4 FORCES, PART 1

In 2008, the submarine Onondaga was hauled on land beside Pointe-au-Père's wharf, not far from Rimouski. To achieve this, the 1400-ton submarine was towed up a 4° slope with the help of pulleys. In all, the steel cable pulled 18 times on the pulley attached to the submarine. The submarine was on small carts rolling on rails, thereby limiting the friction between the ground and the submarine so that it will be neglected here. With what force did they have to pull on the cable in order to move the submarine at a constant speed?

Site historique maritime de la Pointe-au-Père

Discover the answer to this question in this chapter.

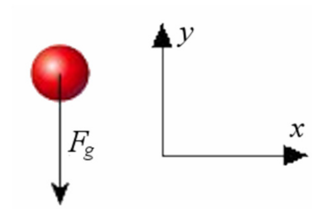
4.1 GRAVITATIONAL FORCE

Formula of the Force

Since falling objects accelerate, there must be a force acting on them. This force is the force of gravity. Since all objects fall with the same acceleration of 9.8 m/s², regardless of their mass, the formula of the gravitational force can easily be found with Newton's 2nd law.

For a free-falling object, the only force exerted is the gravitational force. The direction of the force is obviously downwards. Therefore, the equation of forces is

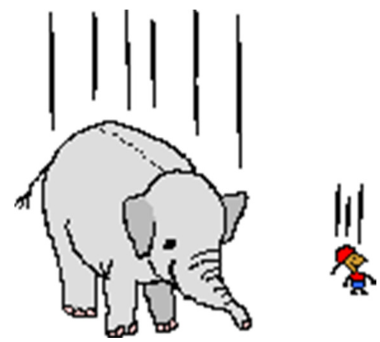
$$\sum F_y = ma_y$$
$$-F_g = m \cdot -9.8 \tfrac{m}{s^2}$$
$$F_g = m \cdot 9.8 \tfrac{m}{s^2}$$
$$F_g = mg$$



Thus, the magnitude of the gravitational force must be equal to *mg* for all objects to fall with an acceleration of 9.8 m/s².

So, the larger the mass of the object, the greater the gravitational force. It makes sense because more massive objects are harder to lift.

It is easy to see why all objects have the same acceleration. The greater force on a larger object is exactly offset by the fact that the larger object is more difficult to accelerate.



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Units of *g*

The *g* in the gravitational force formula is not the acceleration of the object even if the units are m/s² (*g* corresponds to the acceleration only if the object is in free fall). Gravitational force is always *mg* no matter the value of the acceleration of the object.

Actually, this *g* is the magnitude of the gravitational field near the Earth (we will further explore this concept in the next chapter).

To emphasize that this is not an acceleration, N/kg are used for the *g* when used to calculate the gravitational force.

$$g = 9.8 \tfrac{N}{kg}$$

Those N/kg are equivalent to m/s², but the change of units highlights the conceptual difference between the g that measures the acceleration of free-falling objects and the g that gives the magnitude of the gravitational field to the Earth's surface.

A g of 9.8 N/kg means that there is a force of 9.8 N for each kilogram of the object.

So, the gravitational force is as follows.

Gravitational Force (F_g) or Weight (w) (Formula Valid only near the Surface of the Earth)

1) Magnitude of the force

$$w = mg \qquad \text{where } g = 9.8 \tfrac{N}{kg}$$

2) Direction of the force

Downwards (towards the centre of the Earth)

3) Application point of the force

From the centre of mass of the object.
(See the chapter on the centre of mass.
For now, take a point roughly at the centre of the object.)

Mass and Weight

Gravitational force on objects is also called weight, which should not be confused with mass. The mass is in kilograms and represents the amount of matter in an object while the weight is in newtons and represents the gravitational force acting on the object.

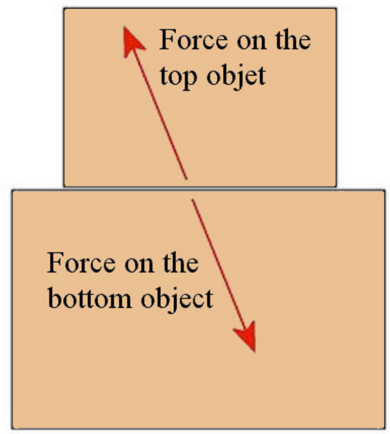
Mass is an intrinsic property of an object, which means that objects always have the same mass regardless of their position in the universe. On the other hand, the weight of an object changes depending on its position in the universe. A 10 kg object in space very far from all planets and stars would have no weight (there would be no gravitational force since there are no planets or nearby stars to make gravitational force) but it remains a 10 kg object. To move this 10 kg object with an acceleration of 2 m/s² still require a 20 N force, even if the weight of the object vanishes.

Isaac Newton was the first to make the first real distinction between mass and weight in 1685.

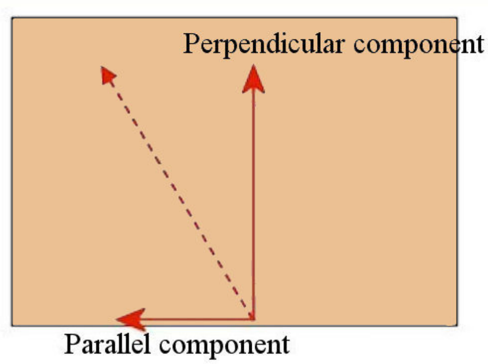
4.2 THE NORMAL FORCE

What Is the Normal Force?

When two objects touch each other, there is a force between the two objects. Generally, the forces can be represented as shown in the diagram.



There are two forces according to Newton's third law. If the top object exerts a force on the bottom object, then the bottom object exerts a force of the same magnitude and opposite direction on the top object. Now, consider only the force exerted on the top object. This force can be resolved into two components: the component perpendicular to the contact surface and the component parallel to the contact surface.

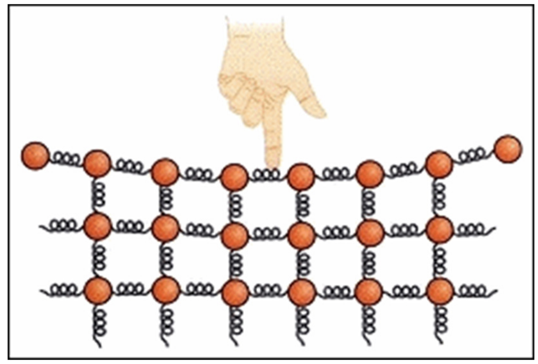


The parallel component will be studied in the next chapter (it is the friction force). For now, the focus will be on the perpendicular component. Since *normal* is synonymous with perpendicular, this component of the force is called the *normal force*. It is denoted F_N. (Sometimes N is used, but it may be confused with the symbol for the newton).

This component represents a repulsion force between objects in contact. To understand why objects repulse each other, imagine you are standing motionless on a mattress. Your presence then crushes the springs of the mattress. If you exert a force on the springs of the mattress directed downwards to compress them, then the springs of the mattress exert a force on you directed upwards according to Newton's third law. It is this force that cancels the force of gravity acting on you. This force must exactly cancel the force of gravity. If the force exerted by the springs is smaller than the gravitational force, there is a net force on you directed downwards. You then accelerate downwards and further compress the springs, thereby increasing the force they exert on you. If the force exerted by the springs is greater than the gravitational force, there is a net force on you directed upwards. You then accelerate upwards and decompress the springs thereby decreasing the force they exert on you. Both effects thus lead to a state where the force exerted by the springs on you exactly cancels your weight.

This may be surprising, but this is exactly what happens if you push on a surface with your finger. When you push on the surface, there is a slight deformation, even if the object is very rigid. Since objects have elasticity, they act like a spring: by compressing the object, the forces between the atoms exert a force opposed to the deformation.

In fact, the forces between the atoms can be represented by springs, as in this diagram. When you push on the surface, the atoms move closer together, which compress all the springs between the atoms. If your finger makes a downwards force on the atoms to compress the springs, then, by Newton's third law, the atoms exert an upward force on your finger. This latter force is the normal force.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As the contact between objects can only compress the objects in contact, the normal force can only be a repulsion between two objects in contact. It can never be an attraction.

So, if you are standing still on the ground. Your feet exert a force on the ground, which compresses it a bit (much less than a mattress, but it compresses). If your feet exert a force on the ground downwards, then the ground exerts an equally large upwards force on your feet according to Newton's third law. This force, the normal force, just cancel the force of gravity acting on you so that you can stay still. This video gives you the same explanation. http://www.youtube.com/watch?v=aJc4DEkSq4I

Here's a summary of the characteristics of the normal force.

Normal Force (N or F_N)

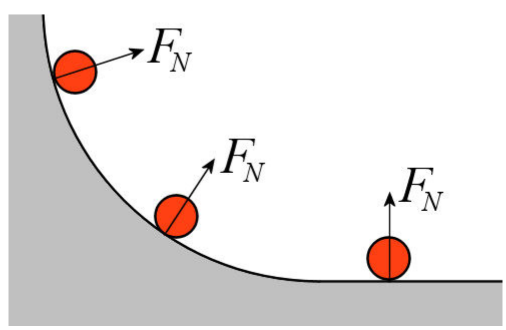
1) Magnitude of the force
   To be determined with Newton's laws

2) Direction of the force
   Repulsion between objects, perpendicular to the contact surface

3) Application point of the force
   Contact surface between objects.

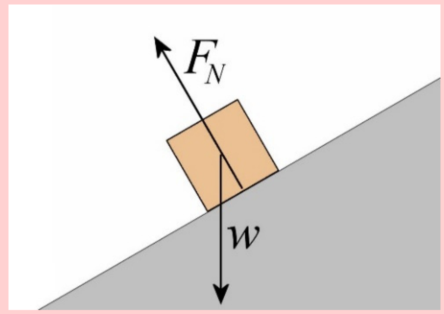
The force is always perpendicular to the surface of contact between the objects, as for these three balls.

The force on the ball is always a repulsive force (the ground pushes on the ball), and this force is always perpendicular to the contact surface (the ground in this case).



Common mistake: Thinking that the Weight and the Normal Force Are Related by Newton's Third Law

This would mean that the normal force would always have the same magnitude as the weight and be directed in the opposite direction. It can sometimes be true (as it is the case for an object stationary on a horizontal ground), but it is false most of the time. The example of an object on an inclined plane without friction illustrates this. The forces on the object are the weight (downwards) and the normal force (perpendicular to the contact surface). It can be seen that these two forces are not in opposite direction to each other. Therefore, they cannot be associated by Newton's third law.



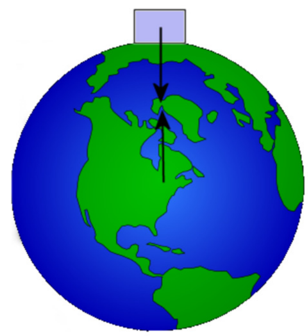
If the normal force is not associated with the weight by Newton's third law, what force is associated with the weight? Just apply the trick mentioned earlier to find it: the force associated is obtained by inverting the two objects in the following sentence:

The Earth exerts a gravitational force on the box.

to obtain

The box exerts a gravitational force on the Earth.

The force associated is, therefore, the gravitational attraction exerted by the box on the Earth.



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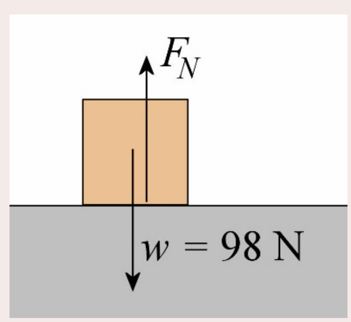
Calculation of the Normal Force

There's no formula to calculate the magnitude of the normal force directly. Newton's second law must be used to find it. Here are some examples.

Example 4.2.1

A 10 kg box is resting on the ground. What is the magnitude of the normal force exerted by the ground on the box?



Forces Acting on the Object

Two forces are acting on the box.

1) The weight (w) directed downwards whose magnitude is
   10 kg · 9.8 N/kg = 98 N.

2) A normal force (F_N). This is a force of repulsion exerted by the ground and is therefore directed upwards.

Sum of the Forces

The sum of the forces is

$$\sum F_y = -98N + F_N$$

Newton's Second Law

Since the box is not accelerating, Newton's second law gives

$$\sum F_y = ma_y \quad \rightarrow \quad -98N + F_N = 0$$

Solving the Equation

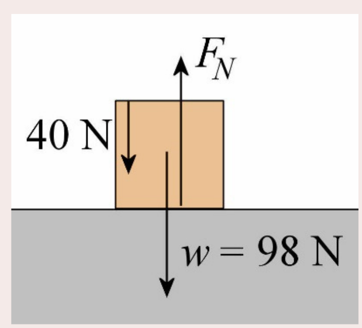
The normal force is

$$-98N + F_N = 0$$
$$F_N = 98N$$

Example 4.2.2

A 10 kg box is resting on the ground. A 40 N force is applied downwards on the box. What is the magnitude of normal force exerted by the ground on the box?

Forces Acting on the Object



Three forces are acting on the box.

1) The 98 N weight (w) directed downwards.
2) A normal force (F_N) directed upwards.
3) A 40 N force directed downwards.

Sum of the Forces

The sum of the forces is

$$\sum F_y = -98N + F_N - 40N$$

Newton's Second Law

Since the box is not accelerating, Newton's second law gives

$$\sum F_y = ma_y \quad \to \quad -98N + F_N - 40N = 0$$

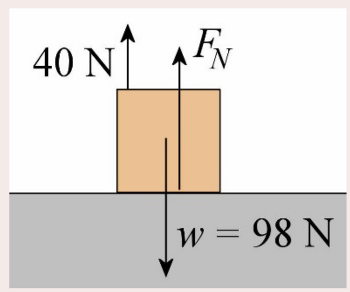
Solving the Equation

The normal force is

$$-98N + F_N - 40N = 0$$
$$F_N = 138N$$

Example 4.2.3

A 10 kg box is resting on the ground. A 40 N force is applied upwards on the box. What is the magnitude of normal force exerted by the ground on the box?



Forces Acting on the Object

Three forces are acting on the box.

1) The 98 N weight (w) directed downwards.
2) A normal force (F_N) directed upwards.
3) A 40 N force directed upwards.

Sum of the Forces

The sum of the forces is

$$\sum F_y = -98N + F_N + 40N$$

Newton's Second Law

Since the box is not accelerating, Newton's second law gives

$$\sum F_y = ma_y \quad \to \quad -98N + F_N + 40N = 0$$

Solving the Equation

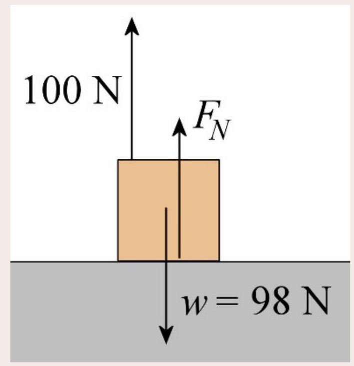
The normal force is

$$-98N + F_N + 40N = 0$$
$$F_N = 58N$$

In these last two examples, the normal force did not have the same magnitude as the weight. This shows once again that the weight and the normal force cannot be related by Newton's third law.

Example 4.2.4

A 10 kg box is resting on the ground. A 100 N force is applied upwards on the box. What is the magnitude of the normal force exerted by the ground on the box?



