.

Sum of the Forces

The sum of the x-component of the forces is (with an x-axis directed upwards)

$$\sum F_x = -19.6N + T$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_1 a_x \qquad \rightarrow \qquad -19.6N + T = 2kg \cdot a$$

Block with mass *m*

<u>*Forces Acting on the Object*</u>

There are 3 forces acting on the box of mass *m*.

1) The weight (*mg*) directed downwards.
2) A normal force (F_N) perpendicular to the slope surface.
3) The tension force (*T*) directed uphill.

<u>*Sum of the Forces*</u>

The table of force is (with an *x*-axis directed downhill)

Forces	*x*	*y*
Weight	*mg* cos (-70°)	*mg* sin (-70°)
Normal force	0	F_N
Tension force	-*T*	0

The sum of *x*-components of the forces is

$$\sum F_x = mg\cos(-70°) - T$$

<u>*Newton's Second Law*</u>

As the velocity is in the direction of the *x*-axis or in the direction opposed to the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_2 a_x \qquad \rightarrow \qquad mg\cos(-70°) - T = ma$$

Solving the equations

The two equations are then

$$-19.6N + T = 2kg \cdot a$$
$$mg\cos(-70°) - T = ma$$

a) Since the acceleration is -2 m/s², we have

$$-19.6N + T = -4N$$
$$mg\cos(-70°) - T = m \cdot \left(-2\tfrac{m}{s^2}\right)$$

The first equation allows us to find T = 15.6 N. If this value is substituted in the second equation, we have

$$mg\cos(-70°) - 15.6N = m \cdot \left(-2\tfrac{m}{s^2}\right)$$
$$mg\cos(-70°) + m \cdot \left(2\tfrac{m}{s^2}\right) = 15.6N$$
$$m\left(g\cos(-70°) + 2\tfrac{m}{s^2}\right) = 15.6N$$
$$m \cdot \left(5.352\tfrac{m}{s^2}\right) = 15.6N$$
$$m = 2.915kg$$

b) If the tension is 25 N, we have

$$-19.6N + 25N = 2kg \cdot a$$
$$mg\cos(-70°) - 25N = ma$$

The first equation allows us to find a_x = 2.7 m/s². If this value is substituted in the second equation, we have

$$mg\cos(-70°) - 25N = m \cdot \left(2.7\tfrac{m}{s^2}\right)$$
$$mg\cos(-70°) - m \cdot \left(2.7\tfrac{m}{s^2}\right) = 25N$$
$$m\left(g\cos(-70°) - 2.7\tfrac{m}{s^2}\right) = 25N$$
$$m \cdot \left(0.6518\tfrac{m}{s^2}\right) = 25N$$
$$m = 38.36kg$$

23. There are 3 objects here.

20 kg Block

Forces Acting on the Object

There are 2 forces acting on the 20 kg block.

1) The 196 N weight directed downwards.
2) The tension force (T_1) directed upwards.

Sum of the Forces

The sum of the x-component of the forces is (with an x-axis directed upwards)

$$\sum F_x = -196N + T_1$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_1 a_x \qquad \rightarrow \qquad -196N + T_1 = 20kg \cdot a$$

80 kg Block

Forces Acting on the Object

There are 4 forces acting on the 80 kg block.

1) The 784 N weight directed downwards.
2) A normal force (F_N) directed upwards.
3) The tension force (T_1) directed towards the right.
4) The tension force (T_2) directed towards the left.

Sum of the Forces

The sum of the x-component of the forces is (with an x-axis directed to the left)

$$\sum F_x = -T_1 + T_2$$

(The sum of the y-component of the forces is useless here.)

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_2 a_x \qquad \rightarrow \qquad -T_1 + T_2 = 80kg \cdot a$$

30 kg Block

Forces Acting on the Object

There are 2 forces acting on the 30 kg block.

1) The 194 N weight directed downwards.
2) The tension force (T_2) directed upwards.

