3 NEWTON'S LAWS

The engine of a 500 kg rocket sled generates a 20 000 N force. There is a 5 000 N friction force opposing the motion of the rocket sled. How long does it take for this sled to travel 1 km if it starts from rest?

www.dailymail.co.uk/news/article-1106231/Murphys-Law-rules-Sixty-years-mishap-WAS-Mr-Murphy.html

Discover the answer to this question in this chapter.

3.1 FORCES

In the two previous chapters, a description of the motion of objects was given without worrying about what was causing the motion. In other words, we were doing *kinematics*. We will now see how an object can be set in motion or kept in equilibrium.

It is quite clear that a force must be exerted on an object at rest to set it in motion. This force can be a push or a pull. For example, a grocery cart must be pushed to set it in motion.

montagepages.fuselabs.com/public/HalimaSaeed/forcesinScience/2618b61d-0a68-481f-9ff9-4392f7e1ad9c.htm

There are several types of forces in nature. Here are some of these forces.

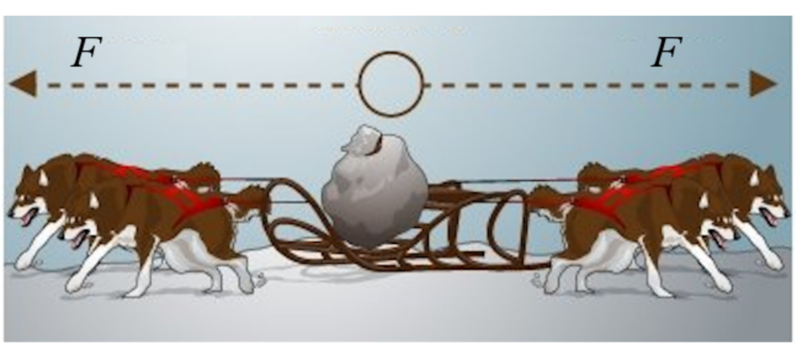
- The gravitational force
- Muscular forces
- Friction forces
- The electric force
- The magnetic force

All of these forces can set objects in motion.

Intuitively, we can say that a *force is an action (pushing or pulling) exerted to change the state of an object.*

We will now try to determine more precisely what the effect of these forces is when they act on an object. This branch of physics is called *dynamics* (since *dynamis* means, force in Greek).

We can quickly determine what happens when there are several forces acting on an object. In this case, the resultant force of these forces must be found. The magnitudes of the forces acting on an object are not simply added because the direction of the force is of crucial importance. The following situation shows why.



science.howstuffworks.com/innovation/scientific-experiments/newton-law-of-motion3.htm

The huskies are pulling with a force F on each side of the sled. It is obvious that the resulting force is not $2F$ and that the forces rather cancel each other. This shows that the direction of the force is important, and, therefore, that forces are vectors.

The net force acting on an object is thus the vector sum of the forces acting on the object.

Net Force or Resulting Force

$$\vec{F}_{net} = \sum \vec{F}$$

The effect of this net force acting on an object must now be looked at. In other words, the laws of dynamics must be found. The effect of forces has been studied for nearly 2500 years, but the true laws of dynamics were found just under 350 years ago. Why did it take nearly 2000 years for the true laws of dynamics to be found?

3.2 BEFORE NEWTON

Warning

The following section is a historical background. The theories in this section are no longer accepted. However, much can be learned with this background as these theories seem so logical at first glance that they are often used intuitively. By knowing and understanding why these theories were rejected, their use can more easily be avoided.

(For a more detailed version of this history, click here https://physique.merici.ca/mechanics/BeforeNewtonv2.pdf)

Speed and Force Are Linked

Early theories of dynamics date from the days of ancient Greece. According to these ancient theories, the force is associated with the speed of the object. The natural state of the object is rest. To change this state, a force must be applied. A force is therefore needed to move an object. As the force increases, the object moves faster. As a result, force was quickly associated with the speed of objects. In a theory associating net force and speed, the following elements are the basis of the theory.

Effect of a Force if Net Force Is Linked With Speed (Erroneous Theories Accepted for Nearly 2,000 Years)

$$v = 0 \text{ if } F = 0$$
$$v = \text{constant if } F = \text{constant}$$
$$\text{If } F \text{ increases, then } v \text{ increases}$$

(I'm using speed in this section, but bear in mind that the distinction between speed and velocity was not yet made at that time). The exact formula linking the force with the speed is not given because there were many variants. Let's just say that they know that friction (often called resistance) decreases the speed of the object, but they are not sure if they must divide the force by the resistance or must subtract the resistance from the force.

The idea of associating force and speed seems quite logical. If a table is pushed, it moves at a certain speed. If you push harder, it moves faster. If you stop pushing, the table stops. The link between force and speed seems obvious. This association between force and speed should not be so bad since it was the only theory of motion for over 2000 years. During this period, nobody proposed to associate forces to something other than the speed of an object. Even today, it is easy enough to find someone who still makes this association. Many studies show that most people use an intuitive physics which associate force and speed. For example, many people think that if the force exerted to push a table is doubled, the table will move twice as fast. This is a clear association between speed and force. Do the test around you to see the answer.

However, the association between the force and the speed quickly leads to serious problems. For example, the motion of launched objects was difficult to explain.

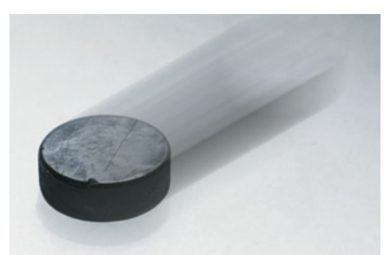
The Problem of Thrown Objects

To illustrate the difficulties of the theory with projectiles, imagine that a hockey puck is pushed with a stick and that the puck then slid on the ice for some distance.

Initially, the puck is pushed with the stick. During this phase of the motion, there is no problem with the theory: the puck moves because the stick pushes it. Since there is a force, the puck can move.

galleryhip.com/hockey-puck-on-ice.html

Here's the problem: why does the puck continue to move when it is no longer in contact with the stick? As soon as the contact between the puck and hockey stick ceases, there is no longer any force acting on the puck (assuming there is no friction). According to the theory, the puck should then be at rest since there is no force acting on the puck. The puck should then suddenly stop as soon as it loses contact with the stick. Obviously, this is not what happens.

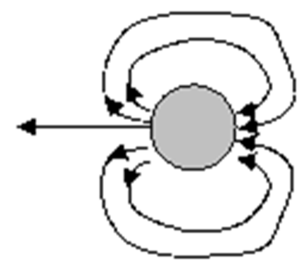


inst.eecs.berkeley.edu/~cs61c/sp10/assignments/ice/

As the puck continues to move when it is no longer in contact with the stick, there must be a force acting on the puck allowing it to continue moving according to the theory associating force and speed. It remained to find that force.

A Force Made by Air?

Aristotle, who made one of the first theories of motion associating force and speed, proposed that the force must be exerted by the air. There have been several mechanisms proposed, but air must be responsible for the force. In a simple version, the puck pushes the air while moving forwards. The air then circles the puck to fill the void created behind the puck by its motion. By striking the rear of the puck, this air pushes the puck, thereby creating the force that allows the puck to move forwards!



www.mathpages.com/home/kmath641/kmath641.htm

An Impregnated Force?

Others think that the force come from the object itself. This is the theory of impregnated force. (This impregnated force took different names at different times and places, the most famous being *impetus*.)

This theory specifies that by applying a force on an object, the object is filled with force, i.e., the object is impregnated with force. This is the theory of the impregnated force. Let's go back to the example of the puck to illustrate this theory. When the puck is pushed with the stick, the puck is impregnated with force. Once the force of the stick on the puck ceases, this impregnated force in the puck is the force that allows the puck to keep moving forwards. Since it was thought that the impregnated force was slowly self-dissipating, much like a hot object that cools, the speed of the sliding puck slowly decreases as the impregnated force dissipates. The cause of motion no longer comes from outside the object, it now comes from the object itself.

A Theory That Never Overcame its Difficulties

Up to the 17^{th} century, physics did not progress.

The theory of air vortices is stalling. The vortices seem too complex to predict anything, and these air movements are not detected around moving objects.

As for the theory of impregnated force, it seems to contradict itself. On the one hand, the theory says that a force pushing an object increases the impregnated force, which causes v to increase. This means that the force would therefore be related to the change in speed (this is actually the beginning of the association between force and acceleration.) On the other hand, the theory says that force is related to the speed *v*. Thus, we have a contradiction between a force associated with speed and a force associated with changing speed.