Forces Acting on the Object

Three forces are acting on the box.

1) The 98 N weight (w) directed downwards.
2) A normal force (F_N) directed upwards.
3) A 100 N force directed upwards.

Sum of the Forces

The sum of the forces is

$$\sum F_y = -98N + F_N + 100N$$

Newton's Second Law

Since the box is not accelerating, Newton's second law gives

$$\sum F_y = ma_y \quad \rightarrow \quad -98N + F_N + 100N = 0$$

Solving the Equation

The normal force is

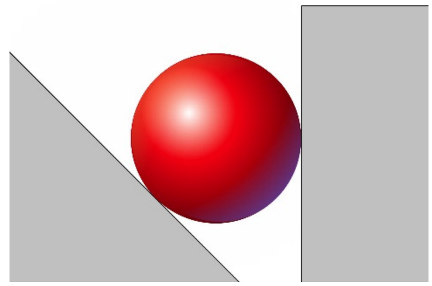
$$-98N + F_N + 100N = 0$$
$$F_N = -2N$$

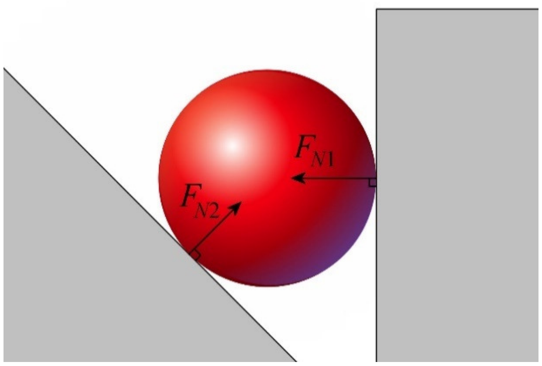
This answer is impossible; the normal force cannot have a negative value. If that were the case, it would mean that the normal force would be in the direction opposite to the direction indicated in the diagram. However, since the normal force is a repulsive force between the objects, its only possible direction was already indicated correctly in the diagram. If it is negative (so in the opposite direction), the normal force then becomes an attraction between objects, and this can never happen with normal forces.

This situation is, therefore, impossible. Of course, a 100 N force can act upwards on a 10 kg box. It is, however, impossible to say that the box remains at rest. If you lift a box weighing 98 N with a force of 100 N, there is a resultant force of 2 N upwards with no other force to cancel it. The box then accelerates upwards in this situation. It is no longer in contact with the surface, and the normal force is zero.

There is a normal force exerted at each contact with other objects

Normal forces are contact forces. At every spot where the object touches another object, there is a normal force exerted. This normal is always a repulsive force and is always perpendicular to the contact surface. The example of this ball illustrates this.

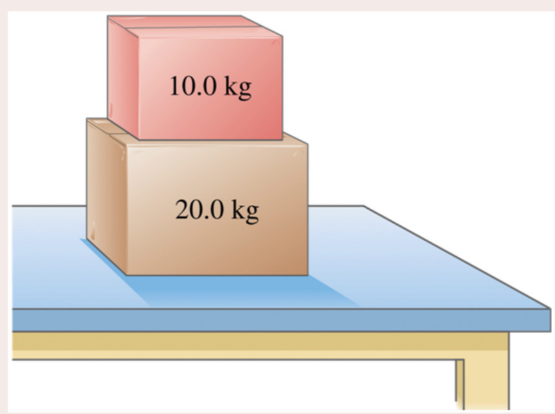




This ball touches two other objects, which means that two normal forces are acting on this ball. As normal forces must always be perpendicular to the contact surface, the normal forces are in the following directions.

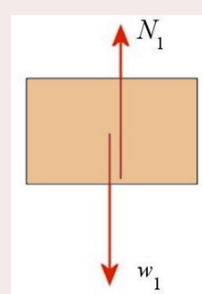
Example 4.2.5

Draw all the forces acting on the two boxes in the diagram. (You do not have to find the magnitudes of the forces.)



www.alpcentauri.info/chapter_4_problems1.html

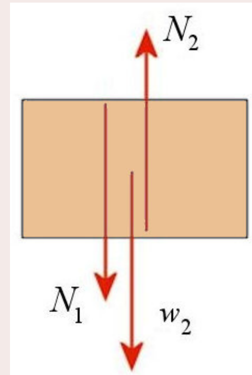
Let's start with the top box. As the box has mass, it has a weight directed downwards. As it touches the box below it, there is a normal force directed upwards. This force is upwards since the top box and the bottom box repel each other.



The forces are therefore:

1) The weight (w_1) directed downwards.
2) The normal force directed upwards exerted by the bottom box (N_1).

Let's follow with the bottom box. There is, of course, a weight directed downwards. As the box touches two things (the top box and the table) there are 2 normal forces. Since these are repulsive forces, the table exerts a normal force directed upwards, and the top box exerts a downwards force. The forces are therefore:



1) The weight (w_2) directed downwards.
2) The normal force directed upwards exerted by the ground (N_2).
3) The normal force directed downwards exerted by the top box (N_1).

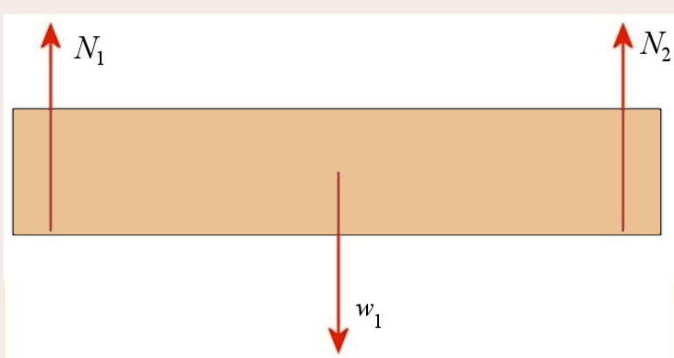
Note that the normal force exerted by the top box is also called N_1. It was unnecessary to use a new symbol such as N_3 since this force has the same magnitude as the normal force made on the top box. This is true because of Newton's third law: if the bottom box exerts the normal force N_1 directed upwards on the top box, then the top box exerts a normal force N_1 directed downwards on the bottom box.

Example 4.2.6

Draw all the forces acting on the three boxes in the diagram. (You don't have to find the magnitudes of the forces.)

Let's start with the top box. There is a gravitational force directed downwards. As this box touches two other objects (the left and right boxes), there are also two normal forces acting on the top box. These forces are directed upwards since the boxes on the left and the right repel the top box. The forces on the top box are then:

1) The weight (w_1) directed downwards.
2) The normal force directed upwards exerted by the box on the left (N_1).
3) The normal force directed upwards exerted by the box on the right (N_2).

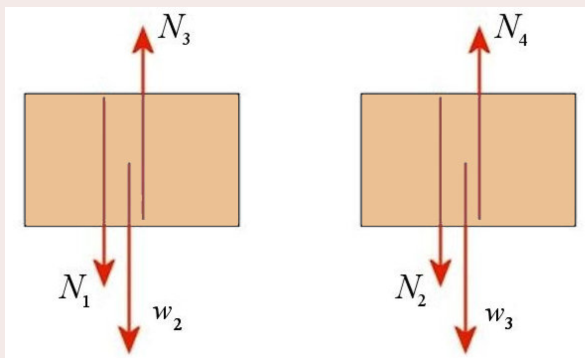


Now let's examine the boxes supporting the top box. Each of these boxes has a weight directed downwards. As these boxes also touch two things (the floor and the top box), two normal forces are acting on each of these boxes. The forces on the box on the left are:

1) The weight (w_2) directed downwards.
2) The normal force directed upwards exerted by the ground (N_3).
3) The normal force directed downwards exerted by the top box (N_1).

The forces on the box on the right are:

1) The weight (w_3) directed downwards.
2) The normal force directed upwards exerted by the ground (N_4).
3) The normal force directed downwards exerted by the top box (N_2).



4.3 THE TENSION FORCE

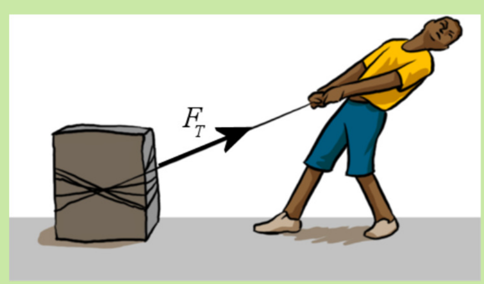
The tension force is the force exerted by a rope or a string. It can be stated from the outset that a string can only pull an object in the direction of the string; it cannot push or apply force in a direction perpendicular to the string because the string simply bends if one tries to apply such forces.

Tension Force (T or F_T)

1) Magnitude of the force
   Given or to be determined with Newton's laws.

2) Direction of the force
   The rope pulls in the direction of the rope.

3) Application point of the force
   Where the rope is fixed to the object.

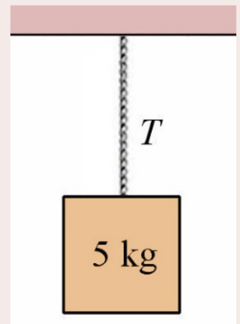


www.mstworkbooks.co.za/natural-sciences/gr9/gr9-ec-01.html

Calculation of the Tension Force

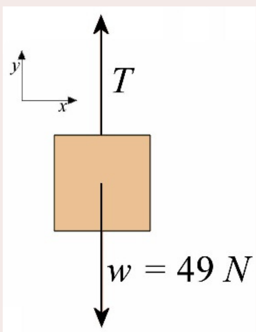
There's no formula to calculate the magnitude of the tension force directly. Newton's second law must be used to find it. Here are some examples.

Example 4.3.1



What is the tension force exerted by the rope that supports this 5 kg box at rest?

Forces Acting of the Object



There are 2 forces acting on the box.

1) The 49 N weight (w) directed downwards.
2) The tension force (T) directed upwards (since a rope must pull).

There is no normal force because the box does not touch anything (excepting the rope).

Sum of the Forces

The sum of the forces is

$$\sum F_y = -49N + T$$

Newton's Second Law

Since the box is not accelerating, Newton's second law gives

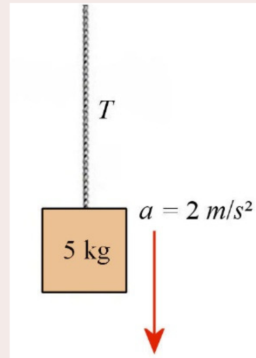
$$\sum F_y = ma_y \quad \rightarrow \quad -49N + T = 0$$

Solving the Equation

The tension is

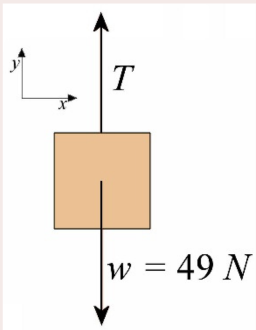
$$-49N + T = 0$$
$$T = 49N$$

Example 4.3.2



What is the tension force exerted by the rope that supports this box if it accelerates downwards at 2 m/s²?

Forces Acting of the Object



There are 2 forces acting on the box.

1) The 49 N weight (w) directed downwards.
2) The tension force (T) directed upwards.

Sum of the Forces

The sum of the forces is

$$\sum F_y = -49N + T$$

Newton's Second Law

Since the y-component of the acceleration is -2 m/s², Newton's second law gives

$$\sum F_y = ma_y \quad \rightarrow \quad -49N + T = 5kg \cdot \left(-2\tfrac{m}{s^2}\right)$$

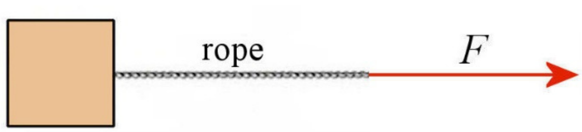
Solving the Equation

The tension is

$$-49N + T = 5kg \cdot \left(-2\tfrac{m}{s^2}\right)$$
$$-49N + T = -10N$$
$$T = 39N$$

The Forces Exerted by a String Is the Same at Each End (If the Mass of the Rope Is Neglected)

It will now be shown that the force exerted by the string on the block in the diagram is the same as the force exerted at the other end of the rope. In other words, we want to know if some force is lost in the string.



Consider first the forces on the string. As the string pulls on the block towards the right, the block must pull on the string towards the left according to Newton's third law.

The equation of the forces acting on the string is

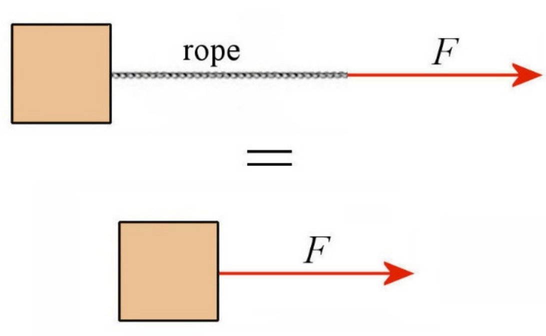
$$F_1 - F_2 = m_{string} a$$

If the mass of the string is neglected, by assuming $m_{string} = 0$ (this will always be assumed unless stated otherwise), the equation becomes

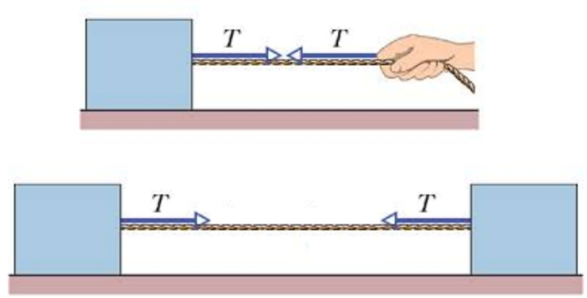
$$F_1 - F_2 = 0$$
$$F_1 = F_2$$

The two forces at each end of the string must then have the same magnitude. This pair of force stretches the string and puts it under tension. This is why this force is called the tension force or the tension of the string and is denoted T or F_T.

Since the force exerted by the box on the string directed towards the left has a magnitude T, the force exerted by the rope on the box is directed towards the right and also has a magnitude T. This is the same force as the one exerted at the other end of the string. This shows that no force is lost in the string. (This is no longer true if the mass of the string is not neglected. In this case, a part of the force is used to accelerate the string, and some of the force is lost.)



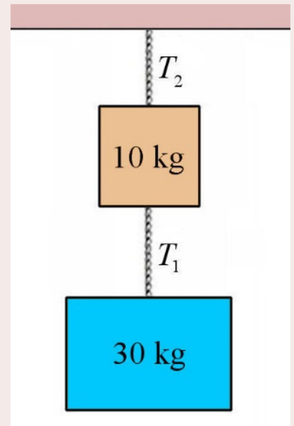
This also means that the force exerted by a string has the same magnitude at each end of the string. Thus, if a string connects two objects, the string pulls on each object with forces of equal magnitude.



www.physicsforums.com/threads/direction-of-tension-force.806174/

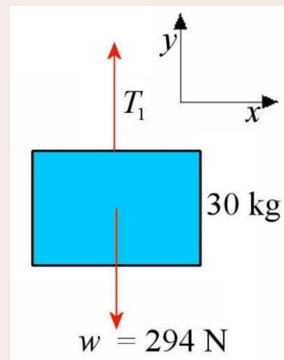
Example 4.3.3

What are the tensions of each of the two strings in the situation shown in the diagram?