Sum of the Forces

The sum of the x-component of the forces is (with an x-axis directed downwards)

$$\sum F_x = 294N - T_2$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_3 a_x \qquad \rightarrow \qquad 294N - T_2 = 30kg \cdot a$$

Solving the Equations

The three equations are then

$$-196N + T_1 = 20kg \cdot a$$
$$-T_1 + T_2 = 80kg \cdot a$$
$$294N - T_2 = 30kg \cdot a$$

a) This system can be resolved by adding these three equations.

$$\left(-196N + T_1\right) + \left(-T_1 + T_2\right) + \left(294N - T_2\right) = 20kg \cdot a_x + 80kg \cdot a_x + 30kg \cdot a$$
$$-196N + 294N = \left(20kg + 80kg + 30kg\right) \cdot a$$
$$98N = 130kg \cdot a$$
$$a = 0.7538 \tfrac{m}{s^2}$$

b) With the acceleration, the tensions can now be found. For T_1, we have

$$-196N + T_1 = 20kg \cdot a$$
$$-196N + T_1 = 20kg \cdot 0.7538 \tfrac{m}{s^2}$$
$$T_1 = 211.1N$$

For T_2, we have

$$294N - T_2 = 30kg \cdot a$$
$$294N - T_2 = 30kg \cdot 0.7538 \tfrac{m}{s^2}$$
$$T_2 = 271.4N$$

24. There are 2 objects here.

20 kg Block

Forces Acting on the Object

There are 3 forces acting on the 20 kg block.

1) The 196 N weight directed downwards.
2) A normal force (F_{N1}) perpendicular to the slope surface.
3) The tension force (T) directed uphill.

Sum of the Forces

The table of force is (with an x-axis directed downhill)

Forces	*x*	*y*
Weight	196 N · cos (-60°)	196 N · sin (-60°)
Normal force	0	F_{N1}
Tension	-*T*	0

The sum of the *x*-component of the forces is

$$\sum F_x = 196N \cdot \cos\left(-60°\right) - T$$

(The sum of the *y*-component of the forces is useless here.)

Newton's Second Law

As the velocity is in the direction of the *x*-axis or in the direction opposed to the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_1 a_x \qquad \rightarrow \qquad 196N \cdot \cos\left(-60°\right) - T = 20kg \cdot a$$

12 kg Block

Forces Acting on the Object

There are 3 forces acting on the 12 kg block.

1) The 117.6 N weight directed downwards.
2) A normal force (F_{N2}) perpendicular to the slope surface.
3) The tension force (T) directed uphill.

Sum of the Forces

The table of force is are (with an *x*-axis directed uphill)

Forces	*x*	*y*
Weight	117.6 N · cos (-150°)	117.6 N · sin (-150°)
Normal force	0	F_{N2}
Tension	*T*	0

The sum of the *x*-component of the forces is

$$\sum F_x = 117.6N \cdot \cos(-150°) + T$$

(The sum of the *y*-component of the forces is useless here.)

Newton's Second Law

As the velocity is in the direction of the *x*-axis or in the direction opposed to the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_2 a_x \qquad \rightarrow \qquad 117.6N \cdot \cos(-150°) + T = 12kg \cdot a$$

Solving the Equations

The two equations are then

$$196N \cdot \cos(-60°) - T = 20kg \cdot a$$
$$117.6N \cdot \cos(-150°) + T = 12kg \cdot a$$

This system can be solved by adding the equations.

$$\left(196N \cdot \cos(-60°) - T\right) + \left(117.6N \cdot \cos(-150°) + T\right) = 20kg \cdot a + 12kg \cdot a$$
$$196N \cdot \cos(-60°) + 117.6N \cdot \cos(-150°) - T = (20kg + 12kg)a$$
$$98N + -101.84N = 32kg \cdot a$$
$$a = -0.12 \tfrac{m}{s^2}$$

Therefore, the tension is

$$196N \cdot \cos(-60°) - T = 20kg \cdot a$$
$$196N \cdot \cos(-60°) - T = 20kg \cdot \left(-0.12 \tfrac{m}{s^2}\right)$$
$$T = 100.4N$$

25. a) The acceleration is found by considering the carrier as a single 820 kg object.

Forces Acting on the Object

There are 3 forces exerted on the carrier.

1) The 8036 N weight directed downwards.
2) A normal force (F_N) directed upwards.
3) An 800 N force exerted by the tractor.