To get out of these difficulties, it was necessary to associate force with acceleration.

3.3 NEWTON'S FIRST LAW

When force is linked with acceleration, we have the following rules.

The Effect of a Force if Net Force is Linked With Acceleration

$$a = 0 \text{ if } F = 0$$
$$a = \text{constant if } F = \text{constant}$$
If F increases, then a increases

A more precise link is made with Newton's first 2 laws. Let's start with Newton's 1st law.

The Law

When force is associated with acceleration, an absence of force means that the acceleration is zero. This means that the velocity is constant when there is no force.

Newton's First Law or Law of Inertia

If there is no net external force exerted on an object, then
the velocity of the object is constant (magnitude and direction).

The force exerted on the object must be made by another object, hence the name *external force*. The force cannot come from within the object itself (as was the case with the impregnated force). The object cannot change its own motion. This is what inertia means, hence the other name *law of inertia* given to this law.

Observations That Illustrate the Law

It can be seen that the velocity remains the same when the forces that act on the object are eliminated. The space station is obviously a great place to do this kind of demonstration because objects can move without having to slide on another surface and without falling with gravity. Only a small drag force (frictional force made by the air) acting in a direction opposed to the motion remains. A situation where the forces are zero is almost achieved in the space station. The motion of an object on which there is no force can then be observed.
https://www.youtube.com/watch?v=gtw6UcHYqHg
https://www.youtube.com/watch?v=pRnslxr3eKc

It can also be seen that objects continue moving at the same velocity in the absence of force by examining what is happening during a car accident. When your car is moving at constant velocity, there is no net force on you. No force is needed to push you so that you can move forward. Besides, you do not feel a force pushing you when you are in a car moving at a constant velocity. The forces exerted by the seat (which are there to cancel gravitation) are the same forces as the forces acting on you when the car is at rest, the feeling is the same. You feel additional forces only when the velocity of the car changes. Then, a force on you is needed so that you can accelerate with the car. Imagine now that the car hits a wall. Once it comes in contact with the wall, a force is exerted on the car to slow it down. If the driver is not wearing a seatbelt, nothing is tying him back to the seat, and no force can be exerted on him. He then continues its motion at a constant velocity while the car is stopping. In the end, he runs into the steering wheel and the windshield. Look closely at this crash test dummy in this video to see that it continues moving with its initial velocity during a collision.
https://www.youtube.com/watch?v=d7iYZPp2zYY

Discovery of the Law

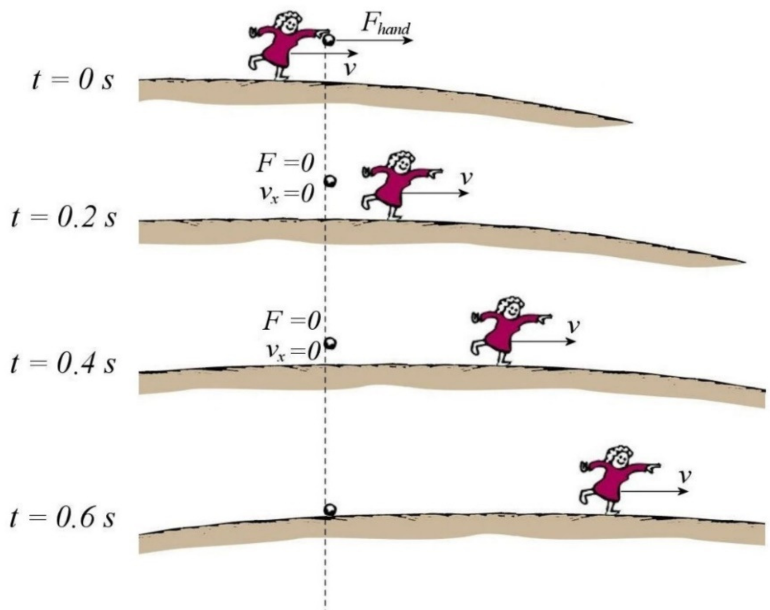
(Click here to see a more detailed version of this history
https://physique.merici.ca/mechanics/Discovery1stLawv2.pdf)

The discovery of Newton's first law was a major change in the history of physics. This law gives a completely different physics compared to physics that associates force with velocity. To illustrate why, let's see what is happening according to Newton's 1st law when a puck is pushed. When the puck is touching the stick, a force exerted by the stick is speeding up the puck. The puck, initially at rest, then picks up velocity as the stick pushes it. When the contact between the stick and the puck stops, there is no longer any force acting on the puck, and it stops accelerating. It, therefore, continues to slide at a constant velocity (if there is no friction). According to Newton's 1st law, it is then quite normal for the puck to continue its motion. It is no longer necessary to invoke air movements or an impregnated force for the puck to move. While there was a lot of debate about the origin of force that pushes projectiles forward in the old theories, this force disappears completely when the force is linked with acceleration. With Newton's 1st law, mv is no longer a force, and one of the main difficulties of mechanics disappeared at once.

Newton did not discover Newton's 1st law. When Newton published his laws, the law of inertia had been almost unanimously accepted for nearly 50 years. Newton knows this and he does not say that it was he the one who discovered this law.

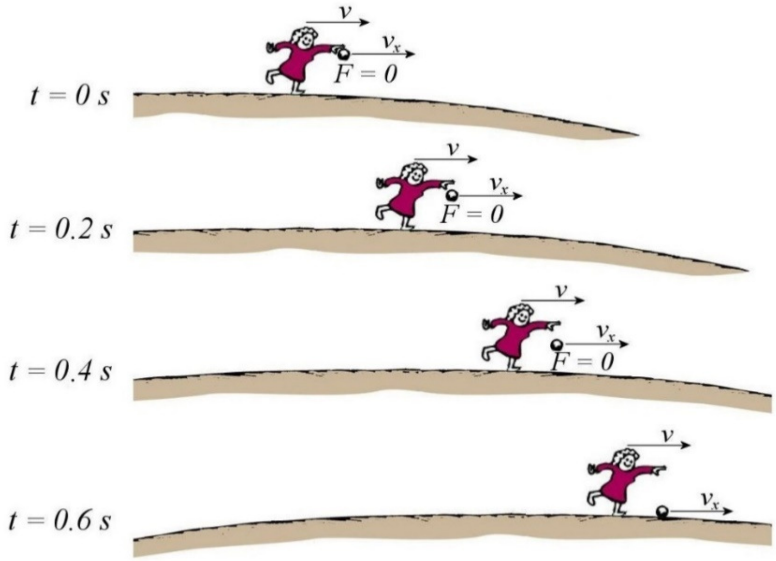
It was not until the beginning of the 17th century that the first arguments in favour of Newton's 1st law appeared. Galileo and Descartes (helped by his friend Beeckman) were the first to arrive at this law. The thoughts that led Descartes to the 1st law are not well known but we know that Galileo was brought to this law by his conviction that the Earth rotates around the Sun. If the Earth revolves around the Sun, then the law of inertia must be true. Let's see why.

In 1543, Copernicus had proposed a model in which the Earth revolves around the Sun (whereas it was previously believed that the Sun was revolving around the Earth). There was, however, a major objection: according to the physics that associate force with velocity, this motion of the Earth would have important effects. To understand the problem, imagine someone dropping a stone on the ground while the Earth is moving. What should we observe according to the physics associating force and velocity? As long as the stone is in the person's hand, it is possible to say that the person's hand makes a force on the stone that allows it to move at the same velocity as the Earth. However, when the person releases the stone, there can no longer be any horizontal force acting on the stone. Thus, the stone can no longer move forward. The stone should therefore stop while the Earth continues its motion. We will then have the following situation, if we assume that the duration of the fall of the stone is 0.6 seconds.



The person on Earth would therefore see the stone fall behind him (if the person is looking in the direction of the motion of the Earth). If the Earth moves at only 10 m/s and it took 0.6 seconds for the stone to fall, that would mean that the Earth have advanced 6 m while the stone remained at the same x-position. The stone would therefore have fallen 6 m behind the person. In fact, the distance would be much greater. Since the Earth moves at nearly 30 km/s around the Sun. The stone would fall about 18 km behind the person. Obviously, objects do not fall like that when dropped. By reasoning in this way, it is possible to deduce a whole series of effects that we should observe if the Earth moves or rotates on itself. Let us particularly note this wind that should be continuously blowing on the surface of the Earth. The air surrounding the Earth would not be able to move with the Earth, and the Earth would then move in air at rest. This would give the impression that there is a continuous wind blowing in the opposite direction of the Earth's motion. As these effects are not observed, most of the scientists of the time conclude that it is impossible for the Earth to be moving or rotating. This is an important problem for those, like Galileo, who thought that Copernicus's system was true and that the Earth really revolves around the Sun. That objection had to be eliminated.