When there are several objects, the problem can be solved by finding the equations of forces for each object separately.

30 kg Block



Forces Acting on the Object

There are 2 forces acting on this block.

1) A 294 N weight (w) directed downwards.
2) A tension force (T_1) directed upwards.

Sum of the Forces

The sum of the forces is

$$\sum F_y = -294N + T_1$$

Newton's Second Law

Since the block is not accelerating, Newton's second law gives

$$\sum F_y = ma_y \quad \rightarrow \quad -294N + T_1 = 0$$

Solving the Equation

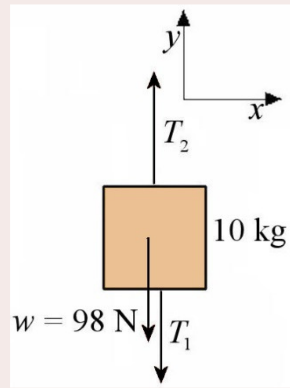
The tension is

$$-294N + T_1 = 0$$
$$T_1 = 294N$$

10 kg Block

There are 3 forces acting on this block.

1) A 98 N weight (w) directed downwards.
2) A tension force (T_1) directed downwards.
3) A tension force (T_2) directed upwards.



Sum of the Forces

The sum of the forces is

$$\sum F_y = -98N - T_1 + T_2$$

Newton's Second Law

Since the block is not accelerating, Newton's second law gives

$$\sum F_y = ma_y \quad \rightarrow \quad -98N - T_1 + T_2 = 0$$

Solving the Equation

The equation is

$$-98N - T_1 + T_2 = 0$$

As the string that connects the two blocks should exert the same force of tension at each end of the string, then T_1 = 294 N. The tension force T_2 is then

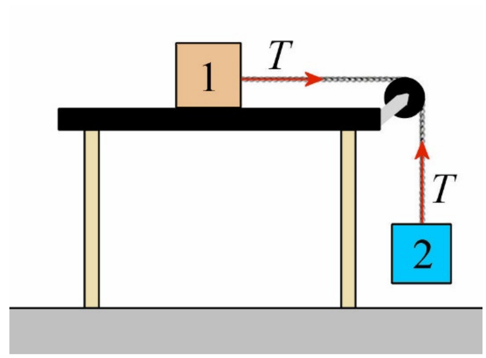
$$-98N - 294N + T_2 = 0$$

And the solution is

$$T_2 = 392N$$

Therefore, the tension of the string 1 is 294 N, and the tension of the string 2 is 392 N. This means that the string 1 pulls with a force of 294 N at each of its ends and that the string 2 pulls with a force of 392 N at each of its ends.

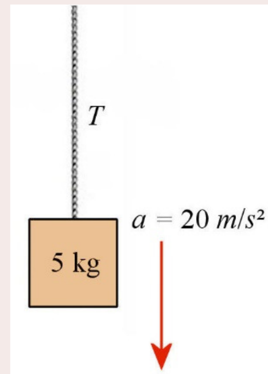
Note that the force made at each end of a string remains of the same magnitude even if the string passes through pulleys (if the mass of the pulley is neglected).



The Tension Force Cannot Be Negative

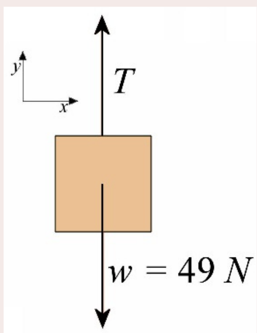
When the force of tension is calculated using Newton's laws, the answer cannot be negative since the direction of the force (a string always pulls) was already taken into account. A negative answer means that the force is in the opposite direction than it had been assumed. This would mean that the rope pushes, which is impossible.

Example 4.3.4



What is the tension force exerted by the string that supports the box in the diagram if it accelerates downwards at 20 m/s²?

The Forces Acting of the Object



There are 2 forces acting on the box.

1) The 49 N weight (w) directed downwards.
2) The tension force (T) directed upwards.

Sum of the Forces

The sum of the forces is

$$\sum F_y = -49N + T$$

Newton's Second Law

Since the y-component of the acceleration is -20 m/s², Newton's second law gives

$$\sum F_y = ma_y \quad \rightarrow \quad -49N + T = 5kg \cdot \left(-20\tfrac{m}{s^2}\right)$$

Solving the Equation

The tension is

$$-49N + T = 5kg \cdot \left(-20\tfrac{m}{s^2}\right)$$
$$-49N + T = -100N$$
$$T = -51N$$

This value is not possible. This situation is, in fact, impossible since the box cannot have an acceleration greater than 9.8 m/s² downwards if the weight is the only force directed downwards.

4.4 APPLICATIONS OF NEWTON'S LAWS

Following the problem-solving method presented in chapter 3 is important. Some precision can now be given.

Forces Acting on the Object

At the first step of the method, the forces acting on the object must be found. As 3 forces are now known, here is how all the forces acting on the object can be found.

Forces Acting on an Object

a) Weight
There is a force of gravity on all objects unless it is explicitly stated to neglect the mass.

b) Normal force
To find out if there is a normal force, ask yourself if the object touches another object (except a string). If so, there is a repulsive force acting on the object at every contact with another object.

c) Tension force
All strings exert a tension force.

d) Any other force explicitly given in the problem.

Choice of Axes

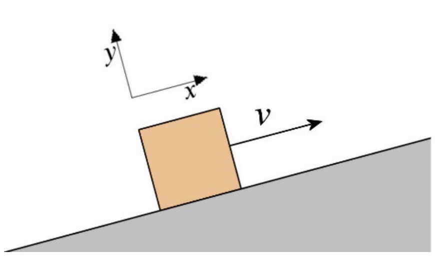
At the 3rd step of the method, axes must be chosen to resolve the forces into components. Let's see how these axes must be chosen.

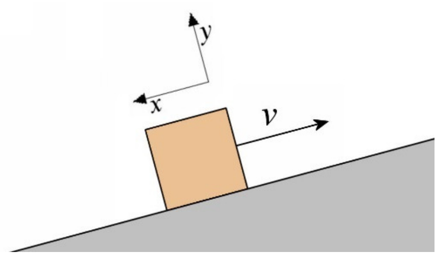
Object at Rest

If the object remains at rest, any direction can be used for the axes, as long as the x-axis is perpendicular to the y-axis.

Object Moving in a Straight Line

In chapters 4 and 5, objects move only in a straight line. In this case, the acceleration can only be in the direction of the velocity or in the opposite direction to the velocity. Thus, if the object is moving or is going to move in a straight line, the solution is greatly simplified if one axis is in the direction of the motion of the object or in the opposite direction to the motion of the object.





If the x-axis is put in the direction of the velocity (diagram to the left) or in the opposite direction to the velocity (diagram to the right), then all the acceleration is in the direction of the x-axis while the y-component of the acceleration will vanish. Therefore, the components of the acceleration are

$$a_x = a \qquad\qquad a_y = 0$$

If the y-axis is set in the direction of the velocity or in the opposite direction to the velocity, then the components of the acceleration are

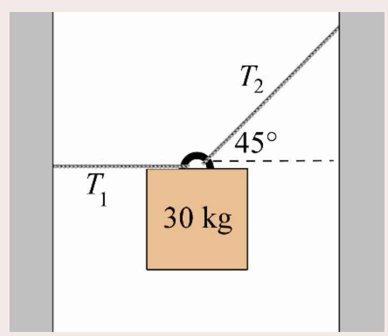
$$a_x = 0 \qquad\qquad a_y = a$$

Even if the axis is set in the opposite direction to the velocity, do not add a negative sign in front of a. This will instead change the sign of the value of a.

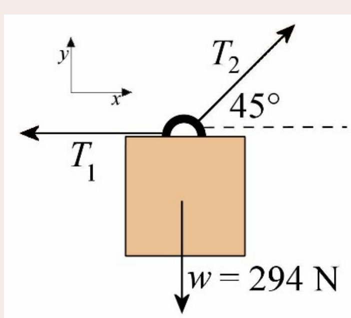
Examples with Only One Object

Example 4.4.1

What are the tension forces exerted by the two ropes supporting this mass?



The tension forces can be found using Newton's second law by considering the force acting on the 30 kg object.



Forces Acting on the Object

There are 3 forces acting on the mass.

1) The 294 N weight (w) directed downwards.
2) The tension force exerted by rope 1 (T_1).
3) The tension force exerted by rope 2 (T_2).

Sum of the Forces

To sum the forces, a table like this one can be used.

Forces	x	y
Weight	0	-294 N
Rope 1	$-T_1$	0
Rope 2	$T_2 \cos 45°$	$T_2 \sin 45°$

In this table, all the components of the forces are found according to the rules given earlier. To sum the x and y-components of the forces, just add the components in each column.

(Here's an important note: by using the angle between the force and the x-axis, you will have only cosines for x-component and only sines for the y-component. Moreover, you can never have different angles for x and y for the same force. For example, the angle is 45° for both components of T_2 here.)

The sum of the x-component of the force is (sum of the x column in the table)

$$\sum F_x = 0 - T_1 + T_2 \cos 45°$$

The sum of the y-component of the force is (sum of the y column in the table)

$$\sum F_y = -294N + T_2 \sin 45°$$

Newton's Second Law

The 30 kg mass is not accelerating. Therefore, the components of the acceleration are zero. The equations are thus

$$\sum F_x = ma_x \quad \rightarrow \quad 0 - T_1 + T_2 \cos 45° = 0$$
$$\sum F_y = ma_y \quad \rightarrow \quad -294N + T_2 \sin 45° = 0$$

Solving the Equations

The second equation can be solved to obtain T_2.

$$-294N + T_2 \sin 45° = 0$$
$$T_2 = 415.78N$$

This value can be substituted into the x-component equation, and the resulting equation can be solved for T_1.

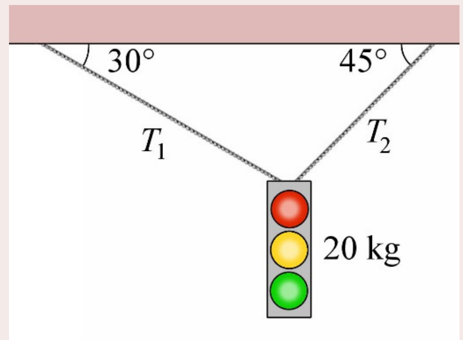
$$-T_1 + T_2 \cos 45° = 0$$

$$-T_1 + 415.78N \cos 45° = 0$$
$$T_1 = 294N$$

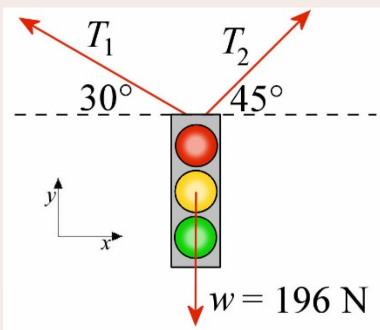
The tension force of the ropes are then 294 N (rope 1) and 415.78 N (rope 2).

Example 4.4.2

What are the tension forces exerted by the ropes which support this traffic light?



The tension forces can be found using Newton's second law by considering the force acting on the traffic light.



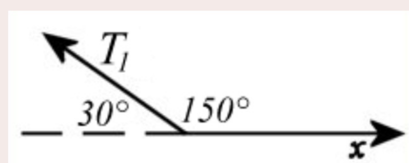
Forces Acting on the Object

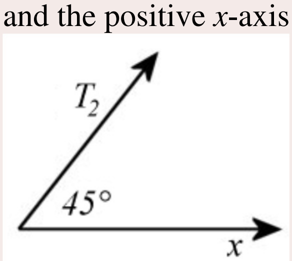
There are 3 forces acting on the traffic light.

1) The 196 N weight (w) directed downwards.
2) The tension force of rope 1 (T_1).
3) The tension force of rope 2 (T_2).

Sum of the Forces

It is important to find the correct angles for the tension forces. To resolve a vector into components, the angle between the force with the positive x-axis. For T_1, the angle between the force and the positive x-axis is 150°.





For T_2, the angle is 45°.

The table of force is then

Forces	x	y
Weight	0	-196 N
Rope 1	$T_1 \cos 150°$	$T_1 \sin 150°$
Rope 2	$T_2 \cos 45°$	$T_2 \sin 45°$

The sum of the x-component of the force is

$$\sum F_x = 0 + T_1 \cos 150° + T_2 \cos 45°$$

The sum of the y-component of the force is

$$\sum F_y = -196N + T_1 \sin 150° + T_2 \sin 45°$$

Newton's Second Law

Since the traffic light is not accelerating, the components of the acceleration are zero. Thus, the two equations are

$$\begin{aligned} \sum F_x = ma_x \quad &\rightarrow \quad T_1 \cos 150° + T_2 \cos 45° = 0 \\ \sum F_y = ma_y \quad &\rightarrow \quad -196N + T_1 \sin 150° + T_2 \sin 45° = 0 \end{aligned}$$

Solving the Equations

We have two equations and two unknowns. This system of equations can be solved with the method of your choice (substitution, Gauss-Jordan, Cramer or any other, remember your linear algebra course…). Here, the substitution method will be used. Thus, the first equation is solved for T_2

$$T_2 = \frac{-T_1 \cos 150°}{\cos 45°}$$

and the result is substituted in the second equation. Then, T_1 can be found.

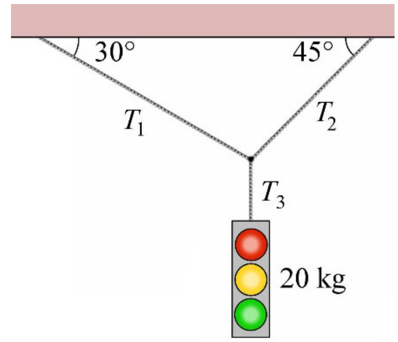
$$\begin{gathered} T_1 \cos 150° + T_2 \cos 45° = 0 \\ -196N + T_1 \sin 150° + \frac{-T_1 \cos 150°}{\cos 45°} \sin 45° = 0 \\ -196N + T_1 \left(\sin 150° - \cos 150° \tan 45° \right) = 0 \\ T_1 = \frac{196N}{\sin 150° - \cos 150° \tan 45°} \\ T_1 = 143.5N \end{gathered}$$

Using this value, T_2 can be found.

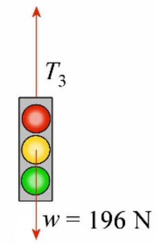
$$\begin{aligned} T_2 &= \frac{-T_1 \cos 150°}{\cos 45°} \\ &= 175.7N \end{aligned}$$

Once the answers are obtained, it is usually a good idea to substitute these values in the original two equations (sum of the x and y-components of the forces) to check if they work…

These examples may seem long, but they were done in great detail. Later, we will go a little faster. If you already feel comfortable to skip some of the steps, such as the table of force, go ahead.



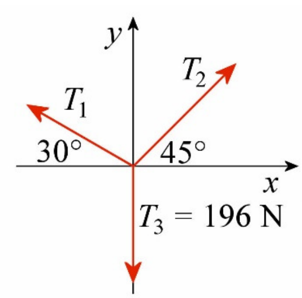
Here's a slight variation of the last example. Suppose that the traffic light is now suspended in this way. (There's an additional rope in this version, linking the traffic light to the node that connects the ropes.)