Sum of the Forces

The sum of the x-component of the forces is (with an x-axis directed to the right)

$$\sum F_x = 800N$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_1 a_x \qquad \rightarrow \qquad 800N = 820kg \cdot a$$

Solving the Equations

The acceleration is

$$800N = 820kg \cdot a$$
$$a = 0.9756 \tfrac{m}{s^2}$$

b) T_1 is found by considering the force on the last carriage.

Forces Acting on the Object

There are 3 forces exerted on this carriage.

1) The 2352 N weight directed downwards.
2) A normal force (F_N) directed upwards.
3) The tension force (T_1) directed towards the right.

Sum of the Forces

The sum of the x-component of the forces is (with an x-axis directed to the right)

$$\sum F_x = T_1$$

Newton's Second Law

As the x-component of the acceleration is 0.9756 m/s², Newton's second law gives

$$\sum F_x = m_1 a_x \qquad \rightarrow \qquad T_1 = 240kg \cdot 0.9756 \tfrac{m}{s^2}$$

Solving the Equations

The tension is

$$T_1 = 240kg \cdot 0.9756 \tfrac{m}{s^2}$$
$$T_1 = 234.1N$$

T_2 is found by considering the last two carriages as a single 400 kg object.

Forces Acting on the Object

There are 3 forces exerted on this carriage.

1) The 3920 N weight directed downwards.
2) A normal force (F_N) directed upwards.
3) The tension force (T_2) directed towards the right.

Sum of the Forces

The sum of the x-component of the forces is (with an x-axis directed to the right)

$$\sum F_x = T_2$$

Newton's Second Law

As the x-component of the acceleration is 0.9756 m/s², Newton's second law gives

$$\sum F_x = m_1 a_x \qquad \rightarrow \qquad T_2 = 400kg \cdot 0.9756 \tfrac{m}{s^2}$$

Solving the Equations

The tension is

$$T_2 = 400kg \cdot 0.9756 \tfrac{m}{s^2}$$
$$T_2 = 390.2N$$

T_3 is found by considering the last three carriages as a single 620 kg object.

Forces Acting on the Object

There are 3 forces exerted on this carriage.

1) The 6076 N weight directed downwards.
2) A normal force (F_N) directed upwards surface.
3) The tension force (T_3) directed towards the right.

Sum of the Forces

The sum of the *x*-component of the forces is (with an *x*-axis directed to the right)

$$\sum F_x = T_3$$

Newton's Second Law

As the *x*-component of the acceleration is 0.9756 m/s², Newton's second law gives

$$\sum F_x = m_1 a_x \qquad \rightarrow \qquad T_3 = 620kg \cdot 0,9756 \tfrac{m}{s^2}$$

Solving the Equations

The tension is

$$T_3 = 620kg \cdot 0.9756 \tfrac{m}{s^2}$$
$$T_3 = 604.9N$$

26. There are 2 objects here.

Block A

Forces Acting on the Object

There are 4 forces acting on block A.

1) The weight ($m_a g$) directed downwards.
2) A normal force (F_N) directed upwards.
3) The tension force (T) directed towards the right.
4) The force (F) directed towards the left.

Sum of the Forces

The sum of the x-component of the forces is (with an x-axis directed to the left)

$$\sum F_x = -T + F$$

(The sum of the y-component of the forces is useless here.)

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_A a_x \qquad \rightarrow \qquad -T + F = m_A a$$

Block B

Forces Acting on the Object

There are 2 forces acting on block B.

1) The weight ($m_b g$) directed downwards.
2) The tension force (T) directed upwards.