However, Galileo understands that the problem is eliminated if it is assumed that the object keeps its velocity when there is no force. Let's go back to the example of the stone. When the stone is released, there is no horizontal force on the stone, which means, according to Galileo, that the horizontal speed of the stone must remain the same. Thus, the x-component of the velocity of the stone always remains the same as the velocity of the Earth and it advances at the same rate as the Earth. This causes it to hit the ground in front of the person, right below the hand of the person, exactly as if the Earth was not moving.



slid.es/tofergregg/gravity-and-fluid-dynamics/fullscreen#/22

Using similar arguments, it can be shown that it would be impossible to see a difference between what is happening on the surface of a motionless Earth and on the surface of an Earth moving at a constant velocity. The motion of the Earth would be imperceptible. (This is the relativity principle that we'll see later in this section).

Galileo then shows that some other phenomena can be explained simply if it is assumed that the velocity is constant in the absence of force. This is how he comes to resolve the motion of the projectile into a horizontal motion at constant velocity (since there is no horizontal force) and a vertical motion at a constant acceleration.

In fact, the idea that velocity remained constant in the absence of force eliminated so many difficulties in physics that the majority of scientists of the time were convinced that the law of inertia must be true shortly after the publication of the treatises of Galileo (1632 and 1638) and Descartes (1644). No need to look for forces of obscure origin for objects to move at a constant velocity. No need to look for the force that moves the hockey puck forward once it leaves the stick. Everything becomes much simpler with Newton's 1st law.

The law was so well accepted that when Huygens (1656) and Newton (1687) took up this law as the starting point of their mechanics a few years later, they no longer even had to justify this choice. All they had to do was say that everyone agrees on this law now considered obvious.

Can the 1st Law Be Proven?

It is very difficult to prove experimentally that Newton's 1st law is true. To do this, we would have to look at what is happening under conditions in which there are no external forces. Is this possible? Not really.

As early as 1643, Roberval argued that the law was not proven because it is an extrapolation of what we think would happen if there were no forces. For example, the law says that the velocity would be constant if there were no friction, but we have never been able to verify this because we have never been able to completely eliminate friction. Even in the space station, there are external forces acting since there is still air friction and gravity made by the Earth (which is almost as great as on the surface of the Earth as we will see in Chapter 6). So, if we have never been able to verify this law and if it is just an extrapolation of what we think would happen under certain impossible conditions, how do we know that the law is true?

Newton's 1st law is actually the starting axiom of modern mechanics. You don't have to prove it because it represents what is assumed to be true. It defines the motion that is considered to be the reference motion that does not have to be explained by forces. We have simply decided that there is no need to explain how a straight-line motion at constant velocity is done. It is only when an object does not follow this reference motion that we say that a force acts on the object.

Thus, the transformation of physics in the 16th and 17th centuries is in fact a change of the motion considered to be the natural motion. Previously, rest was considered to be the reference motion that does not require explanation. Thus, as soon as the object was not in this state (so as soon as the object was moving), the motion had to be explained by a force acting on the object. With Newton's 1st law, the constant velocity motion is now considered to be the reference motion that does not require explanation. It is no longer necessary to look for the force that moves the object forward at constant velocity, but the forces acting on the object when the velocity of the object is not constant must be found. By changing the reference motion, the definition of force also changes because we say that there is a force acting when the object does not follow the reference motion.

The choice of the natural motion of reference as the constant velocity motion then seems completely arbitrary. That's true, but the choice has allowed us to make a great leap forward. When the reference motion chosen was rest, forces had to be found to explain a constant velocity motion. The problem with this choice was that there was no logic to predict the value of the force from the configuration of a system. For example, it was impossible to predict how an object would fall because it was impossible to make a simple formula of the force of gravity. A falling object could have any velocity at a height of 5 m (since its velocity could change depending on the height at which it was released). How then could we obtain a simple formula of the gravitational force acting on the object when it is at a height of 5 m when force is associated with velocity if the velocity can take about any value at this place? In addition to this difficulty in finding the formulas of the forces, it was very difficult to understand where the force was coming from. What was the force acting on the puck so that it can move forward after leaving the stick? This choice of rest as a reference motion ultimately led to a whole series of practically insurmountable difficulties and a non-workable theory.

When the constant velocity motion is taken as a reference, a simpler and more coherent physics is obtained, and this theory allows us to find force formulas and make predictions. With this reference motion, the gravitational force on an object is a simple constant force. It is a much simpler formula, and it can be used to predict how an object will fall. With constant velocity motion as the reference motion, it is possible to find formulas for several forces (such as electrical forces, forces made by springs, forces made by air pressure and many others) and understand the origin of these forces. In short, everything becomes much simpler with this choice.

This simplification of physics is also at the origin of the rapid acceptance of Newton's 1st law in the middle of the 17th century. This law so simplified the solution of several problems in physics that virtually everyone quickly accepted the law. The law was not proven, but by accepting it, it was now possible to make prediction.

Newton's 1st Law and the Relativity Principle

Newton's 1st law can be obtained from the principle of relativity. The law must be true if the principle of relativity is true.

The principle of relativity specifies that there are no new effects that appear when moving at a constant velocity. Everything happens as if we were at rest.

For example, we know how to pour a cup of coffee in our house (which is at rest). If, one day, you become a flight attendant and you must pour coffee into a cup in a plane moving at a constant velocity, do not change your habits: pour the coffee on the plane in motion exactly the same way as you do it at home. There is no need to pour the coffee a bit in front of the cup, thinking that the coffee will move a little towards the rear of the aircraft because of its motion. Pool (the game) is played exactly the same way on a plane travelling at a constant velocity as in a pool hall. There is no "correction" to do because of the motion of the plane.

In this video, a ball is dropped from a moving truck. A board was installed on the side of the truck to give a reference for the motion of the ball. The ball stays in the middle of the board as it falls, exactly as what would happen if the ball was released while the vehicle was stopped.
https://www.youtube.com/watch?v=_ky-ITbNfeY

All this also means that if an observer is locked inside a plane in motion at a constant velocity without any window, it is impossible for the observer to tell whether the plane is moving or not. If no new effect appears when the plane is in motion, everything appears

exactly the same way whether the aircraft is moving or not. No experiment can show that it is moving.
https://www.youtube.com/watch?v=uJ8l4kh_jto

This also means that if you close your eyes in a car in motion (if you're not driving of course), you won't be able to say whether the car is moving or not. You surely think that any moron can tell whether the car is moving or is stopped, and that this is not true. Actually, it is true. You would know that the car moves because the bumps on the road will shake the car a little. However, we're saying here that the motion is undetectable if the velocity is constant. If that is the case, you must imagine that you are travelling on a road without any bumps because these bumps accelerate the car in every direction. So you must imagine that you are travelling on a freshly paved road and that there is no bump whatsoever. Then, it becomes more difficult to tell if the car is moving. You still hear the air passing along the car, but this could be a wind blowing on the car at rest... It would be more convincing with a spaceship. Then, there would be no sound from the wind or the road and it would be impossible to tell whether the spaceship is moving or not if your eyes are closed.

The final argument that should convince you that nothing changes when moving at a constant velocity is the fact that we are a planet moving around the Sun at 29.8 km/s and nothing peculiar happens. Even better, the Sun revolves around the centre of the Galaxy at 240 km/s, and we do not notice this motion. Everything happens as if the Earth were at rest.

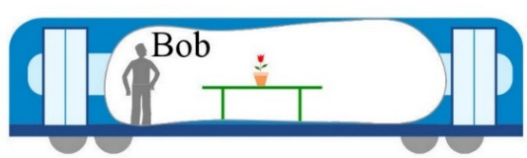
If there are no new effects, it is because the laws of physics are the same for all observers, regardless of their velocity. There are no new forces that come into play, which means that the laws are exactly the same. This brings us to the Galilean principle of relativity (actually stated by Euler).