In this variant, the tension force of the third rope can be found with the sum of the forces on the traffic light (diagram to the left).

It is then quite easily found that the tension force T_3 must be 196 N.

Then the forces acting on the node connecting the three strings are considered. The following forces act on the node (diagram).

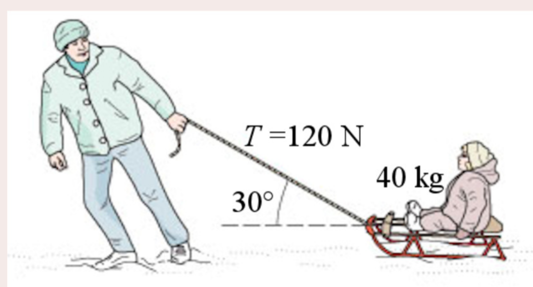


These forces are the same forces as the forces acting on the traffic light in the preceding example. The tension forces T_1 and T_2 are therefore the same as they were in the example.

All that to say that if there is a node connecting several strings, the solution of the problem is often found by summing the forces acting on the node.

Example 4.4.3

Lucien pulls with a force of 120 N on Adele's sled as shown in the diagram. It is assumed that there is no friction. (40 kg is the mass of Adele and the sled.)



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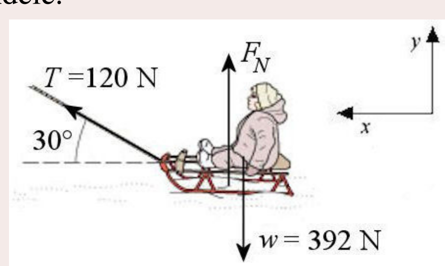
a) What is Adele's acceleration?
b) What is the magnitude of the normal force between the ground and the sled?

The acceleration and the normal force can be found using Newton's second law by considering the force acting on the sled and Adele.

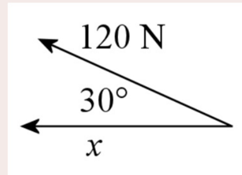
Forces Acting on the Object



There are 3 forces acting on the sled and Adele.

1) The 392 N weight (w) directed downwards.
2) A normal force (F_N) directed upwards.
3) The 120 N tension force (T) made by the rope.

Sum of the Forces



To resolve into components, the angle of the tension force must be known. With the x-axis used, the angle is 30°.

Then, the table of forces is

Forces	***x***	***y***
Weight	0	-392 N
Normal force	0	F_N
Tension force	120 N cos 30° = 103.92N	120 N sin 30° = 60 N

The sum of the x-component of the force is

$$\sum F_x = 103.92N$$

The sum of the y-component of the force is

$$\sum F_y = -392N + F_N + 60N$$

Newton's Second Law

As the velocity is in the direction of the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Newton's Second law therefore gives

$$\begin{aligned} \sum F_x = ma_x \quad &\rightarrow \quad 103,92N = 40kg \cdot a \\ \sum F_y = ma_y \quad &\rightarrow \quad -392N + F_N + 60N = 0 \end{aligned}$$

Solving the Equations

An answer can now be given to the two questions asked. This first equation gives the acceleration.

$$103.92N = 40kg \cdot a$$

$$a = 2,598 \tfrac{m}{s^2}$$

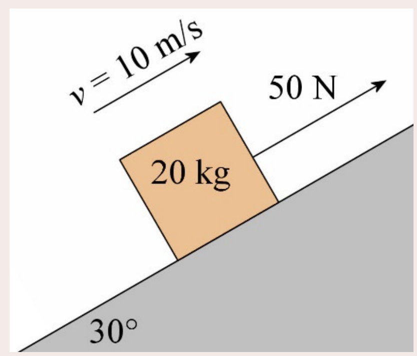
The second equation gives the magnitude of the normal force.

$$-392N + F_N + 60N = 0$$
$$F_N = 332N$$

Common mistake: Thinking that the normal force is always *mg* for an object on a horizontal surface

The last example shows that normal force is 332 N while the weight is 392 N. In this case, the normal force is smaller than the weight. It is smaller because Lucien pulls a little upwards with his rope. It, therefore, supports some of the weight, and this decreases the normal force. Thus, the normal force is not necessarily equal to the weight, even on a horizontal surface. Several students who want to go too quickly would not have done the sum of the vertical forces and would have simply assumed that the normal force is equal to the weight. That would be a mistake.

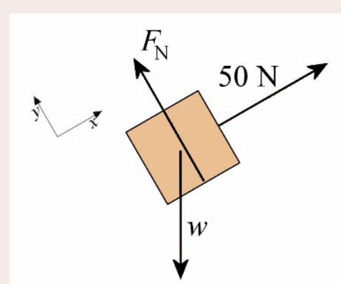
Example 4.4.4



This 20 kg box is moving uphill at 10 m/s on a slope. There is a 50 N force pulling the box towards the top of the slope. What will the speed of this box be 2 seconds later if there is no friction?

To find the speed 2 seconds later, the acceleration of the box is needed. Newton's 2nd law is used to find it by considering all the forces acting on the box.

Forces Acting on the Object



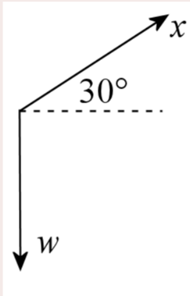
There are 3 forces acting on the box.

1) The 196 N weight (w) directed downwards.
2) The normal force (F_N) exerted by the slope.
3) The 50 N force.

The x-axis is tilted in the direction of the slope since the box is moving in this direction.

Sum of the Forces

The normal force is then directly in the direction of the y-axis and the 50 N force is directly in the direction of the x-axis. So, it is not difficult to find the components of



these forces. On the other hand, the situation is more complicated for the weight. Let's draw the force and x-axis (diagram to the left).

The angle between the x-axis and the force of gravity is -120 N (Remember the sign convention. Here, the positive direction is counterclockwise since it is in the rotation direction going from the positive x-axis towards the positive y-axis. When rotating from the positive x-axis towards the force, the rotation is clockwise. As this is in the opposite direction of the positive direction here, the angle is negative).

Thus, the table of force is

Forces	x	y
Weight	196 N cos -120° = -98 N	196 N sin -120° = -169.7 N
Normal Force	0	F_N
F	50 N	0

The sum of the x-component of the force is

$$\sum F_x = -98N + 50N$$

The sum of the y-component of the force is

$$\sum F_y = -169.7N + F_N$$

Newton's Second Law

As the velocity is in the direction of the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Newton's Second law therefore gives

$$\sum F_x = ma_x \quad \rightarrow \quad -98N + 50N = 20kg \cdot a$$
$$\sum F_y = ma_y \quad \rightarrow \quad -169.7N + F_N = 0$$

Solving the Equations

The acceleration can then be found with the first equation.

$$-98N + 50N = 20kg \cdot a$$
$$a = -2.4 \tfrac{m}{s^2}$$

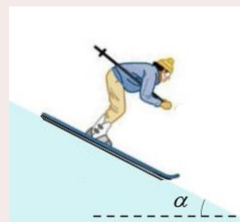
Since the velocity is positive (the axis chosen is in the direction of velocity), a negative acceleration means that this object is slowing down.

The velocity 2 seconds later can now be found.

$$\begin{aligned} v &= v_0 + at \\ &= 10\tfrac{m}{s} + -2.4\tfrac{m}{s^2} \cdot 2s \\ &= 5.2\tfrac{m}{s} \end{aligned}$$

The box is still moving uphill (since the velocity is positive), but it has actually slowed down, as predicted.

Example 4.4.5



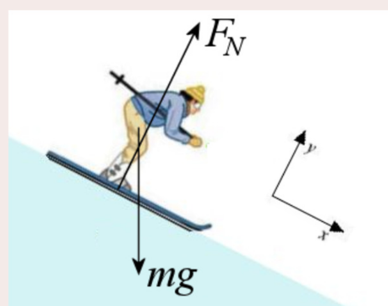
A skier of mass *m* is on a slope inclined at an angle α. What is the acceleration of the skier and what is the normal force exerted on the skier if there is no friction?

The acceleration and the normal force can be found using Newton's second law by considering the force acting on the skier. Even if there are no numerical values in this example, the solution is still done by using the problem-solving method given earlier.

Forces Acting on the Object

There are 2 forces acting on the skier.

1) The weight (mg) directed downwards.
2) The normal force (F_N) exerted by the slope.



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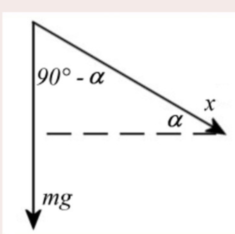
Sum of the Forces



The *x*-axis is tilted in the downhill direction because this is the direction of the motion of the skier.

The angle between the *x*-axis and the force of gravity is $-(90° - \alpha)$

The components of the forces are then

Forces	*x*	*y*
Weight	$mg\cos(-(90°-\alpha))$	$mg\sin(-(90°-\alpha))$
Normal force	0	F_N

Therefore, the sums of the components of the forces are

$$\sum F_x = mg\cos\left(-\left(90°-\alpha\right)\right)$$
$$\sum F_y = mg\sin\left(-\left(90°-\alpha\right)\right) + F_N$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis, the *x*-component of the acceleration is a and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Newton's Second law therefore gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad mg\cos\left(-\left(90°-\alpha\right)\right) = ma$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad mg\sin\left(-\left(90°-\alpha\right)\right) + F_N = 0$$

Solving the Equations

The acceleration is obtained with the first equation.

$$\cancel{m}g\cos\left(-\left(90°-\alpha\right)\right) = \cancel{m}a$$
$$a = g\cos\left(-\left(90°-\alpha\right)\right)$$
$$a = g\cos\left(90°-\alpha\right)$$
$$a = g\sin\alpha$$

This is the desired result. Some trigonometric identities were used to simplify it. These identities will be looked at more closely after this example. Finally, the normal force can be found with the second equation.

$$mg\sin\left(-\left(90°-\alpha\right)\right) + F_N = 0$$
$$F_N = -mg\sin\left(-\left(90°-\alpha\right)\right)$$
$$F_N = mg\sin\left(90°-\alpha\right)$$
$$F_N = mg\cos\alpha$$

Interesting note: the acceleration of an object on a frictionless slope is independent of the mass. In a toboggan race on a frictionless track, all the racers arrive at the same time at the bottom of the slope, regardless of their mass if they start simultaneously. This is not really shocking because the gravitational acceleration is the same for everybody.

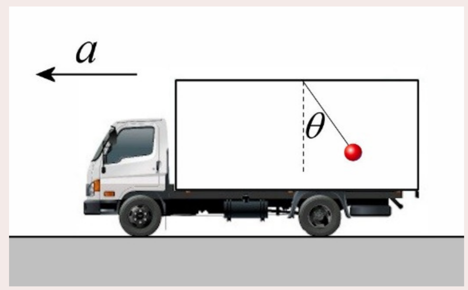
Reminder of some trigonometric identities

The following formulas are sometimes useful to simplify the solution of a problem without numerical data.

$$\cos(-x) = \cos x \qquad \cos(90° - x) = \sin x \qquad \cos(180° - x) = -\cos x$$
$$\sin(-x) = -\sin x \qquad \sin(90° - x) = \cos x \qquad \sin(180° - x) = \sin x$$
$$\cos(90° + x) = -\sin x \qquad \cos(180° + x) = -\cos x$$
$$\sin(90° + x) = \cos x \qquad \sin(180° + x) = -\sin x$$

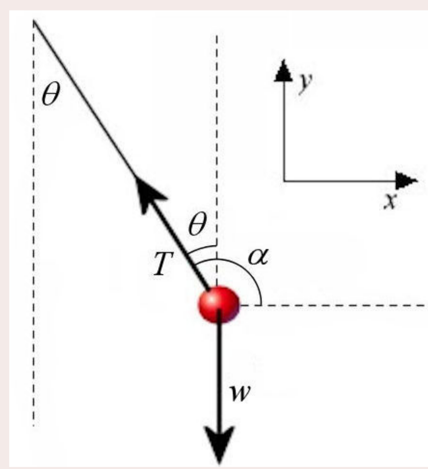
Example 4.4.6

A mass is hung with a string from the roof of the box of a truck. What is the angle between the pendulum and the vertical when the truck is accelerating at 5 m/s²?



www.canstockphoto.com/delivery-cargo-truck-13682465.html

The angle made by the rope is found by finding the direction of the tension force acting on the mass. The direction of the tension is α and the angle between the rope and the vertical is $\alpha - 90°$. (Note that an x-axis in the opposite direction to the velocity is used, just to do something different…) The direction of tension can be found with Newton's 2nd Law by considering the forces acting to the mass.



Forces acting on the Object

There are 2 forces acting on the mass at the end of the string.

1) The weight (mg) directed downwards.
2) The tension force (T) of the string.

Sum of the Forces

The table of force is then

Forces	x	y
Weight	0	$-mg$
Tension force	T_x	T_y

When the magnitude and direction of a vector is not known, as for the tension here, the solution is always easier to obtain using the components T_x and T_y rather than $T\cos(\alpha)$ and $T\sin(\alpha)$. Once the components obtained, the direction of the vector can be easily found.

The sums of the components of the forces are therefore

$$\sum F_x = T_x$$
$$\sum F_y = -mg + T_y$$

Newton's Second Law

As the x-axis is in the opposite direction of the velocity (which is towards the right because the mass must follow the truck), the x-component of the acceleration is -5 m/s² and the y-component of the acceleration vanishes. Thus

$$a_x = -5\tfrac{m}{s^2} \qquad\qquad a_y = 0$$

(As the acceleration is towards the left and the x-axis is towards the right, the acceleration is negative.)

Newton's Second law therefore gives

$$\sum F_x = ma_x \quad \rightarrow \quad T_x = m\cdot\left(-5\tfrac{m}{s^2}\right)$$
$$\sum F_y = ma_y \quad \rightarrow \quad -mg + T_y = 0$$

Solving the Equations

The equations give directly the components of the tension force.

$$T_x = m\cdot\left(-5\tfrac{m}{s^2}\right)$$
$$T_y = mg$$

Therefore, the direction of the tension force is

$$\tan\alpha = \frac{T_y}{T_x}$$
$$\tan\alpha = \frac{mg}{m\cdot\left(-5\tfrac{m}{s^2}\right)}$$
$$\tan\alpha = \frac{g}{\left(-5\tfrac{m}{s^2}\right)}$$
$$\alpha = 117.03°$$

(Since the divider is negative, 180° were added to the answer given by the calculator when the inverse tan function was calculated.)

Finally, the angle θ can be calculated.

$$\begin{aligned}\theta &= \alpha - 90° \\ &= 117.03° - 90° \\ &= 27.03°\end{aligned}$$

Examples with Several Objects

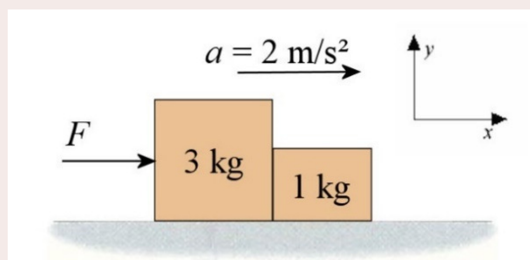
When there are several objects in the system, two options are available:

A) Treat all the objects as a single object.
B) Treat all the objects separately.