Sum of the Forces

The sum of the x-component of the forces is (with an x-axis directed upwards)

$$\sum F_x = -m_B g + T$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = m_B a_x \qquad \rightarrow \qquad -m_B g + T = m_B a$$

Solving the Equations

The two equations are then

$$\begin{aligned} -T + F &= m_A a \\ -m_B g + T &= m_B a \end{aligned}$$

If these equations are added, we get

$$\begin{aligned} (-T + F) + (-m_B g + T) &= m_A a + m_B a \\ F - m_B g &= (m_A + m_B) a \end{aligned}$$

We also know that the acceleration is -1 m/s² when the force is 100 N and that the acceleration is 2 m/s² when the force is 200 N. We, therefore, have the two following equations.

$$\begin{aligned} 100N - m_B g &= (m_A + m_B) \cdot \left(-1\tfrac{m}{s^2}\right) \\ 200N - m_B g &= (m_A + m_B) \cdot \left(2\tfrac{m}{s^2}\right) \end{aligned}$$

If we solve for m_A in the first equation.

$$\begin{aligned} 100N - m_B g &= (m_A + m_B) \cdot \left(-1\tfrac{m}{s^2}\right) \\ 100N - m_B \cdot 9.8\tfrac{m}{s^2} &= -m_A \cdot 1\tfrac{m}{s^2} + -m_B \cdot 1\tfrac{m}{s^2} \\ 100N - m_B \cdot 8.8\tfrac{m}{s^2} &= -m_A \cdot 1\tfrac{m}{s^2} \\ m_A &= 8.8 \cdot m_B - 100kg \end{aligned}$$

And substitute in the second equation, we have

$$200N - m_B g = (m_A + m_B)\cdot\left(2\tfrac{m}{s^2}\right)$$
$$200N - m_B g = (8.8\cdot m_B - 100kg + m_B)\cdot\left(2\tfrac{m}{s^2}\right)$$
$$200N - m_B g = (9.8\cdot m_B - 100kg)\cdot\left(2\tfrac{m}{s^2}\right)$$
$$200N - m_B \cdot 9.8\tfrac{m}{s^2} = m_B \cdot 19.6\tfrac{m}{s^2} - 200N$$
$$400N = m_B \cdot 29.4\tfrac{m}{s^2}$$
$$m_B = 13.61kg$$

Then, m_A is

$$m_A = 8.8\cdot m_B - 100kg$$
$$m_A = 19.73kg$$

27. As the rope is attached to the pulley, let's examine the forces on the lower pulley.

Forces Acting on the Object

There are 4 forces acting on the pulley.

1) A tension force directed downwards, equal to the weight of the 100 kg mass. This is therefore a 980 N force, directed downwards.
2) 3 times the tension force T directed upwards exerted by the string passing through the pulleys.

Sum of the Forces

The sum of the y-component of the forces is (with a y-axis directed upwards)

$$\sum F_y = -980N + 3T$$

Newton's Second Law

Since the acceleration is zero, we have

$$\sum F_y = ma_y \qquad \rightarrow \qquad -980N + 3T = 0$$

Solving the Equations

The tension is

$$-980N + 3T = 0$$
$$T = 326.7N$$

28. *Forces Acting on the Object*

There are 3 forces acting on the object composed of the pulley and the 5 kg block.

1) The 49 N weight directed downwards.
2) Twice the tension force *T* directed upwards exerted by the string.

Sum of the Forces

The sum of the *y*-component of the forces is (with a *y*-axis directed upwards)

$$\sum F_y = -49N + 2T$$

Newton's Second Law

As the velocity is in the direction of the *y*-axis or in the direction opposed to the *y*-axis, the *x*-component of the acceleration is 0 and the *y*-component of the acceleration is *a*. Thus

$$a_x = 0 \qquad\qquad a_y = a$$

Therefore, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -49N + 2T = 5kg \cdot a$$

Solving the Equations

a) If the acceleration is zero, we have

$$-49N + 2T = 0$$
$$T = 24.5N$$

b) If the tension is 20 N, the acceleration is

$$-49N + 2T = ma_y$$
$$-49N + 2 \cdot 20N = 5kg \cdot a_y$$
$$a_y = -1.8 \tfrac{m}{s^2}$$

29. *Forces Acting on the Object*

There are 3 forces acting on the bottom pulley.

1) A tension force directed downwards, equal to the weight of the 25 kg mass. This is therefore a 245 N force, directed downwards.
2) Twice the tension force T directed upwards exerted by the string passing through the pulleys.