The Relativity Principle

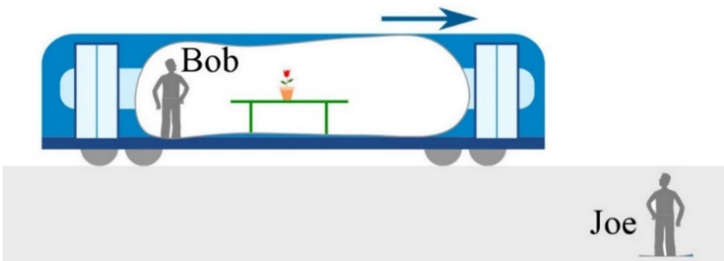
The laws of physics are the same for all the observers moving at a constant velocity.

Newton's first law must be true according to the principle of relativity. To understand why, let's look at what is happening in a train that is going at a constant velocity according to one observer who is on the train and another observer who is on the side of the track. According to the principle of relativity, the physics is the same for both observers, and this means that the forces must be the same for both observers.

Let's say there's a flowerpot sitting on a table on a train. For a person on the train (we'll call him Bob), the pot is at rest and there is no net force acting on the pot.



That's what Newton's First Law says, but it's also what the theories that associated force with velocity said.



www.mathsisfun.com/physics/relativity.html

Now. Let's take the point of view of an observer on the ground (we'll call him Joe) watching the train pass by.

If there is no force on the flowerpot according to Bob, then there must be no force according to Joe (since the laws of physics must be the same). Joe then sees the flowerpot moving at constant velocity when there is no force acting on the flowerpot. Taking Joe's point of view, we can conclude that the velocity can be constant when there is no force. This is what Newton's 1st law says, but it is contrary to what the theories that associate force with velocity were saying. We arrive at Newton's 1st law if the principle of relativity is true. Pierre Gassendi gave practically the same arguments (it was a boat rather than a train) in 1642 to justify that velocity could be constant in the absence of force.

1st Law Mistakes

Many people are still making mistakes with Newton's 1st law and think that a force must be acting when an object moves with a constant velocity.

Sometimes, people will say that a moving object continues its motion in a straight line because a force of inertia acts on the object. It is true that inertia causes the object to continue its motion, but there is no such thing as a force of inertia.

Common mistake: thinking that a force of inertia exists

Many errors of Newton's 1st law are found in science fiction movies. In space, there is no friction and, if there is no star or planet close by to exert a gravitational force, the only force acting on a spaceship is the force exerted by the engine. In the movie, there is often a scene where the engine is broken, and the spaceship stops. This is, of course, an error. If the engine stops, there is no longer any force exerted, and thus the spaceship should continue at a constant velocity! Moreover, why was the engine running in the first place? The engine should be run at the start to give some velocity to the spaceship; then the engine should be stopped letting the ship moves at a constant velocity, and finally the engine should be used to slow down the spacecraft. This is basically how they proceeded to send astronauts to the Moon. If the engine were running all the time, the spaceship would accelerate endlessly. Then the spaceship would be going pretty fast on arrival, and it would not be easy to stop it. If the trip lasts two weeks, they cannot speed up for two weeks and then slow down in two minutes; imagine the acceleration then! Rather, if they absolutely want to use the engine all the way, it would be better to accelerate for a week and then to slow down for a week.

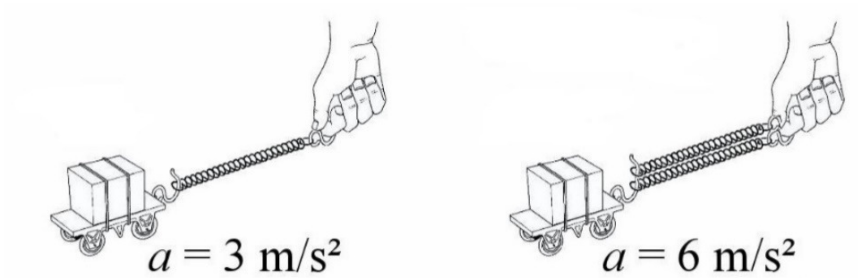
There is also a big Newton's first law error in this scene of *Gravity.*
https://physique.merici.ca/mecanique/Gravity.wmv
Everything is fine for the first 39 seconds of this clip. At the end of those 39 seconds, George Clooney holds the end of the rope and has a constant velocity (which is the same velocity as Sandra Bullock's velocity). Then, things start to go wrong. From that moment on, there is a significant problem with Newton's first law. In the movie. George must continuously hold the rope to maintain its velocity. In reality, the rope is not useful since no force is required for George to move at a constant velocity once he has the same velocity as Sandra. When George let go of the rope, he should continue to move at the same velocity and stay at the same distance from Sandra. Yet, in the film, George begins to move away from Sandra as if a force was required to continue to move at a constant velocity. (If they were in the atmosphere, the scene would make sense because the force made by the rope would compensate for air friction. As there is no air in space, this scene is completely wrong.)

3.4 NEWTON'S SECOND LAW

The Link Between Force and Acceleration

With the first law, the motion at constant speed is chosen as the reference motion that doesn't need to be explained with force. This means that if the object does not have a constant velocity, then a net force must be acting on the object. Newton's 2nd law clarifies the link between net force and acceleration.

To find the link between net force and acceleration, we can start with the following experiment.

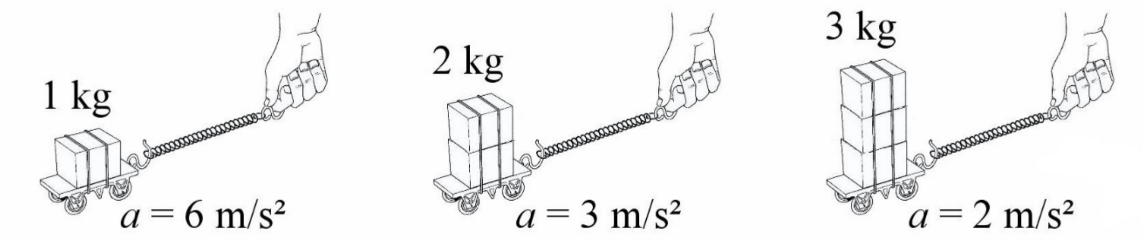


www.nuffieldfoundation.org/practical-physics/constant-and-varying-forces-between-trucks

The cart is pulled making sure that the stretching of the springs is the same for all three springs. This ensures that the force exerted by the spring is the same since the force exerted by a spring depends on its stretching. As we observe that a force 2 times greater (when there are 2 springs) generates an acceleration 2 times greater, it can be concluded that the acceleration is proportional to the net force.

$$a \propto F_{net}$$

However, the acceleration does not depend only on the force acting on the object; it also depends on the mass of the object. To see this link, let's look at what happens when there is an identical force made by a spring acting on three objects of different mass (1 kg, 2 kg, 3 kg). The accelerations are then measured, and let's say that the force has a certain magnitude so that the accelerations indicated in the diagram are obtained.



These results clearly show that the acceleration decreases as mass increases. So, we should have the following relationship.

$$a \propto \frac{F_{net}}{m}$$

This is what we can see in this video
https://www.youtube.com/watch?v=sPZ2bjW53c8
(Correctly, we do not see the acceleration, we only observe the final velocity of the object. Since the final velocity is smaller if the acceleration is smaller and the distance over which the object accelerates is always the same, it can be concluded that the acceleration is smaller when the mass is larger.)

Thus, we arrive at

$$F_{net} \propto ma$$

This means that

$$F_{net} = kma$$

where k is a constant of proportionality. It only remains to choose the value of k. The simplest choice was made, i.e. $k = 1$.

The result is Newton's second law.

Newton's Second Law

$$\sum \vec{F} = m\vec{a}$$

In components:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

This law is actually the definition of force. There are some arbitrary aspects to this definition. An association between force and acceleration was chosen, as specified in Newton's 1st law and a proportionality constant of $k = 1$ was chosen.

Measurement of Force: The Newton

As a force is the result of a multiplication of mass and acceleration, the unit of force is kgm/s^2. Since it takes a long time to write this down, a new name was given to this unit: the newton (N).

The Newton

$$1N = 1\tfrac{kgm}{s^2}$$

To give you a rough idea of what one newton is, it takes about 10 N to lift a 1 kg mass.

Discovery of the Law

It was said previously that in the theory of impregnated force, a force applied to an object adds or removes some impregnated force. There is some confusion in this theory because there are 2 types of force. There is an impregnated force that is proportional to *mv,* and there is an applied force that makes *mv* change (therefore linked to $\Delta(mv)$). (Obviously, no one writes *mv* and $\Delta(mv)$ at that time because the concept of mass is not yet invented, but the meaning comes down to that). The applied force therefore makes *mv* change, and if *mv* changes, then *v* changes. Clearly, as early as the 14th century, there is a link between the applied force and acceleration, but no one explicitly mentions this link. Scaliger (1557) and Benedetti (1585) were the first to say that the applied force is related to acceleration, but they did not specify how velocity changed when a force is applied.