In the following examples, these two ways of considering the system will be employed according to what is asked.

Example 4.4.7

The boxes in the diagram are pushed with a force F. The boxes then accelerate at 2 m/s² towards the right (and there is no friction between the ground and the boxes).

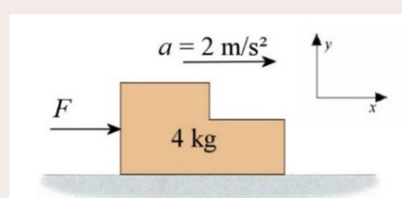


a) What is the magnitude of the force F?

To calculate this, the two boxes can be considered as a single 4 kg object. The force can then be found with Newton's 2nd law by considering the forces acting on this 4 kg block.

Forces Acting on the Object

There are 3 forces on this 4 kg object.



1) The 39.2 N weight directed downwards.
2) The normal (F_N) force directed upwards.
3) The force F directed towards the right.

Sum of the Forces

The sums of the forces are

$$\sum F_x = F$$
$$\sum F_y = -39.2N + F_N$$

Newton's Second Law

As the velocity is in the direction of the x-axis, the x-component of the acceleration is 2 m/s² and the y-component of the acceleration vanishes. Thus

$$a_x = 2\tfrac{m}{s^2} \qquad\qquad a_y = 0$$

Newton's Second law therefore gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad F = 4kg \cdot 2\tfrac{m}{s^2}$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -39.2N + F_N = 0$$

Solving the Equations

The second equation could allow us to find the normal force exerted by the ground on the boxes, but this information is not needed. The force F can be found with the first equation.

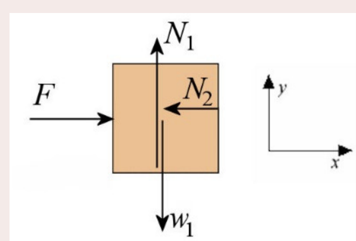
$$F = 4kg \cdot 2\tfrac{m}{s^2}$$
$$= 8N$$

b) What is the normal force between the two boxes?

When asked to find a force between two objects, you must imperatively split the system and consider the forces acting on one of the objects.

Let's see the results if the 3 kg box is used.

Forces Acting on the Object



There are 4 forces acting on the 3 kg box.

1) The 29.4 N weight (w_1) directed downwards.
2) A normal force made by the ground (N_1) directed upwards.
3) A normal force made by the 1 kg box (N_2) directed towards the left.
4) The 8 N force (F) directed towards the right.

Sum of the Forces

The sums of the forces are

$$\sum F_x = F - N_2$$
$$\sum F_y = -29.4N + N_1$$

Newton's Second Law

As the velocity is in the direction of the x-axis, the x-component of the acceleration is 2 m/s² and the y-component of the acceleration vanishes. Thus

$$a_x = 2\tfrac{m}{s^2} \qquad\qquad a_y = 0$$

Newton's Second law therefore gives

$$\sum F_x = ma_x \qquad \to \qquad 8N - N_2 = 3kg \cdot 2\tfrac{m}{s^2}$$
$$\sum F_y = ma_y \qquad \to \qquad -29,4N + N_1 = 0$$

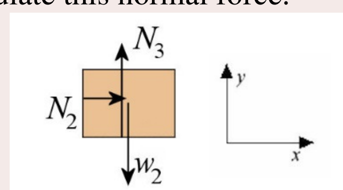
Solving the Equations

N_2 can then be calculated with the first equation.

$$8N - N_2 = 3kg \cdot 2\tfrac{m}{s^2}$$
$$8N - N_2 = 6N$$
$$N_2 = 2N$$

The 1 kg box could also have been considered to calculate this normal force.

Forces Acting on the Object



There are 3 forces acting on the 1 kg box.

1) The 9.8 N weight (w_2) directed downwards.
2) A normal force (N_3) made by the ground directed upwards.
3) A normal force (N_2) made by the 3 kg box directed to the right.

The same symbol was used for the normal force made by the 3 kg box on the 1 kg box and for the normal force made by the 1 kg box on the 3 kg box since these two forces are related by Newton's third law and must have the same magnitude.

The force F does not act on this box. If the person pushes on the 3 kg box, he is not touching the 1 kg box.

Sum of the Forces

The sums of the forces are

$$\sum F_x = N_2$$
$$\sum F_y = -9.8N + N_3$$

Newton's Second Law

As the velocity is in the direction of the x-axis, the x-component of the acceleration is 2 m/s² and the y-component of the acceleration vanishes. Thus

$$a_x = 2\tfrac{m}{s^2} \qquad\qquad a_y = 0$$

Newton's Second law therefore gives

$$\sum F_x = ma_x \quad \rightarrow \quad N_2 = 1kg \cdot 2\tfrac{m}{s^2}$$
$$\sum F_y = ma_y \quad \rightarrow \quad -9.8N + N_3 = 0$$

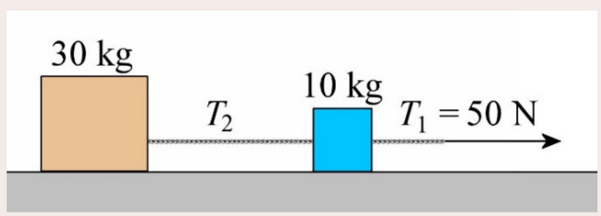
Solving the Equations

N_2 is found with the first equation.

$$N_2 = m_2 a$$
$$= 1kg \cdot 2\tfrac{m}{s^2} = 2N$$

Example 4.4.8

Two boxes are pulled with a string as illustrated in the diagram. (There is no friction between the ground and the boxes.) What is the tension of the string connecting the two boxes?



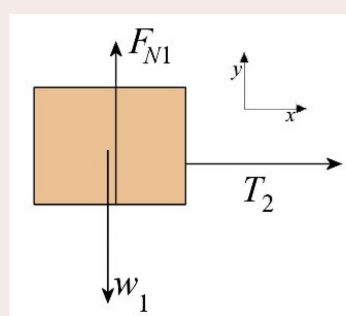
Since tension is a force between blocks, the forces acting on each block must be found. Then, the tension can be found with Newton's 2nd law.

30 kg block

Forces Acting on the Object

There are 3 forces acting on the 30 kg box.

1) The 294 N weight directed downwards.
2) A normal force (F_{N1}) made by the ground directed upwards.
3) The tension force (T_2) directed towards the right.



Sum of the Forces

As the tension force is a horizontal force, only the sum of the *x*-component of the force can be considered.

$$\sum F_x = T_2$$

Newton's Second Law

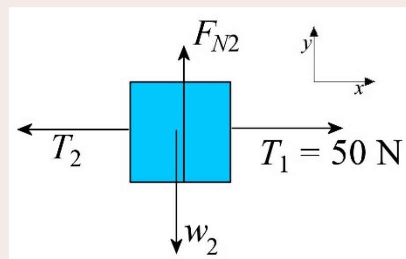
As the velocity is in the direction of the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

The *x*-component of Newton's Second law gives

$$\sum F_x = ma_x \quad \rightarrow \quad T_2 = 30kg \cdot a$$

10 kg block



Forces Acting on the Object

There are 4 forces acting on the 10 kg box.

1) The 98 N weight directed downwards.
2) A normal force (F_{N2}) made by the ground directed upwards.
3) The tension force (T_2) directed towards the left.
4) The tension force (T_1 = 50 N) directed towards the right.

Sum of the Forces

The sum of the *x*-component of the force is

$$\sum F_x = 50N - T_2$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

The *x*-component of Newton's Second law gives

$$\sum F_x = ma_x \quad \rightarrow \quad 50N - T_2 = 10kg \cdot a$$

Solution of the equations

We thus have these 2 equations.

$$T_2 = 30kg \cdot a$$
$$50N - T_2 = 10kg \cdot a$$

Adding those two equation, the result is

$$(T_2) + (50N - T_2) = 30kg \cdot a + 10kg \cdot a$$
$$50N = 40kg \cdot a$$
$$a = 1.25 \tfrac{m}{s^2}$$

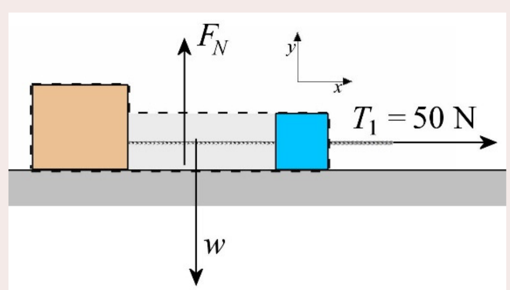
This trick, which consists of adding the equations, will always work (until we arrive at Chapter 12). When this is done, all the forces between two objects of the system and all the tensions of the ropes connecting two objects of the system will cancel each other. If they do not cancel, something is wrong with your axes or with your equations of Newton's second law.

Once the acceleration is obtained, the acceleration can easily be found. The first equation gives

$$\begin{aligned} T_2 &= 30kg \cdot a \\ &= 30kg \cdot 1.25 \tfrac{m}{s^2} \\ &= 37.5N \end{aligned}$$

Note that the acceleration could also have been found by considering a 40 kg mass pulled by a 50 N force. In this case, the equation for Newton's 2nd law would have given

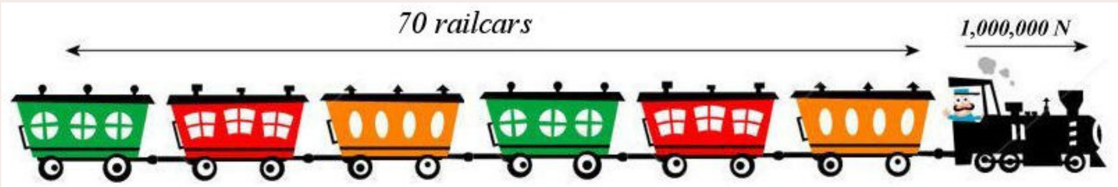
$$\sum F_x = ma_x \quad \rightarrow \quad 50N = 40kg \cdot a$$



This gives an acceleration of 1.25 m/s². With this acceleration, the tension could then have been found with the equation of forces in *x* acting on one of the blocks.

Example 4.4.9

A train, initially at rest, is composed of a 100-ton locomotive and of 70 10-ton railcars. To move this train, a 1,000,000 N force is exerted by the locomotive. Then, what is the tension in the coupler between the 10th and 11th railcars?



www.shutterstock.com/pic-45673156/stock-vector-train-and-wagon-alternatives-with-different-cartoon-characters.html

The tension can be found by splitting the train into two parts.

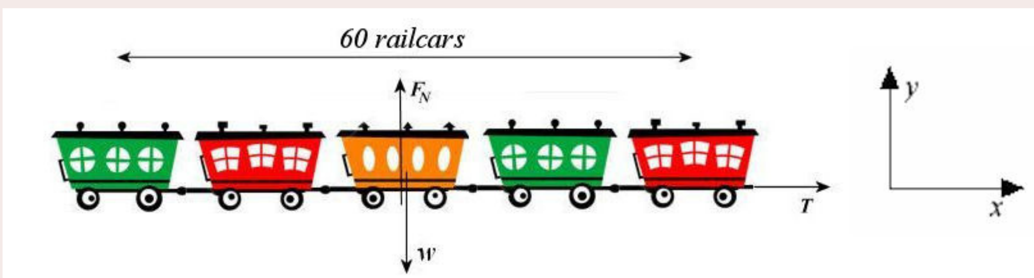
1) The last 60 railcars.
2) The locomotive and the first 10 railcars.

By examining the force acting on any of these parts, the tension of the coupler can be found with Newton's second law. We'll consider the rear of the train.

Forces Acting on the Object

There are 3 forces acting on this part of the train.

1) The weight (w) directed downwards.
2) A normal force exerted by the ground (F_N) directed upwards.
3) The tension force (T) of the coupler directed towards the right.



Sum of the Forces

The sums of the forces are

$$\sum F_x = T$$
$$\sum F_y = -w + F_N$$

Newton's Second Law

As the velocity is in the direction of the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Thus, Newton's second law gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad T = 600{,}000kg \cdot a$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad -P + F_N = 0$$

Solving the equations

There is a little problem. We're looking for T but the tension cannot be found without knowing a and there is no way to obtain this acceleration with the second equation. Actually, a can be found by examining the forces on the entire train

Forces Acting on the Object

There are 3 forces acting on the train.

1) The weight directed downwards.
2) A normal force exerted by the ground directed upwards.
3) The force of 10^6 N made by the locomotive directed towards the right.

Sum of the Forces

The sum of the x-component of the force is

$$\sum F_x = 10^6 N$$

Newton's Second Law

As the velocity is in the direction of the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Thus, Newton's second law gives

$$\sum F_x = ma_x \quad \rightarrow \quad 10^6 N = (800{,}000kg) \cdot a$$

Solving the Equations

With this equation, the acceleration can be found.

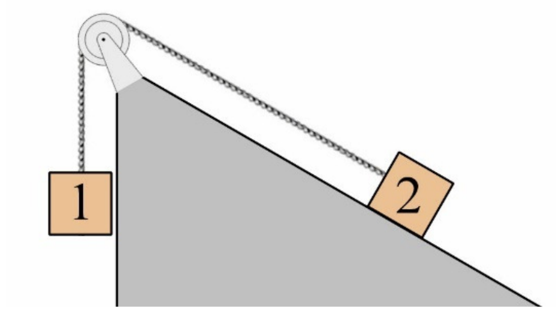
$$10^6 N = (800{,}000kg) \cdot a$$
$$a = 1.25 \tfrac{m}{s^2}$$

With the acceleration, the tension in the coupler can be found.

$$\begin{aligned} T &= 600,000kg \cdot a \\ &= 600,000kg \cdot 1.25\tfrac{m}{s^2} \\ &= 750,000N \end{aligned}$$

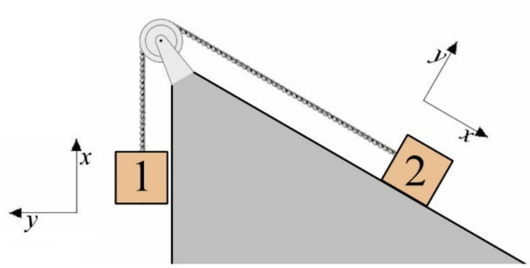
How to Choose the Axes if the Objects in the System Move in Different Directions

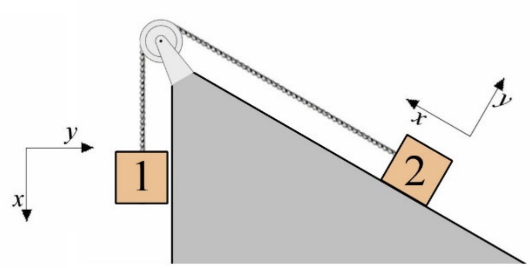
In some systems, there may be objects moving in different directions. Here is an example of such a system.



Let's assume that the mass of block 1 is large enough for block 1 to move downwards. In this case, block 2 will obviously move towards the top of the slope. Thus, this is a system in which objects move in different directions.

But it was said that an axis in the direction of velocity or in the opposite direction to the velocity must be used. Which direction should be chosen then if the velocities of the two blocks are in different directions? In fact, different axes system can be used for each block. For example, axes systems with the *x*-axis in the direction of speed for each object can be used or axis systems with the *x*-axis in the opposite direction to the speed for each object can be used.