Sum of the Forces

The sum of the y-component of the forces is (with a y-axis directed upwards)

$$\sum F_y = -245N + 2T$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_y \qquad \to \qquad -245N + 2T = 0$$

Solving the Equations

The tension of the rope is

$$-245N + 2T = 0$$
$$T = 122.5N$$

The sum of the forces on the bucket is then

$$\sum F_y = ma_y$$
$$T - mg = 0$$
$$122.5N - mg = 0$$
$$m = 12.5kg$$

30. *Forces Acting on the Object*

There are 3 forces acting on Romeo.

1) The 392 N weight directed downwards.
2) Twice the tension force T = 250 N directed upwards.

Sum of the Forces

The sum of the y-component of the forces is (with a y-axis directed upwards)

$$\sum F_y = -392N + 2T$$

Newton's Second Law

As the velocity is in the direction of the y-axis or in the direction opposed to the y-axis, the x-component of the acceleration is 0 and the y-component of the acceleration is a. Thus

$$a_x = 0 \qquad\qquad a_y = a$$

Therefore, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -392N + 2T = 40kg \cdot a$$

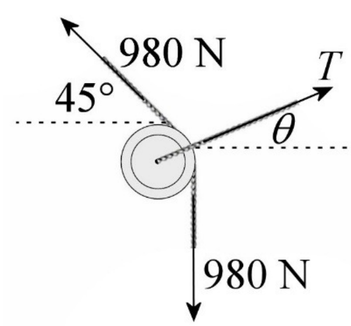
Solving the Equations

The acceleration is

$$\begin{gathered} -392N + 2T = 40kg \cdot a \\ -392N + 2 \cdot 250N = 40kg \cdot a \\ a = 2.7 \tfrac{m}{s^2} \end{gathered}$$

31. *Forces Acting on the Object*

There are 3 forces acting on the pulley.



1) The tension of the rope that supports the 100 kg mass, which is 980 N
2) The tension of the rope on the left, which is also 980 N since it's the same rope.
3) The tension of the rope on the right (T).

Sum of the Forces

The sums of the forces are (with an x-axis towards the right towards the right and a y-axis directed upwards)

$$\sum F_x = 980N \cdot \cos 135° + T_x$$
$$\sum F_y = -980N + 980N \cdot \sin 135° + T_y$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\begin{aligned}
&\sum F_x = ma_x \\
&\qquad \rightarrow 980N \cdot \cos 135° + T_x = 0 \\
&\sum F_y = ma_y \\
&\qquad \rightarrow -980N + 980N \cdot \sin 135° + T_y = 0
\end{aligned}$$

Solving the Equations

Thus

$$\begin{aligned}
T_x &= -980N \cdot \cos 135° \\
&= 692.96N
\end{aligned}$$

and

$$\begin{aligned}
T_y &= 980N - 980N \cdot \sin(135°) \\
&= 287.03N
\end{aligned}$$

Therefore, the tension is

$$\begin{aligned}
T &= \sqrt{T_x^2 + T_y^2} \\
&= \sqrt{(692.96N)^2 + (287.03N)^2} \\
&= 750N
\end{aligned}$$

and the angle is

$$\begin{aligned}
\theta &= \arctan \frac{T_y}{T_x} \\
&= \arctan \frac{287.03N}{692.96N} \\
&= 22.5°
\end{aligned}$$

32. Pulley C

Forces Acting on the Object

There are 3 forces acting on pulley C.

1) 2 times the tension F directed downwards.
2) The tension force T directed upwards.

(F is the tension of the rope passing through pulleys C and A, while T is the tension of the rope passing through the pulley B.)

Sum of the Forces

The sum of the y-component of the forces is (with a y-axis directed upwards)

$$\sum F_y = -2F + T$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_y \qquad \to \qquad -2F + T = 0$$

Pulley A

Forces Acting on the Object

There are 4 forces acting on pulley A.

1) A tension force directed downwards, equal to the weight of the 10 kg mass. This is therefore a 98 N force directed downwards.
2) 2 times the tension F directed upwards.
3) The tension force T directed upwards.