The discovery of Newton's 1st law will make it possible to clarify the link between force and acceleration.

First, the 1st law says that there is no force when an object travels at constant velocity. This clearly means that the impregnated force is not a force. The impregnated force (related to *mv*) will therefore slowly disappear, and they will focus more and more on the effect of an applied force.

Second, the 1st law makes it possible to obtain the link between force and acceleration. It is obvious that the change in *mv* increases if the force increases. It seemed obvious to them that the change in *mv* should be proportional to the applied force and to the time of application of the force. This would mean that, in modern notation, we have the following relationship.

$$F\Delta t \propto \Delta(mv)$$

If this equation is divided by Δt, the result is

$$F \propto \frac{\Delta(mv)}{\Delta t}$$
$$F \propto \frac{m\Delta v}{\Delta t}$$

As $\Delta v/\Delta t$ is the acceleration, we have

$$F \propto ma$$

Using this idea, those who were the first to arrive at the law of inertia, Beeckman and Descartes, came, as early as 1618, to the conclusion that free fall must be done with a constant acceleration, even before Galileo published this information. (They even manage to calculate how the velocity would change by taking into account air friction if air friction were proportional to the speed.) Beeckman and Descartes did not publish these results, but this kind of proof became very common after 1630.

However, the idea took some time to develop.

First, many still believe that force is related to velocity. The association between force and acceleration was only a recent and relatively unpopular idea among many other theories that still associated force with velocity. The association between force and velocity seemed to be confirmed by many experiments performed in order to observe the deformations made by a collision. As these deformations increased with velocity, experiments seemed to show that the impact force increased with velocity. Obviously, it would have been difficult to see the link between force and acceleration in collision experiments because measuring the acceleration is almost impossible in this case. Since the velocity was the only measurable quantity with collisions, it was easy to think that there should be a link between force and velocity. From these experiments, many have concluded that the force must be proportional to *mv* or even mv^2.

Second, those who accepted the link between force and acceleration were having difficulties finding the link between those two quantities because they were having problems with the mathematics of accelerated motions. Many did not arrive at the right equation when they wanted to calculate the displacement generated by a constant acceleration! Galileo had succeeded in the calculation, but several others missed it by treating instantaneous velocity in a wrong way. For example, Gassendi missed the calculation, and this led him to invoke a force made by air (as in Aristotle's physics) to explain the difference between his formula of position and the observed positions. This difficulty led many to think that the constant force made by gravity does not produce a constant acceleration motion, and that the relationship between force and acceleration is not as simple as $F \propto a$. Many, like Gassendi, finally correct their error after a few years to finally conclude that the constant gravitational force does indeed generate a constant acceleration motion.

Christiaan Huygens is the first to correctly use the link between force and acceleration to study something else than a simple falling objects. In 1659, he found the formula for the force exerted by a string on an object that makes a circular motion by examining the displacements due to the constant acceleration generated by this force (taking the right formula for constant acceleration displacements). In fact, Huygens began to develop the association between force and acceleration a little more, but he never finished this work. Perhaps today we would be talking about Huygens's laws rather than Newton's laws if he had finished this work. (This may be a good thing that these laws are named after Newton because it's quite difficult to pronounce Huygens's name correctly https://www.youtube.com/watch?v=yH-qQu2ndc8.)

Isaac Newton was the one who, in the end, fully developed the association between force and acceleration first. Discussions with Edmund Halley in 1684 led him to study the orbits of the planets and to publish his results in 1687. Don't search for $F = ma$ in Newton's book. The 2nd law is written in words, not in the form of an equation. Even in words, the wording of the law

A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.

is incomplete (the time of application of the force and the mass should play a role) and a little unclear (he never specified what he means by *change in motion*). Actually, one must examine all the definitions and the corollaries in Newton's book and follow carefully Newton's calculations to truly understand that what he does is equivalent to $F = ma$. You can click on this link to see how Newton used his law.
https://physique.merici.ca/mechanics/FmaNewton.pdf
Note that Newton uses geometric methods to solve problems. This is a method that was widely used at that time and is very different from what we use now. This makes the calculations very difficult to follow for us.

As Newton obtained a multitude of results in agreement with observations, many understood that a useful law of motion had finally been discovered.

Newton says that he did not discover the law, but, curiously, he attributes the discovery of the law to Galileo, who never stated a link between force and acceleration.

Before seeing Newton's second law appear in the usual form, Newton's geometric approach had to be abandoned and a more algebraic approach had to be used. Jakob Hermann was the first to begin a reformulation of Newton's physics using Leibniz's differential calculus. He then became the first, in 1716, to obtain an equation equivalent to $F = ma$ when he wrote that Newton's method of making calculations is equivalent to $F\,dt = dv$ where F is the force per unit of mass. However, Hermann arrived at this form in the middle of a proof and the wording is not very explicit.

At the beginning of the 18th century, Newton's methods were just one method among many used at the time to solve mechanical problems. Newton's geometric method makes calculations very difficult and even Euler, probably the greatest mathematician of all time, has difficulty applying the method. They even wondered if the method can be applied to anything else than gravitation, which was the main subject of Newton's book. However, Euler reformulated Newton's methods (he made several versions between 1736 and 1775) to essentially arrive at the mechanic used today. It was from this moment that Newton's laws became much easier to use and became <u>the</u> method for solving mechanical problems. Ultimately, Euler was the one who clearly formulates $F = ma$. What we use nowadays is actually Euler's mechanics.

Euler is also the one who fixed the value of the proportionality constant $k = 1$ in

$$F = kma$$

(After passing through $F = ma/n$ (1736), $F = 2ma$ (1750) and $F = ma/2g$ (1765) before arriving at $F = ma$ in 1775.)

Applications of Newton's Second Law

Following a problem-solving method in dynamics is important. This method is

1) Find all the forces acting on the object that is studied.

   In this chapter, the forces are given. From the next chapter on, you'll have to find the forces.

2) Resolve these forces into their x and y-components. This implies that an axis system must be chosen. (We will see in the next chapter how to choose these axes. For now, use an x-axis towards the right and an upwards pointing y-axis.)

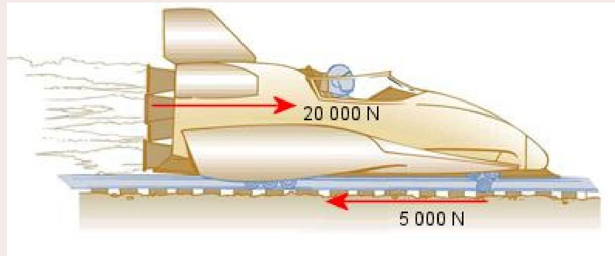
3) Apply Newton's second law

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$

4) Solve these equations to find the unknowns.

Example 3.4.1

The engine of a 500 kg rocket sled generates a 20 000 N force. There is a 5 000 N friction force opposing the motion of the rocket sled. How long does it take for this sled to travel 1 km if it starts from rest?



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The time can be found with the kinematics equations if the acceleration is known. This acceleration is found with Newton's second law.

Forces Acting on the Object

The forces acting on the object are shown in the diagram. (There are more forces acting than those shown but we'll consider only those 2 forces.)

Sum of the Forces

With a horizontal axis pointing towards the right, the x-component of the net force is

$$\sum F_x = 20\,000N + (-5\,000N) = 15\,000N$$

Newton's Second Law

Newton's second law gives

$$\sum F_x = ma_x$$
$$15\,000N = 500kg \cdot a_x$$

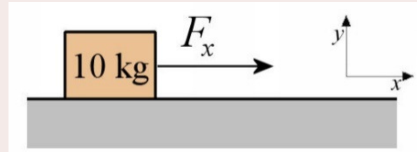
Solving the Equation

$$15\,000N = 500kg \cdot a_x$$
$$a_x = 30\tfrac{m}{s^2}$$

With the acceleration, the solution can now be found. Starting from rest, the time required to travel 1000 m with such an acceleration is

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$
$$1000m = 0m + 0m + \tfrac{1}{2}\cdot\left(30\tfrac{m}{s^2}\right)\cdot t^2$$
$$t = 8.165s$$

Example 3.4.2



A 10 kg box is subjected to a variable force given by

$$F_x = 6\tfrac{N}{s} \cdot t + 2N$$

and there are no other forces acting in the direction of the *x*-axis. The initial velocity of the box is 1 m/s towards the left.

a) What is the acceleration of the box at t = 1 s?