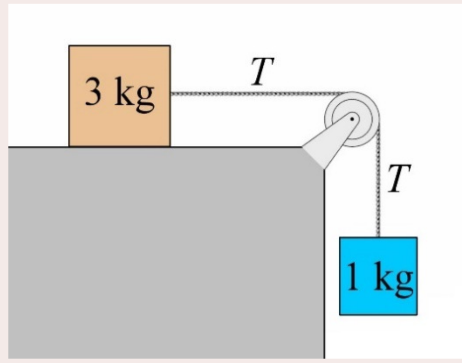




Avoid using axes systems with the *x*-axis in the direction of the velocity for some objects and axes systems with the *x*-axis in the opposite direction to the velocity for other objects. They can be used in theory, but this leads to some complications that we would like to avoid.

If the objects remain at rest, just imagines that they move anyway and choose x-axes that are all in the direction of the velocity of each object or that are all in the opposite direction to the velocity of each object.

Example 4.4.10



What are the tension of the string and the acceleration of the boxes in the system shown in the diagram on the right? (There is no friction between the ground and the 3 kg box.)

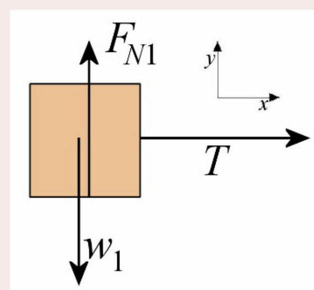
The acceleration and the tension force are found with Newton's second law by considering the forces exerted on each box.

3 kg Block

Forces Acting on the Object

There are 3 forces acting on the 3 kg box.

1) The 29.4 N weight (w_1) directed downwards.
2) A normal force (F_N) made by the ground directed upwards.
3) The tension force of the string (T) directed towards the right.



Sum of the Forces

The sums of the forces are

$$\sum F_x = T$$
$$\sum F_y = -29.4N + F_N$$

An x-axis towards the right (which will be the direction of the velocity) is used.

Newton's Second Law

As the velocity is in the direction of the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

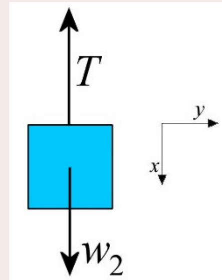
$$a_x = a \qquad\qquad a_y = 0$$

Thus, Newton's second law gives

$$\sum F_x = ma_x \quad\rightarrow\quad T = 3kg \cdot a$$
$$\sum F_y = ma_y \quad\rightarrow\quad -29.4N + F_N = 0$$

1 kg Block

Forces Acting on the Object



There are 2 forces acting on the 1 kg box.

1) The 9.8 N weight (w_2) directed downwards.
2) The tension force of the string (T) directed upwards.

Sum of the Forces

The sum of the force is

$$\sum F_x = 9.8N - T$$

The x-axis is now directed downwards since this is the direction of the velocity of this block. We thus used two different axes systems for each box, but the x-axis is in the direction of the velocity for both blocks.

Newton's Second Law

As the velocity is in the direction of the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Thus, Newton's second law gives

$$\sum F_x = ma_x \quad \rightarrow \quad 9.8N - T = 1kg \cdot a$$

Solving the equations

The following system of equations must be solved to find the acceleration and the tension force.

$$T = 3kg \cdot a$$
$$-29.4N + F_N = 0$$
$$9.8N - T = 1kg \cdot a$$

By adding these first and last of these equations, the acceleration can be found.

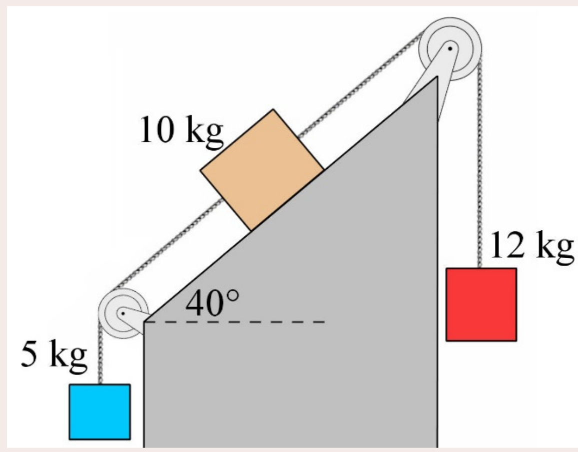
$$(T) + (9.8N - T) = 1kg \cdot a + 3kg \cdot a$$
$$9.8N = 4kg \cdot a$$
$$a = 2.45 \tfrac{m}{s^2}$$

With the acceleration, the tension can be found.

$$\begin{aligned} T &= 3kg \cdot a \\ &= 3kg \cdot 2.45 \tfrac{m}{s^2} \\ &= 7.35N \end{aligned}$$

Note that the tension of the string is not just equal to the weight of the suspended mass. The tension is equal to the weight only if the system is not accelerating. When it is accelerating, the tension of the string is no longer equal to the weight of the suspended mass.

Example 4.4.11

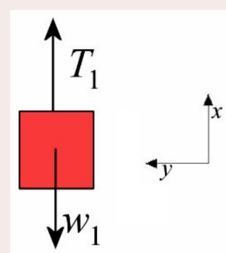


What are the tensions of the strings and the acceleration of the boxes in this system? (There is no friction between the slope and the 10 kg box.)

The acceleration and the tension forces are found with Newton's second law by considering the forces exerted on each box.

12 kg Block

Forces Acting on the Object



There are 2 forces acting on the 12 kg block.

1) The 117.6 N weight (w_1) directed downwards.
2) The tension force of the string (T_1) directed upwards.

Sum of the Forces

The sum of the forces is

$$\sum F_x = -117.6N + T_1$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis or in the direction opposed to the *x*-axis (we don't know if this block will move upwards or downwards), the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

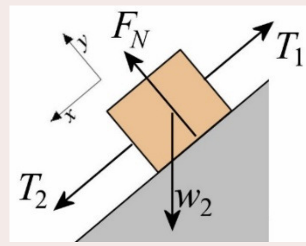
Therefore, Newton's second law gives

$$\sum F_x = ma_x \quad \rightarrow \quad -117.6N + T_1 = 12kg \cdot a$$

10 kg Block

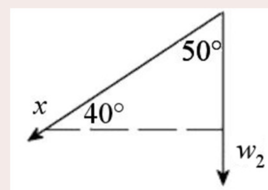
Forces Acting on the Object

There are 4 forces acting on the 10 kg block.



1) The 98 N weight (w_2) directed downwards.
2) The normal force exerted by the slope (F_N).
3) The tension force of the string (T_1) directed uphill.
4) The tension force of the string (T_2) directed downhill.

Sum of the forces



Most of the forces are directly in the direction of an axis except for the weight. The angle between this force and the positive *x*-axis is -50 ° as shown in the diagram.

The table of force for the 10 kg box is

Forces	*x*	*y*
Weight	98 N·cos(-50°)	98 N·sin(-50°)
Normal force	0	F_N
Tension force 1	- T_1	0
Tension force 2	T_2	0

The sums of the forces are therefore

$$\sum F_x = 98N \cdot \cos\left(-50°\right) - T_1 + T_2$$
$$\sum F_y = 98N \cdot \sin\left(-50°\right) + F_N$$

Newton's Second Law

As the velocity is in the direction of the *x*-axis or in the direction opposed to the *x*-axis, the *x*-component of the acceleration is *a* and the *y*-component of the acceleration vanishes. Thus

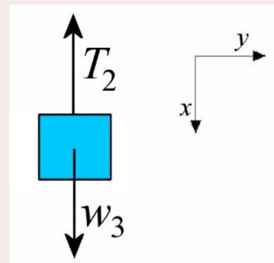
$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \qquad \rightarrow \qquad 98N \cdot \cos(-50°) - T_1 + T_2 = 10kg \cdot a$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad 98N \cdot \sin(-50°) + F_N = 0$$

5 kg Block

Forces Acting on the Object



There are 2 forces acting on the 5 kg block.

1) The 49 N weight (w_3) directed downwards.
2) The tension force of the string (T_2) directed upwards.

Sum of the Forces

The sum of the forces is

$$\sum F_x = 49N - T_2 = m_3 a$$

Newton's Second Law

As the velocity is in the direction of the x-axis or in the direction opposed to the x-axis, the x-component of the acceleration is a and the y-component of the acceleration vanishes. Thus

$$a_x = a \qquad\qquad a_y = 0$$

Therefore, Newton's second law gives

$$\sum F_x = ma_x \quad \rightarrow \quad 49N - T_2 = 5kg \cdot a$$

Solving the Equations

The acceleration and the tension forces can now be found. The following system of three equations must be solved to obtain the values of theses unknowns.

$$-117.6N + T_1 = 12kg \cdot a$$
$$-T_1 + T_2 + 98N \cdot \cos(-50°) = 10kg \cdot a$$
$$49N - T_2 = 5kg \cdot a$$

(The equation for the y-component of Newton's 2^{nd} law of the 10 kg block is omitted because it is not needed to find acceleration and tension.)

As it was done previously, this system can be solved by adding the three equations to eliminate the tension forces.

$$\left(-117.6N+T_1\right)+\left(98N\cdot\cos\left(-50°\right)-T_1+T_2\right)+\left(49N-T_2\right)=12kg\cdot a+10kg\cdot a+5kg\cdot a$$

$$-117.6N+98N\cdot\cos\left(50°\right)+49N=\left(12kg+10kg+5kg\right)\cdot a$$

$$-5.607N=27kg\cdot a$$

$$a=-0.2077\tfrac{m}{s^2}$$

Now that the acceleration is known, the tension forces can be calculated.

$$-117.6N+T_1=12kg\cdot a$$

$$-117.6N+T_1=12kg\cdot\left(-0.2077\tfrac{m}{s^2}\right)$$

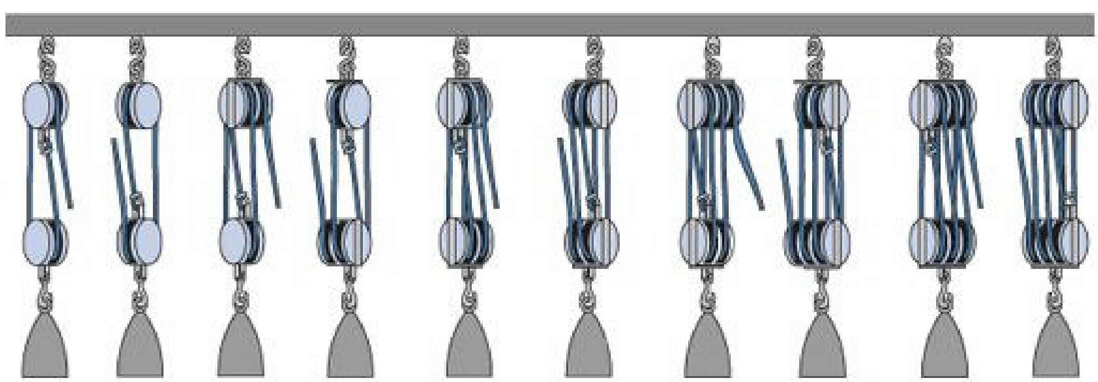
$$T_1=115.11N$$

$$49N-T_2=5kg\cdot a$$

$$49N-T_2=5kg\cdot\left(-0.2077\tfrac{m}{s^2}\right)$$

$$T_2=50.04N$$

The string can also pass several times in the pulley as shown in this diagram.

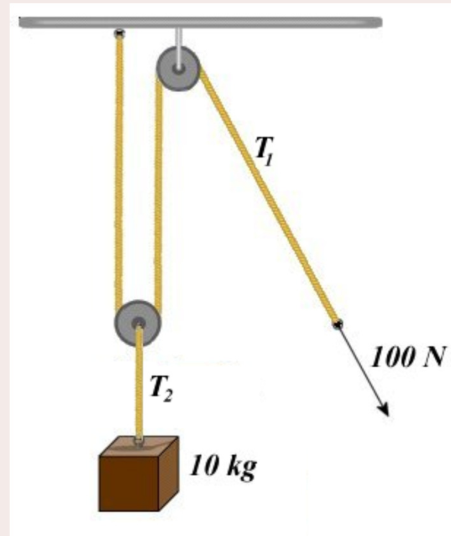


www.justonly.com/blog/?cat=8

In such a case, the string passing through the pulley must be looked at very carefully to find how often the string pulls on the pulley. Sometimes, the equation of the forces exerted on the pulley must be found to calculate a tension force. The mass of the pulley will always be neglected unless stated otherwise. The following examples illustrate these two ideas.

Example 4.4.12

What is the acceleration of the 10 kg crate in this diagram?

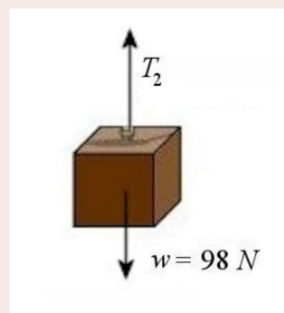


science.howstuffworks.com/transport/engines-equipment/pulley.htm

The acceleration is found with Newton's second law by considering the forces exerted on the 10 kg box.

10 kg Box

Forces Acting on the Object



There are 2 forces acting on the 10 kg box.

1) The 98 N weight (w) directed downwards.
2) The tension force of the string (T_2) directed upwards.

Sum of the Forces

Using a y-axis pointing upwards, the sum of the forces is

$$\sum F_y = -98N + T_2$$

Newton's Second Law

As the velocity is in the direction of the y-axis or opposed to the y-axis (we don't know if this block will move upwards or downwards), the x-component of the acceleration vanishes and the y-component of the acceleration is a. Thus

$$a_x = 0 \qquad\qquad a_y = a$$

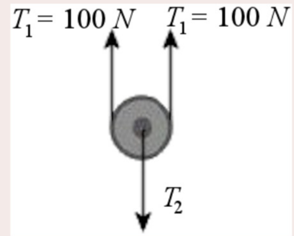
Therefore, Newton's second law gives

$$\sum F_y = ma_y \quad \rightarrow \quad -98N + T_2 = 10kg \cdot a$$

The acceleration cannot be known without knowing the tension T_2. This tension is not known, but the rope with this tension is connected to a pulley. Therefore, we need to look at the forces on the pulley.

Pulley

Forces Acting on the Object



There are 3 forces acting on the pulley.

1) The tension force of the string (T_1) directed upwards.
2) The tension force of the string (T_1) directed upwards.
3) The tension force of the string (T_2) directed downwards.

We will neglect the mass of the pulley (up to Chapter 12). That's why there is no weight.