Sum of the Forces

The sum of the y-component of the forces is (with a y-axis directed upwards)

$$\sum F_y = -98N + 2F + T$$

Newton's Second Law

As there is no acceleration, Newton's second law gives

$$\sum F_y = ma_y \qquad \rightarrow \qquad -98N + 2F + T = 0$$

Solving the Equations

We have the following equations.

$$-2F + T = 0$$
$$-98N + 2F + T = 0$$

The first equation gives

$$T = 2F$$

Using this result if the second equation leads to

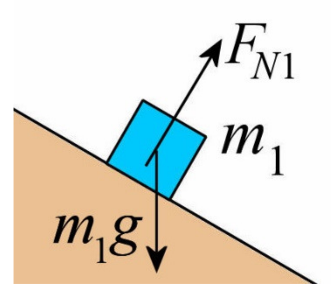
$$-98N + 2F + 2F = 0$$
$$-98N + 4F = 0$$
$$F = 24.5N$$

33. There are 2 objects here.

Small Block

Forces Acting on the Object

There are 2 forces exerted on the small block.



1) The weight (m_1g) directed downwards.
2) A normal force (F_{N1}).

Sum of the Forces

The table of force is (with an x-axis towards the right towards the right and a y-axis directed upwards)

Forces	***x***	***y***
Weight	0	$-m_1g$
Normal Force	F_{N1} cos (90°-θ)	F_{N1} sin (90°-θ)

The sums of the force are, therefore,

$$\sum F_x = F_{N1}\cos(90° - \theta)$$
$$\sum F_y = -m_1 g + F_{N1}\sin(90° - \theta)$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x$$
$$\to F_{N1}\cos\left(90°-\theta\right) = m_1 a$$
$$\sum F_y = ma_y$$
$$\to -m_1 g + F_{N1}\sin\left(90°-\theta\right) = 0$$

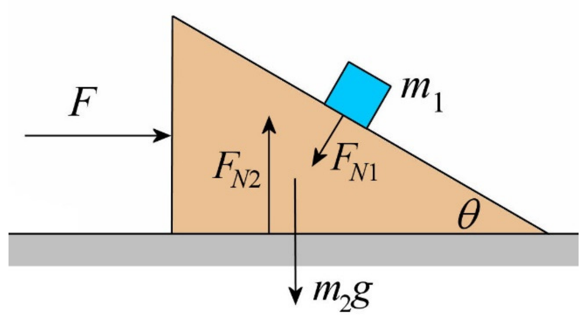
The y-component of the acceleration vanishes since the block does not slide. The x-component of the acceleration is not zero as the triangle and the small block accelerate when the force F is exerted on the triangle.

Triangle

Forces Acting on the Object

There are 4 forces exerted on the triangle.

1) The weight (m_2g) directed downwards.
2) A normal force (F_{N1}) exerted by the small block.
3) A normal force (F_{N2}) directed upwards exerted by the ground.
4) The applied force (F) directed towards the right.



Sum of the Forces

The table of force is (with an x-axis towards the right towards the right and a y-axis directed upwards)

Forces	*x*	*y*
Weight	0	$-m_2 g$
Normal force 1	F_{N1} cos -(90°+θ)	F_{N1} sin -(90°+θ)
Normal force 2	0	F_{N2}
***F* Force**	F	0

The sums of the force are, therefore,

$$\sum F_x = F_{N1}\cos-\left(90°+\theta\right)+F$$
$$\sum F_y = -m_2 g + F_{N1}\sin-\left(90°+\theta\right)+F_{N2}$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis or in the direction opposed to the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x$$
$$\rightarrow F_{N1}\cos-\left(90°+\theta\right)+F = m_2 a$$
$$\sum F_y = ma_y$$
$$\rightarrow -m_2 g + F_{N1}\sin-\left(90°+\theta\right)+F_{N2} = 0$$