Forces Acting on the Object

The only force in the direction of the *x*-axis is $F_x = 6\tfrac{N}{s} \cdot t + 2N$.

Sum of the Forces

With a horizontal axis pointing towards the right, the *x*-component of the net force is

$$\sum F_x = 6\tfrac{N}{s} \cdot t + 2N$$

Newton's Second Law

Newton's second law gives

$$\sum F_x = ma_x$$
$$6\tfrac{N}{s} \cdot t + 2N = 10kg \cdot a_x$$

Solving the Equation

At t = 1 s, we have

$$6\tfrac{N}{s} \cdot 1s + 2N = 10kg \cdot a_x$$
$$8N = 10kg \cdot a_x$$
$$a_x = 0.8\tfrac{m}{s^2}$$

b) What is the velocity at t = 10 s?

The formula for acceleration is

$$6\tfrac{N}{s} \cdot t + 2N = 10kg \cdot a_x$$
$$a_x = 0.6\tfrac{m}{s^3} \cdot t + 0.2\tfrac{m}{s^2}$$

As $a_x = dv_x/dt$, we have

$$\frac{dv_x}{dt} = 0,6\tfrac{m}{s^3} \cdot t + 0,2\tfrac{m}{s^2}$$

With an integral, the formula for the velocity is obtained.

$$\begin{aligned} v_x &= \int \left(0.6\tfrac{m}{s^3} \cdot t + 0.2\tfrac{m}{s^2}\right) dt \\ &= 0.3\tfrac{m}{s^3} \cdot t^2 + 0.2\tfrac{m}{s^2} \cdot t + constant \end{aligned}$$

The constant can be found because we know that the velocity at t = 0 s is -1 m/s.

$$\begin{gathered} -1\tfrac{m}{s} = 0.3\tfrac{m}{s^3} \cdot (0s)^2 + 0.2\tfrac{m}{s^2} \cdot 0s + constant \\ -1\tfrac{m}{s} = constant \end{gathered}$$

So, the formula for velocity is

$$v_x = 0.3\tfrac{m}{s^3} \cdot t^2 + 0.2\tfrac{m}{s^2} \cdot t - 1\tfrac{m}{s}$$

At t = 10 s, the velocity is

$$\begin{aligned} v_x &= 0.3\tfrac{m}{s^3} \cdot (10s)^2 + 0.2\tfrac{m}{s^2} \cdot 10s - 1\tfrac{m}{s} \\ &= 31\tfrac{m}{s} \end{aligned}$$

c) What is the displacement of the object between t = 0 s and t = 10 s?

The formula for the velocity is

$$v_x = 0.3\tfrac{m}{s^3} \cdot t^2 + 0.2\tfrac{m}{s^2} \cdot t - 1\tfrac{m}{s}$$

As $v_x = dx/dt$, we have

$$\frac{dx}{dt} = 0.3\tfrac{m}{s^3} \cdot t^2 + 0.2\tfrac{m}{s^2} \cdot t - 1\tfrac{m}{s}$$

With an integral, the formula for the position is obtained.

$$\begin{aligned} x &= \int \left(0.3\tfrac{m}{s^3} \cdot t^2 + 0.2\tfrac{m}{s^2} \cdot t - 1\tfrac{m}{s}\right) dt \\ &= 0.1\tfrac{m}{s^3} \cdot t^3 + 0.1\tfrac{m}{s^2} \cdot t^2 - 1\tfrac{m}{s} \cdot t + constant \end{aligned}$$

The constant can be found if it assumed that the position at t = 0 s is 0 m.

$$\begin{gathered} 0m = 0.1\tfrac{m}{s^3} \cdot (0s)^3 + 0.1\tfrac{m}{s^2} \cdot (0s)^2 - 1\tfrac{m}{s} \cdot 0s + constant \\ 0m = constant \end{gathered}$$

Therefore, the formula for the position is

$$x = 0.1\tfrac{m}{s^3} \cdot t^3 + 0.1\tfrac{m}{s^2} \cdot t^2 - 1\tfrac{m}{s} \cdot t$$

At t = 10 s, the position is

$$\begin{aligned} x &= 0.1\tfrac{m}{s^3}\cdot(10s)^3 + 0.1\tfrac{m}{s^2}\cdot(10s)^2 - 1\tfrac{m}{s}\cdot 10s \\ &= 100m \end{aligned}$$

Therefore, the displacement is

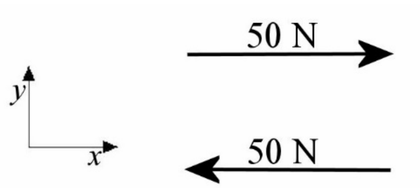
$$\begin{aligned} \Delta x &= x - x_0 \\ &= 100m - 0m \\ &= 100m \end{aligned}$$

Components of the Forces

Before solving problems, you must know how to resolve forces into their x and y components. Here's a brief summary of the way to resolve a force vector into components.

When the forces are directly in the direction of an axis or directly opposed to an axis, the components are easily found. The component is positive if the force is in the direction of the axis, and it is negative if the force is in the opposite direction to the axis.

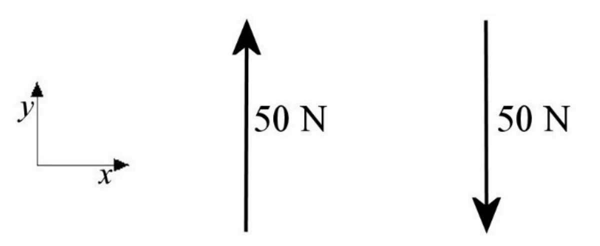
Forces Directed Along the x-axis



$F_x = 50N$ $\quad F_y = 0N$

$F_x = -50N$ $\quad F_y = 0N$

Forces Directed Along the y-axis

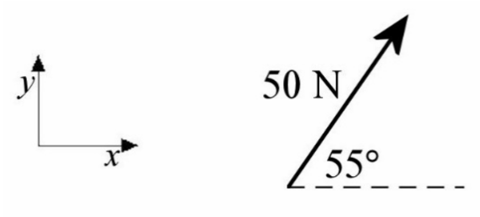


$F_x = 0N$ $\quad F_x = 0N$

$F_y = 50N$ $\quad F_y = -50N$

If the force is not directed along one of the axes, the decomposition formulas seen in Chapter 2 are used.

Forces in Any Direction

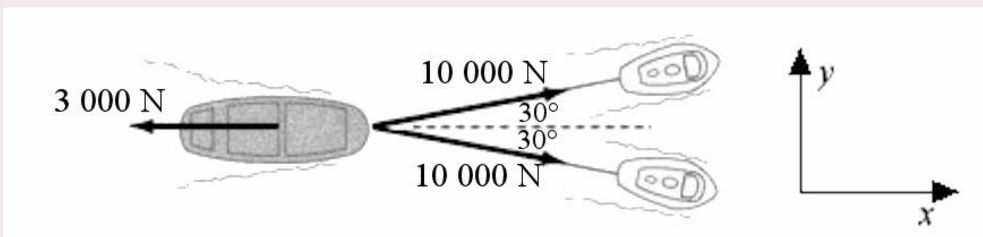


$$F_x = 50N \cdot \cos 55°$$
$$F_y = 50N \cdot \sin 55°$$

To resolve into components using these formulas, remember that you have to find the angle that the force makes with the **positive *x* axis,** and that you have **to find the sign of the angle**. The positive direction for the angle is always in the direction of rotation that runs from the positive *x-axis* to the positive *y-axis.*

Example 3.4.3

A 100-ton barge is pulled by the forces shown in the diagram (the 3000 N force is the friction force exerted by the water). What is the acceleration of the barge?



curricula2.mit.edu/pivot/book/ph0503.html?acode=0x0200

The two components of the acceleration are found from the net force with Newton's second law.

Forces Acting on the Object

The 3 forces acting on the object are shown in the diagram. (There are more forces acting than those shown but we'll consider only those 3 forces.)

Sum of the Forces

Using the axes shown in the diagram, the 3 forces will be resolved into components.