Sum of the Forces

Using a y-axis pointing upwards, the sum of the forces is

$$\sum F_y = 100N + 100N - T_2$$

Newton's Second Law

As the velocity is in the direction of the y-axis or opposed to the y-axis, the x-component of the acceleration vanishes and the y-component of the acceleration is a. Thus

$$a_x = 0 \qquad\qquad a_y = a$$

Therefore, Newton's second law gives

$$\sum F_y = ma_y \quad \rightarrow \quad 100N + 100N - T_2 = m_{pulley} a$$

Neglecting the mass of the pulley (m_{pulley} = 0), this becomes

$$100N + 100N - T_2 = 0$$

Solving the Equations

Now T_2 can be found.

$$100N + 100N - T_2 = 0$$
$$T_2 = 200N$$

This value can then be used to calculate the acceleration.

$$-98N + T_2 = 10kg \cdot a$$

$$-98N + 200N = 10kg \cdot a$$
$$a = 10.2 \tfrac{m}{s^2}$$

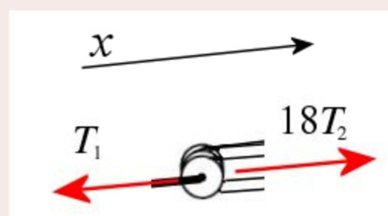
Example 4.4.13

In 2008, the submarine Onondaga was hauled on land beside Pointe-au-Père's wharf, not far from Rimouski. To achieve this, the 1400-ton submarine was towed up a 4° slope with the help of pulleys. In all, the steel cable pulled 18 times on the pulley attached to the submarine. The submarine was on small carts rolling on rails, thereby limiting the friction between the ground and the submarine so that it will be neglected here. With what force did they have to pull on the cable in order to move the submarine at a constant speed?

www.thebattleofatlanticmuseum.ca/page3/page3.html
www.cstephenmurray.com/onlinequizes/physics/simplemachines/pulleybasics.htm

Since the rope we are interested in is connected to a pulley, we begin by examining the forces acting on the pulley.

Pulley



Forces Acting on the Object

There are 19 forces acting on the pulley.

1) 18 times the tension force of the rope (T_2) towards the positive x-axis.
2) The tension force of the rope (T_1) towards the negative x-axis.

(There is no weight since the mass of the pulley is neglected.)

Sum of the Forces

The sum of the forces is

$$\sum F_x = 18T_2 - T_1$$

Newton's Second Law

Since the velocity is constant, there is no acceleration. Therefore, Newton's second law leads to the following equation.

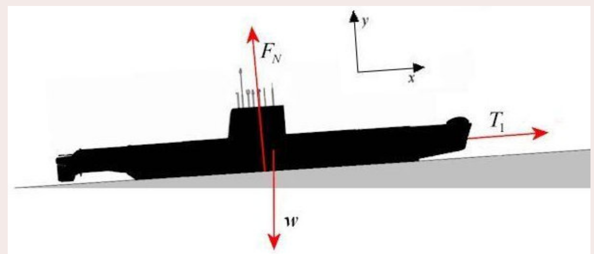
$$\sum F_x = ma_x \quad \rightarrow \quad 18T_2 - T_1 = 0$$

We see that we cannot find T_2 without knowing T_1. As the rope having the tension T_1 is attached to the submarine, we have to go and examine the forces acting on the submarine.

Submarine

Forces Acting on the Object

There are 3 forces acting on the submarine.



1) The weight (mg) directed downwards.
2) The normal force (F_N) exerted by the slope.
3) The tension force of the rope (T_1) towards the positive x-axis.

Sum of the Forces

The table of forces is

Forces	*x*	*y*
Weight	mg cos(-94°)	mg sin(-94°)
Normal force	0	F_N
Tension force	T_1	0

The sum of the x-component of the force is

$$\sum F_x = mg\cos(-94°) + T_1$$

(The sum of the y-component of the force is of no use here.)

Newton's Second Law

Since the velocity is constant, there is no acceleration. Therefore, Newton's second law leads to the following equation.

$$\sum F_x = ma_x \quad \rightarrow \quad mg\cos(-94°) + T_1 = 0$$

Solving the Equations

The tension can now be found with this last equation.

$$mg\cos(-94°)+T_1=0$$
$$T_1=-mg\cos(-94°)$$
$$T_1=-1,400,000kg\cdot 9,8\tfrac{N}{kg}\cdot\cos(-94°)$$
$$T_1=957,058N$$

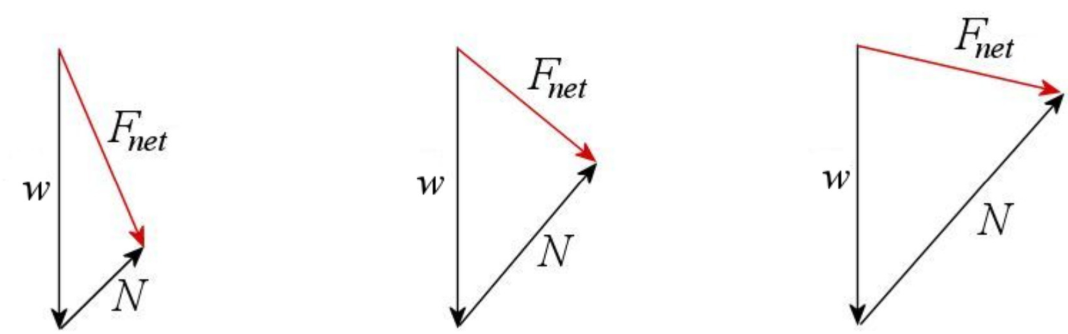
With this value, the tension T_1 can now be found.

$$-T_1+18T_2=0$$
$$T_2=\frac{T_1}{18}$$
$$T_2=53,170N$$

4.5 DIRECTION OF THE NET FORCE

Sometimes, it can be difficult to find the direction of the net force (i.e. the sum of forces) acting on an object if the magnitude of each force is not known precisely. For example, suppose the direction of the net force on a skier going downhill has to be found.

There are not many forces acting on this skier. There are only the weight and a normal force (if friction is neglected). Obviously, the weight is directed downwards, and the normal force has a direction perpendicular to the slope. However, it is hard to find the direction of the resultant force without prior knowledge of the magnitude of these forces. For example, the three following directions can be obtained by changing the magnitude of the normal force.



Fortunately, Newton's second law can help. This law says that

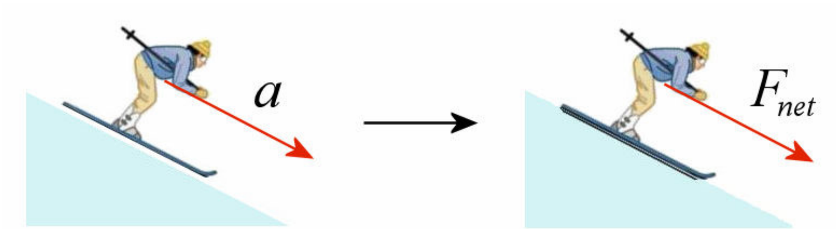
$$\vec{F}_{net}=m\vec{a}$$

In such a vector equation where m is necessarily positive, the direction of the vector F_{net} must be identical to the direction the vector a. It follows that

Direction of the Net Force

The net force is always in the same direction as the acceleration.

It then becomes much easier to find the direction of the net force. As it is obvious that the skier is accelerating downhill, the direction of the net force is also directed towards the bottom of the hill.



SUMMARY OF EQUATIONS

Gravitational Force (F_g) or Weight (w) (Formula Valid only near the Surface of the Earth)

1) Magnitude of the force

$$w = mg \qquad \text{where } g = 9.8 \tfrac{N}{kg}$$

2) Direction of the force

Downwards (towards the centre of the Earth)

3) Application point of the force

From the centre of mass of the object.
(See the chapter on the centre of mass.
For now, take a point roughly at the centre of the object.)

Normal Force (N or F_N)

1) Magnitude of the force

To be determined with Newton's laws.

2) Direction of the force

Repulsion between objects, perpendicular to the contact surface.

3) Application point of the force

Contact surface between objects.

Tension Force (*T* or F_T)

1) Magnitude of the force
   Given or to be determined with Newton's laws.

2) Direction of the force
   The rope pulls in the direction of the rope.

3) Application point of the force
   Where the rope is fixed to the object.

Direction of the Net Force

The net force is always in the same direction as the acceleration.

EXERCISES

4.1 The Gravitational Force

1. What is the gravitational force on a 100 kg person in the following situations?

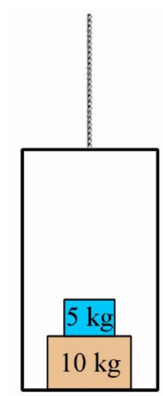
   a) The person is standing on the floor.
   b) The person is in free fall.
   c) The person is standing on the floor and is holding a 20 kg flour bag.

4.2 The Normal Force

2. William, whose mass is 72 kg, is in an elevator.

   a) What is the normal force acting on William if the elevator moves up with a constant speed of 5 m/s?
   b) What is the normal force acting on William when the elevator goes upwards with a speed of 5 m/s if the speed is increasing at a rate of 2 m/s²?
   c) What is the normal force acting on William when the elevator goes upwards with a speed of 5 m/s if the speed is decreasing at a rate of 3 m/s²?

3. Two boxes are placed one on top of the other in an elevator.



   What are the normal forces acting on each block if the elevator goes up at 8 m/s and is slowing down at a rate of 1 m/s²?

4.3 The Tension Force

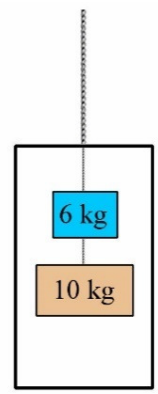
4. A helicopter lifts a 300 kg item during the construction of a dome.

    a) What is the tension force exerted by the rope if the helicopter does not accelerate?
    b) What is the tension force exerted by the rope if the helicopter has a 3 m/s² acceleration directed upwards?
    c) What is the tension force exerted by the rope if the helicopter has a 2 m/s² acceleration directed downwards?

www.chegg.com/homework-help/questions-and-answers/a-6480-kg-helicopter-accelerates-upward-at063-m-s2-while-lifting-a1190-kg-frame-at-a-const-q366433

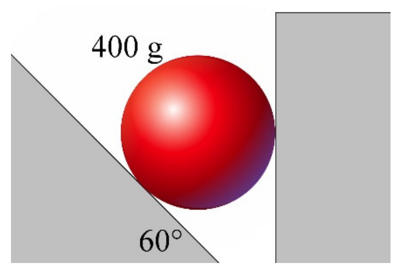
5. Two boxes are suspended from the ceiling of an elevator.

    a) What are the tensions of the strings if the elevator has an acceleration of 2.4 m/s² directed downwards?
    b) What is the maximum upwards acceleration that the elevator can have without breaking the strings (if they break when their tension exceeds 200 N)?

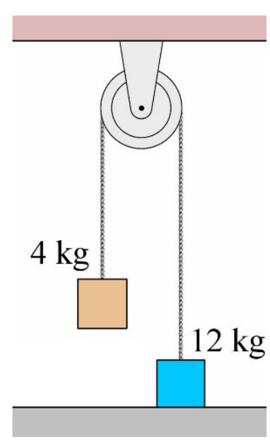


4.4 Applications of Newton's Laws

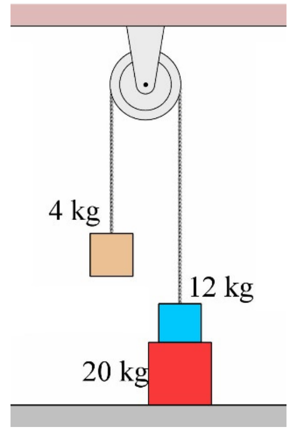
6. What are normal forces acting on this 400 g ball?



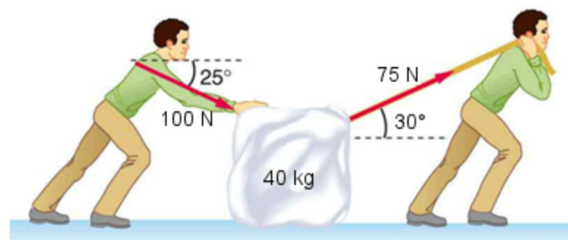
7. What is the normal force made by the ground acting on the 12 kg box in this situation?



8. What are the normal forces acting on the 12 kg and the 20 kg boxes in this situation?



9. Gontran (on the left) and Philemon (on the right) move their big snowball by exerting the forces shown in the diagram. There is no friction between the snowball and ground.

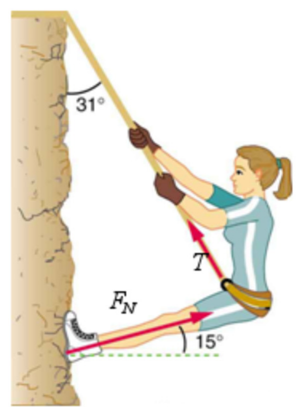


cnx.org/content/m42139/latest/

a) What is the acceleration of the snowball?
b) What is the magnitude of normal force exerted by the ground on the snowball?

10. Irina, whose mass is 60 kg, is climbing a cliff. At some point, she finds herself in the position shown in the diagram.

a) What is the tension of the rope?
b) What is the magnitude of the normal force acting on Irina's feet?



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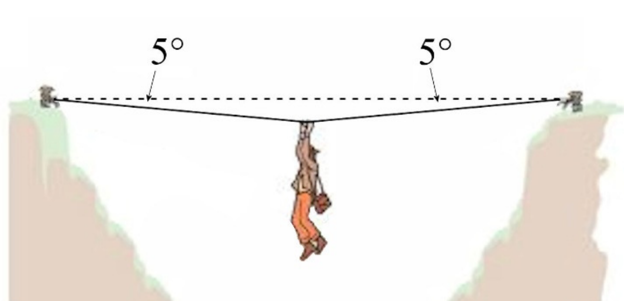
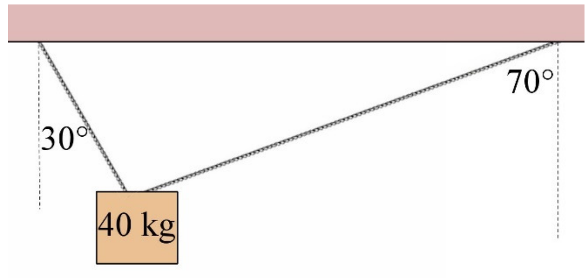
11. Indiana, whose mass is 65 kg, is in the unfortunate position shown in the diagram. What is the tension of the rope?

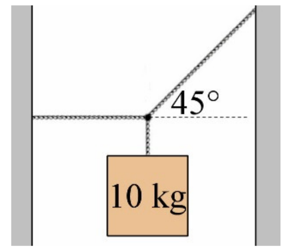


www.bimmerfest.com/forums/showthread.php?p=1903113

12. An object is suspended with two strings as shown in the diagram. What are the tensions of the two strings?

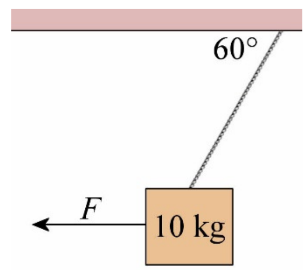


13. An object is suspended as shown in the diagram. What are the tensions of the three strings?



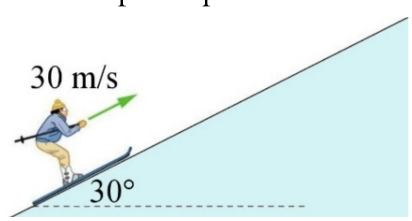
14. A force *F* is maintaining a 10 kg block in the equilibrium position shown in the diagram.

    a) What is the magnitude of the force *F*?
    b) What is the tension of the string?