Solving the Equations

Using trigonometric identities, the equations of the small block become

$$F_{N1}\sin\theta = m_1 a$$
$$-m_1 g + F_{N1}\cos\theta = 0$$

Using trigonometric identities, the equation of the *x*-component of the triangle becomes (the equation for the *y*-component is useless)

$$-F_{N1}\sin\theta + F = m_2 a$$

F_{N1} is found with $-m_1 g + F_{N1}\cos\theta = 0$. It is

$$F_{N1} = \frac{m_1 g}{\cos\theta}$$

Substituting in the other two equations, we obtain

$$\begin{gathered} F_{N1}\sin\theta = m_1 a \\ \frac{m_1 g}{\cos\theta}\sin\theta = m_1 a \\ m_1 g\tan\theta = m_1 a \\ g\tan\theta = a \end{gathered}$$

and

$$\begin{gathered} -F_{N1}\sin\theta + F = m_2 a \\ -\frac{m_1 g}{\cos\theta}\sin\theta + F = m_2 a \\ -m_1 g\tan\theta + F = m_2 a \end{gathered}$$

The two equations are now

$$\begin{gathered} g\tan\theta = a \\ -m_1 g\tan\theta + F = m_2 a \end{gathered}$$

Using a from the first equation into the second, F is found

$$\begin{gathered} -m_1 g\tan\theta + F = m_2 g\tan\theta \\ F = m_2 g\tan\theta + m_1 g\tan\theta \\ F = (m_1 + m_2)\, g\tan\theta \end{gathered}$$

34. We're going to treat the rope as two objects: the part that hangs and the part on the table. We'll assume that these two parts are connected by a small massless string of zero length.

Hanging part

Forces Acting on the Object

The are 2 forces on the hanging part.

1) The weight ($m_1 g$) directed downwards.
2) The tension of the string that connects the two pieces (T) directed upwards.

Sum of the Forces

The equation of the x-component of the forces is (with an axis directed downwards)

$$\sum F_x = m_1 g - T$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \to \qquad m_1 g - T = m_1 a$$

Part on the Table

Forces Acting on the Object

There are 3 forces on the part on the table.

1) The weight ($m_2 g$) directed downwards.
2) A normal force (F_N) directed upwards
3) The tension of the string that connects the two pieces (T) directed towards the right.

Sum of the Forces

The equation of the x-component of the forces is (with an axis directed to the right)

$$\sum F_x = T$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad T = m_2 a$$

Solving the Equations

Adding these two equations, the result is

$$m_1 g - T + T = m_1 a + m_2 a$$
$$m_1 g = (m_1 + m_2)a$$
$$m_1 g = Ma$$

where M is the total mass of the rope.

The problem is that m_1 (the mass of the hanging part) change constantly as the rope slides. Let's suppose that the length of the hanging part of the rope is x. Then, the mass of the hanging part is equal to the total mass multiplied by the proportion of the rope that is hanging.

$$m_1 = M\frac{x}{L}$$

where L is the total length of the rope. Thus, the acceleration is

$$m_1 g = Ma$$
$$x\frac{M}{L}g = Ma$$
$$a = x\frac{g}{L}$$

We now need to find speed knowing that x = 20 cm initially and x = 60 cm at the end. The speed can be found with

$$a = \frac{dv}{dt}$$

Then

$$\frac{dv}{dt} = x\frac{g}{L}$$

The equation is solved in the following manner

$$\frac{dv}{dt} = x\frac{g}{L}$$
$$\frac{dv}{dx}\frac{dx}{dt} = x\frac{g}{L}$$
$$\frac{dv}{dx}v = x\frac{g}{L}$$
$$vdv = x\frac{g}{L}dx$$

If both sides are integrated, the result is

$$\int vdv = \int x\frac{g}{L}dx$$
$$\frac{v^2}{2} = x^2\frac{g}{2L} + Cst$$

Since $v = 0$ when $x = 0.2$ m, the value of the constant can be found

$$0 = (0.2m)^2\frac{g}{2L} + Cst$$
$$Cst = -(0.2m)^2\frac{g}{2L}$$

Therefore, the formula for the speed is

$$\frac{v^2}{2} = x^2\frac{g}{2L} - (0.2m)^2\frac{g}{2L}$$
$$v = \sqrt{\frac{g}{L}\left(x^2 - (0.2m)^2\right)}$$

Thus, the speed when $x = 0.6$ m is

$$v = \sqrt{\frac{9.8\frac{N}{m}}{0.6m}\cdot\left((0.6m)^2 - (0.2m)^2\right)}$$
$$= 2.286\tfrac{m}{s}$$