F_1 (top tug)

$$F_{1x} = 10\,000N \cdot \cos 30° = 8660N$$
$$F_{1y} = 10\,000N \cdot \sin 30° = 5000N$$

F_2 (bottom tug)

$$F_{2x} = 10\,000N \cdot \cos(-30°) = 8660N$$
$$F_{2y} = 10\,000N \cdot \sin(-30°) = -5000N$$

F_3 (friction)

$$F_{3x} = -3000N$$
$$F_{3y} = 0N$$

Therefore, the net force components are

$$\sum F_x = 8660N + 8660N + (-3000N) = 14\,320N$$
$$\sum F_y = 5000N + (-5000N) + 0N = 0N$$

Newton's Second Law

Newton's Second Law leads to these 2 equations.

$$\sum F_x = ma_x \qquad \rightarrow \qquad 14\,320N = 100\,000kg \cdot a_x$$
$$\sum F_y = ma_y \qquad \rightarrow \qquad 0N = 100\,000kg \cdot a_y$$

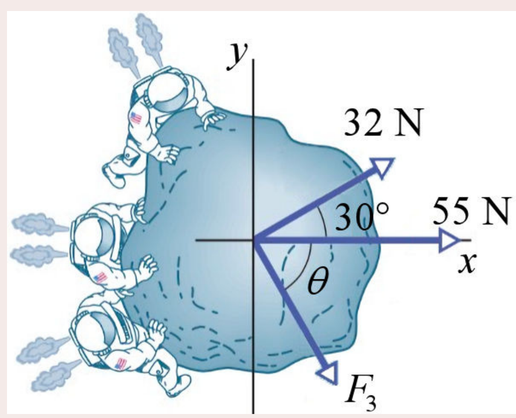
Solving the Equations

The components of the acceleration are

$$a_x = \frac{14\,320N}{100\,000kg} = 0.1432 \tfrac{m}{s^2}$$
$$a_y = \frac{0N}{100\,000kg} = 0 \tfrac{m}{s^2}$$

Example 3.4.4

Three astronauts exert forces on a small 20-ton asteroid. What is the unknown force (magnitude and direction) in the diagram if the asteroid has an acceleration of 0.01 m/s² in the direction of the positive x-axis?



www.chegg.com/homework-help/questions-and-answers/f1-0-f2-5-go-three-astronauts-propelled-jet-backpacks-push-guide-03-120-kg-asteroid-toward-q57871544

Since the acceleration is known, the components of the net force can be found with Newton's second law.

Forces Acting on the Object

The 3 forces acting on the object are shown in the diagram.

Sum of the Forces

Using the axes shown in the diagram, the 3 forces will be resolved into components.

F_1 (bottom astronaut)

$$F_{1x} = 32N \cdot \cos 30° = 27.71N$$
$$F_{1y} = 32N \cdot \sin 30° = 16N$$

F_2 (middle astronaut)

$$F_{2x} = 55N$$
$$F_{2y} = 0N$$

F_3 (bottom astronaut)

$$F_{3x} = F_{3x}$$
$$F_{3y} = F_{3y}$$

When the magnitude or the direction of a force is not known, it is better to write the components F_x and F_y rather than $F\cos\theta$ and $F\sin\theta$. The resulting system of equations is much easier to solve then.

The components of the net force are, therefore,

$$\begin{aligned} \sum F_x &= 27.71N + 55N + F_{3x} \\ &= 82.71N + F_{3x} \end{aligned} \qquad \begin{aligned} \sum F_y &= 16N + 0N + F_{3y} \\ &= 16N + F_{3y} \end{aligned}$$

Newton's Second Law

The components of the acceleration are given.

$$a_x = 0.01\tfrac{m}{s^2} \qquad a_y = 0$$

Thus, Newton's second law gives us those 2 equations.

$$\sum F_x = ma_x \quad \rightarrow \quad 82.71N + F_{3x} = 20\,000kg \cdot 0.01\tfrac{m}{s^2}$$
$$\sum F_y = ma_y \quad \rightarrow \quad 16N + F_{3y} = 0$$

Solving the Equation

From x-component equation, the x-component of the force is obtained.

$$82.71N + F_{3x} = 20\,000kg \cdot 0.01\tfrac{m}{s^2}$$
$$82.71N + F_{3x} = 200N$$
$$F_{3x} = 117.29N$$

From y-component equation, the y-component of the force is obtained.

$$16N + F_{3y} = 0$$
$$F_{3y} = -16N$$

From the components, the magnitude of the force can be found.

$$\begin{aligned} F &= \sqrt{F_x^2 + F_y^2} \\ &= \sqrt{(117.29N)^2 + (-16N)^2} \\ &= 118.4N \end{aligned}$$

From the components, the direction of the force can be found.

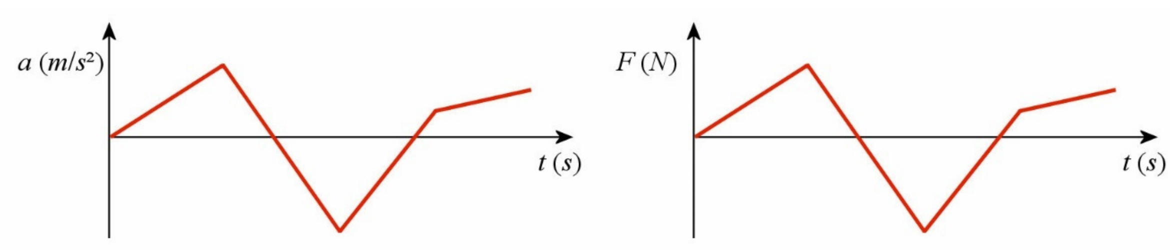
$$\begin{aligned} \theta &= \arctan \frac{F_y}{F_x} \\ &= \arctan \frac{-16N}{117.29N} \\ &= -7.77° \end{aligned}$$

The force applied by the third astronauts is thus a 118.4 N force at -7.77°.

Graphs of Motion

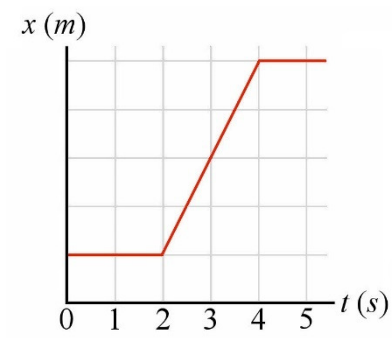
The Net Force Graph is Identical to the Acceleration Graph

Since the net force is directly proportional to the acceleration according to $F_{net} = ma$, the graph of the net force acting on an object as a function of time is identical to the acceleration graph, except that the scaling of the vertical axis is different. For example, here are the graphs of the net force acting on a 5 kg object and of the acceleration of this object.



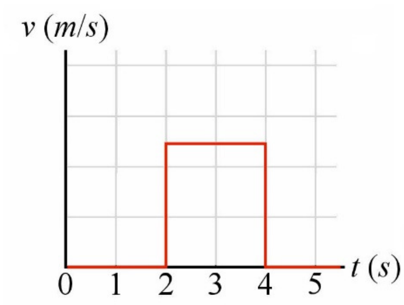
The two graphs are identical, apart from the scaling of the vertical axis.

Impossible Graphs



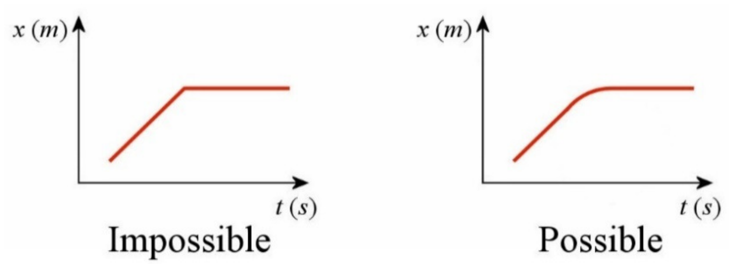
Here's a graph of an object's position as a function of time.

Although this graph looks quite legitimate, it represents an impossible motion. To understand why, the velocity of the object will be plotted.



Between t = 0 s and t = 1 s, the speed is zero (since the slope is zero). Then the velocity of the object is constant and positive from t = 1 s to t = 4 s and the speed finally returns to zero (zero slope) between t = 4 s and t = 5 s.

The acceleration of the object, which is the slope on the velocity graph, can then be found. The acceleration is always zero except at t = 1 s and t = 4 s where it is infinite. However, an infinite acceleration means that the force acting on the object is also infinite, which is impossible. This motion is thus impossible. The problem comes from the sudden change in the value of the slope on the position graph. For the motion to be physically possible, the slope must change gradually.



With a graph of the position where the slope has no sudden jump, a continuous graph is obtained for the speed and the acceleration is never infinite. Note that there can be sudden jumps of the slope in the graphs of velocity and acceleration.