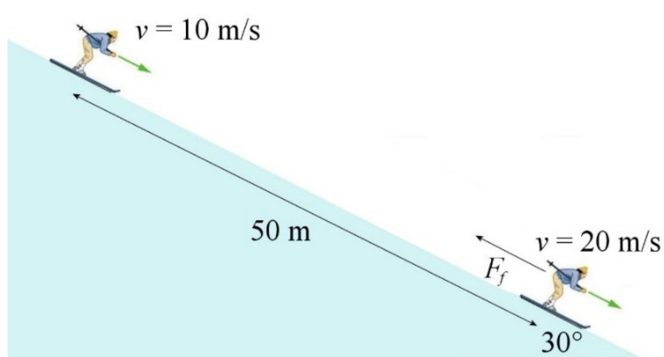
15. Yannick is skiing. At the bottom of a slope, he has an uphill speed of 30 m/s. Neglect friction in this problem.

    a) How far will Yannick travel before stopping?
    b) How much time does it take for Yannick to stop?



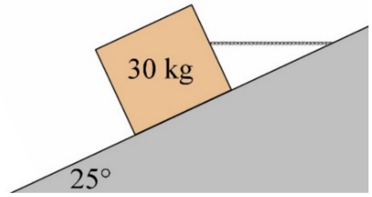
vhcc2.vhcc.edu/ph1fall9/frames_pages/openstax_problems.htm

16. Wolfgang is skiing downhill. While he moves, there's a constant friction force acting in a direction opposite to the velocity. According to what is shown in the diagram, what is the magnitude of the friction force if Wolfgang's mass is 70 kg?



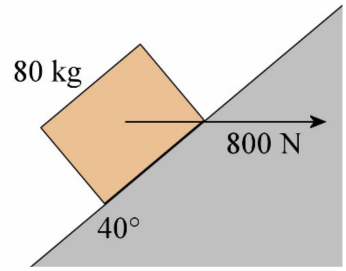
17. A 30 kg block is held in place by a horizontal string on a 25° slope.

    a) What is the tension of the rope?
    b) What is the magnitude of the normal force between the surface and the block?

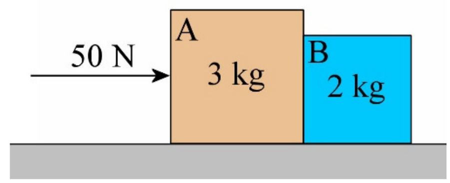


18. An 80 kg crate is pushed with an 800 N horizontal force on a 40° inclined surface.

    a) What is the acceleration of the crate?
    b) What is the magnitude of the normal force acting on the crate?



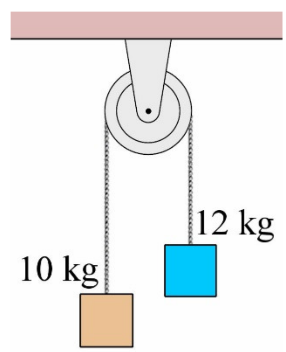
19. The two boxes shown in the diagram are pushed with a 50 N force. There is a 10 N frictional force opposed to the motion of box A and also an 8 N frictional force opposed to the motion of box B.



    a) What is the acceleration of the boxes?
    b) What is the normal force between the two boxes?

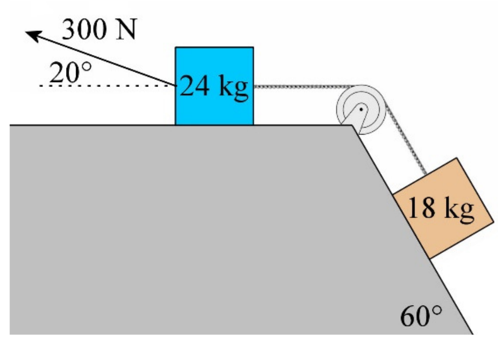
20. Here are two blocks connected by a rope passing over a pulley.

    a) What is the acceleration of the blocks?
    b) What is the tension of the rope?



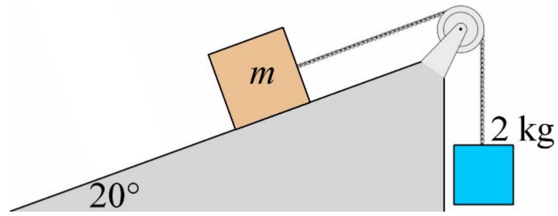
21. A 300 N force is exerted on a 24 kg block connected to an 18 kg block by a string, as shown in this diagram.

    a) What is the magnitude of the acceleration of the blocks?
    b) What is the tension of the rope?
    c) What is the magnitude of the normal force acting on each block?



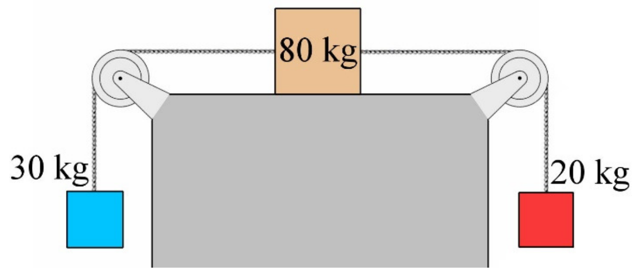
22. Two blocks are connected by a rope as shown in this diagram.

a) What is the mass m of the block on the slope if the 2 kg block has an acceleration of 2 m/s² directed downwards?
b) What is the mass m of the block on the slope if the tension of the rope is 25 N?



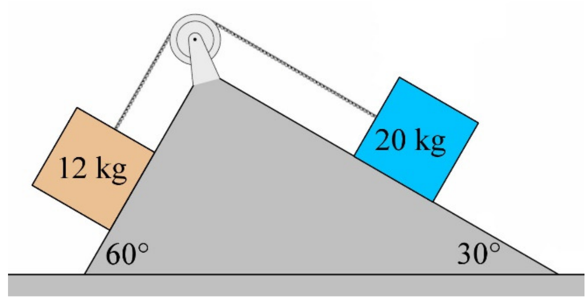
23. Three blocks are connected by ropes as shown in this diagram.

a) What is the magnitude of the acceleration of the blocks?
b) What are the tensions of the two strings?

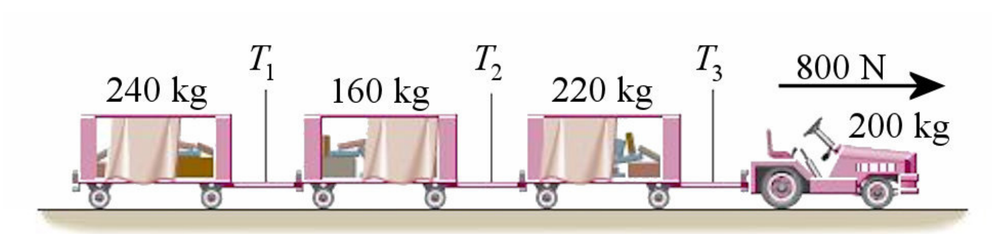


24. Two blocks are connected by a rope as shown in this diagram.

a) What is the acceleration of the blocks?
b) What is the tension of the rope?



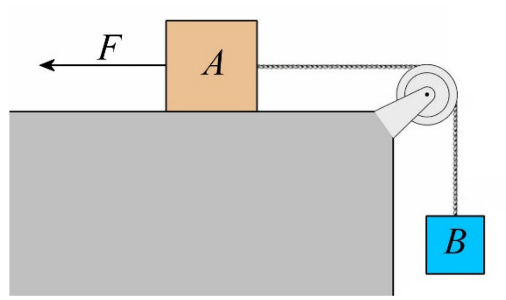
25. The tractor of this airport luggage carrier exerts an 800 N force.



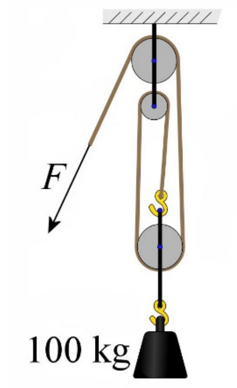
www.chegg.com/homework-help/questions-and-answers/airport-luggage-unloaded-plane-cars-luggage-carrier-drawing-shows-acceleration-carrier-012-q2644085

a) What is the acceleration of the airport luggage carrier?
b) What are the tensions T_1, T_2 and T_3?

26. When block A is pulled with a 100 N force, block B has an acceleration of 1 m/s² directed downwards. When the block A is pulled with a 200 N force, block B has an acceleration of 2 m/s² directed upwards. What are the masses of the two blocks?



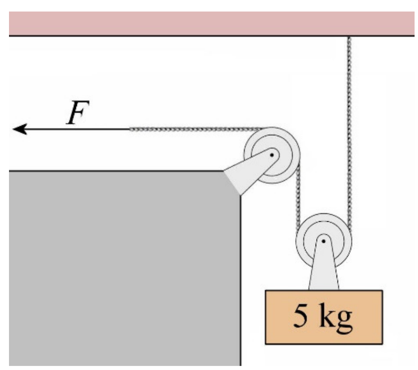
27. What force F should be exerted to keep this system in equilibrium?



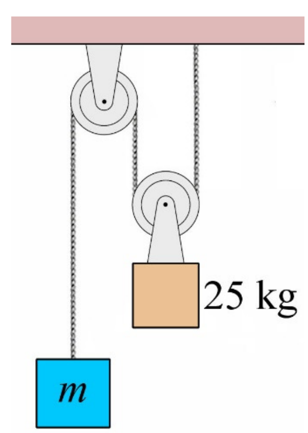
en.wikipedia.org/wiki/Mechanical_advantage_device

28. In the situation shown in the diagram,…

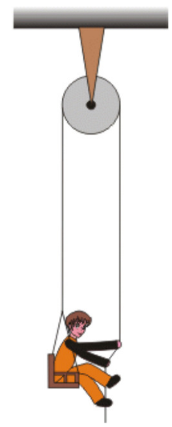
    a) what force F should be exerted to keep this system in equilibrium?
    b) what is the acceleration of the block if the force F has a magnitude of 20 N?



29. What should be the mass of the bucket for this system to be in equilibrium?

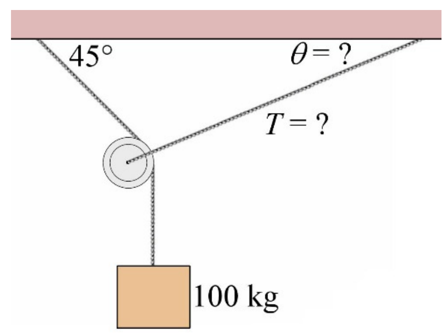


30. What is the acceleration of Romeo, whose mass if 40 kg, if he pulls on the rope with a 250 N force?

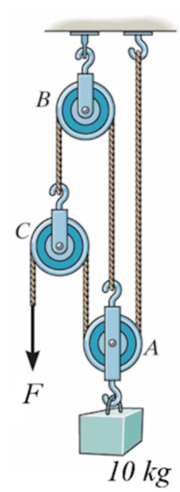


physicstasks.eu/uloha.php?uloha=278

31. In the situation shown in the diagram, what is the tension T and the angle θ?



32. What force F should be exerted to keep this system in equilibrium?

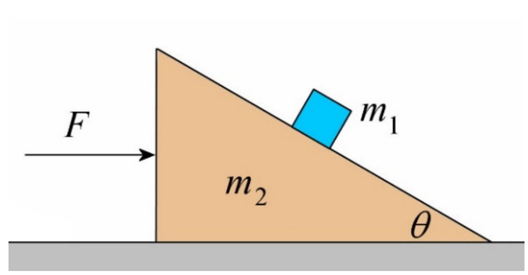


ux.brookdalecc.edu/fac/engtech/mqaissaunee/engi101/jpegs/chapter06/

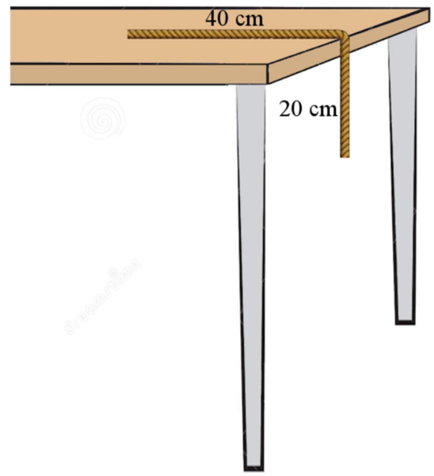
Challenges

(Questions more difficult than the exam questions.)

33. With what force should this triangle be pushed so that the small block does not slide down or up the slope? (There is no friction.)



34. In the situation shown in the diagram, there is no friction between the table and the 5 kg rope sliding on the table. What will the speed of the rope be when the end arrives at the edge of the table?



www.dreamstime.com/royalty-free-stock-image-simple-drawing-table-isolated-white-background-image34921036

ANSWERS

4.1 The Gravitational Force

1. For a, b and c: 980 N

4.2 The Normal Force

2. a) 705.6 N directed upwards b) 849.6 N directed upwards c) 489.6 N directed upwards
3. 5 kg box: a 44 N normal force directed upwards made by the 10 kg box
   10 kg box: a 44 N normal force directed downwards made by the 5 kg box and a 132 N normal force directed upwards made by the ground

4.3 The Tension Force

4. a) 2,940 N b) 3,840 N c) 2,340 N
5. a) T_1 = 118.4 N T_2 = 74 N b) 2.7 m/s²

4.4 Applications of Newton's Laws

6. vertical wall: 6.79 N towards the left
   inclined surface: 7.84 N at 30°
7. F_N = 78.4 N directed upwards
8. 12 kg box: a 78.4 N normal force directed upwards made by the 20 kg box
   20 kg box: a 78.4 N normal force directed downwards made by the 12 kg box and a 274.4 N normal force directed upwards made by the ground
9. a = 3.89 m/s² F_N = 396.8 N
10. T = 590.9 N F_N = 315.0 N
11. 3,654 N

12. Rope to the right: 199 N rope to the left: 374 N
13. Rope attached to the block: 98 N rope to the left: 98 N
    Rope to the right: 138.6 N
14. T = 113.2 N F = 56.58 N
15. a) 91.84 m b) 6.122 s
16. 133 N
17. a) 137.1 N b) 324.4 N
18. a) 1.361 m/s² directed uphill b) 1,115 N
19. a) 6.4 m/s² directed to the right b) 20.8 N
20. a) 0.891 m/s² directed downwards for the 12 kg block, and directed upwards for the 10 kg block
    b) 106.9 N
21. a) 3.075 m/s² towards the left for the 24 kg block b) 208.1 N
    c) 24 kg block: 132.6 N 18 kg block: 88.2 N
22. a) 2.915 kg b) 38.36 kg
23. a) 0.7538 m/s² towards the left for the 80 kg block
    b) rope on the right: 211.1 N rope on the left: 271.4 N
24. a) 0.12 m/s² directed uphill for the 20 kg block
    b) 100.4 N
25. a) 0.9756 m/s² towards the right
    b) T_1 = 234.1 N T_2 = 390.2 N T_3 = 604.9 N
26. m_a = 19.73 kg m_b = 13.61 kg
27. 326.7 N
28. a) 24.5 N b) 1.8 m/s² directed downwards
29. 12.5 kg
30. 2.7 m/s² directed upwards
31. T = 750 N and $\theta = 22.5°$
32. 24.5 N

Challenges

33. $F = (m_1 + m_2)\, g \tan\theta$
34. 2.286 m/s