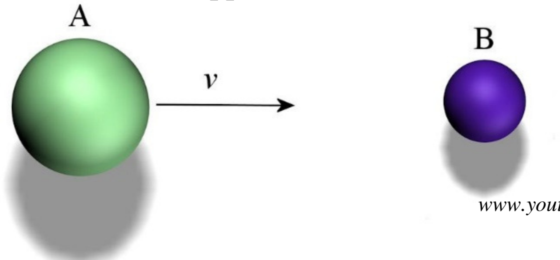
3.5 NEWTON'S THIRD LAW

The Law

The forces are always generated by the interaction between 2 objects. It can be, for example, a contact force generated during a collision between two objects, a gravitational force of attraction between two planets or an electric repulsion force between 2 objects with identical electric charges. For all interactions, the 2 objects are always subjected to a force.

The forces always come from the interaction between 2 objects. Every object that interacts is subjected to a force.

Let's illustrate this with a collision. Suppose that ball A is moving towards a motionless ball B.

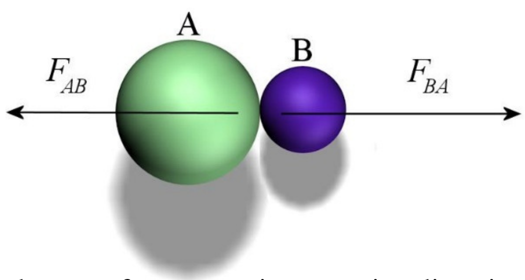


After the collision, ball B moves towards the right while the ball A has lost some speed.

If ball B is now moving towards the right, then there has been an acceleration towards the right, which implies that there was a force directed towards the right on this ball.

If ball A has lost speed, then there has been an acceleration in the opposite direction of the velocity, so towards the left. There has, therefore, been a force towards the left in the collision acting on ball A.

Therefore, the following forces are exerted during the collision. (The force acting on object A made by object B is denoted F_{AB}. The force acting on object B made by object A is denoted F_{BA}.)



You surely notice that the two forces are in opposite directions. The force on ball B is towards the right, and the force on ball A is towards the left. The forces are always in opposite directions like this, no matter the type of force.

The forces always come from the interaction between 2 objects. Every object that interacts is subjected to a force and the forces are always in opposite directions.

But wait, it goes even further. The force acting on object A always has the same magnitude as the force acting on object B. If the forces did not have the same magnitude, then the objects could accelerate on their own without any external force, which would be contrary to Newton's first law (this justification, which will be proven only in chapter 11, is essentially the same as the one given by Newton). We therefore come to the following conclusion.

Newton's Third Law

The forces always come from the interaction between 2 objects. Every object that interacts is subjected to a force and the forces have the same magnitude, and are always in opposite directions.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

It is then said that these two forces are related by Newton's third law.

These two forces have the following properties.

1) **The two forces are of the same type.** If the force on object A is a gravitational force, then the force on object B must also be a gravitational force. Two forces of different kinds, like gravity and a force exerted by a spring, for example, cannot be related by Newton's third law.

2) **The two forces never act on the same object**. If object A exerts a force **on object B**, then object B exerts a force **on object A**. The two forces act on different objects.

Common Mistake: Forgetting These 2 Properties

Forgetting those properties (or ignorance of them) sometimes leads to the following paradox: how can there be any motion if Newton's third law is true? If, for every force, there is an opposite force of equal size, the sum of the forces must always be zero. If a cart is pushed with a force of 50 N, then there is another 50 N force in the opposite direction. The sum of these forces is zero and the cart should not speed up! This argument is not correct because it is important to consider on which object the force acts. If Tony pushes a cart with a 50 N force, then the cart exerts a 50 N force on Tony. To determine the acceleration of the cart, only the forces acting on the cart must be considered. The second 50 N force being applied on Tony must not be considered when calculating the acceleration of the cart. This is what the author of this video has not understood.
https://www.youtube.com/watch?v=X2cis7-SpmY

Newton's 3rd law also means that for every force, another opposite force must exist somewhere in the universe. The forces always come in pairs. It is impossible to have a force without having, somewhere, another force related to it by Newton's third law. There is no exception to this rule.

Newton's 3rd law is often formulated as the law of action and reaction: to every action, there is an equal and opposite reaction. This formulation should be avoided since it suggests that the two forces are not acting simultaneously, that the reaction occurs shortly after the action. The two forces related by Newton's third law always act simultaneously and are perfectly symmetrical. There is no force that takes precedence over the other force.

How Can the Two Forces Related by Newton's Third Law Be Found?

If the forces come in pairs related by Newton's third law, how can these two forces be found? Here's the trick, illustrated by an example. Suppose we have the following force:

The Earth exerts a force of gravity on the Moon.

To find the related force, simply switch the two objects (Earth and Moon) in the sentence to obtain:

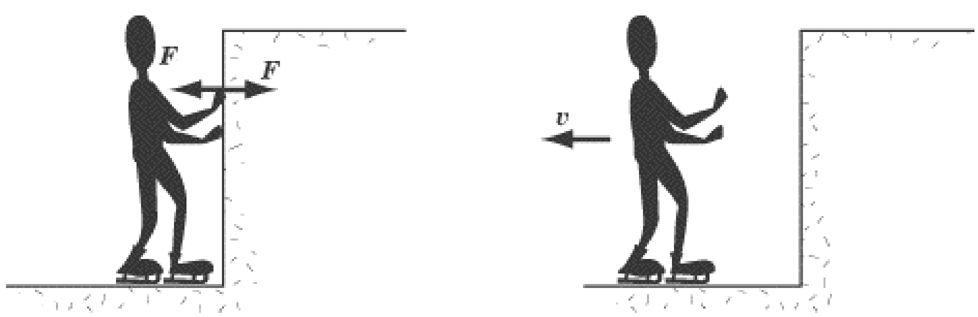
The Moon exerts a force of gravity on the Earth.

These are the two forces related by Newton's third law. By doing this, we ensure that we have the same type of force and that the two related forces do not act on the same object.

Demonstrations of the Law

A collision is featured in this first video.
https://www.youtube.com/watch?v=T8eLWEeQXkc
When the person on the left collides with the person on the right, the latter experiments a force towards the right which gives him an acceleration while the collision lasts, which eventually gives him some speed. A force also acts on the person on the left since he slows down. If he has slowed down, then the force must be opposed to his motion and so is directed towards the left. This shows that there is a force on each person in this collision and that these forces are in opposite directions as predicted by Newton's third law.

If a person pushes on a wall, then the wall also pushes the person according to Newton's third law. Yet, this does not seem to have any effect since the wall does not eject the person. The presence of the force exerted by the wall is less obvious because the friction made by the ground prevents him from moving. If friction is eliminated, by using skates on ice for example, then the effect becomes more obvious. If the skater pushes on the board of the ice rink, he exerts a force on the board and, according to Newton's third law, the board exerts a force on the skater in the opposite direction. This force speeds him up so that he moves away from the board.



www.sparknotes.com/testprep/books/sat2/physics/chapter6section2.rhtml

Here's a demonstration of this (with a skateboard) in this video.
https://www.youtube.com/watch?v=Em0R896soTA

Newton's third law also explains what is happening in this video.
https://www.youtube.com/watch?v=pyVTNyzQ1bI
(Daryl is on the chair to the left and Raphael is on the chair to the right.)

When Daryl pushes Raphael's chair, the two chairs are accelerating. Raphael's chair accelerates since Daryl exerted a force on that chair. Daryl's chair is also accelerating because, according to Newton's third law, Raphael's chair exerted a force towards the left on Daryl when Daryl exerted a force towards the right on Raphael's chair. It is this force that propelled Daryl's chair towards the left.

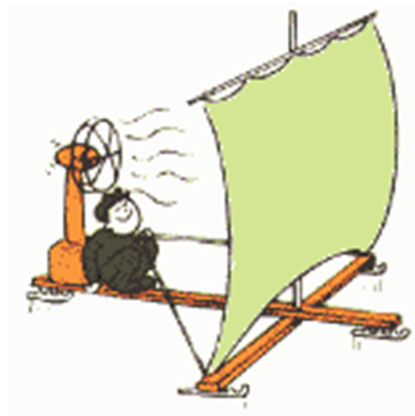
If, like me, you were involved in a fight in high school and did not really know how to fight, get some comfort by telling yourself that you have given great blows to the fist of your opponent with your face (my greetings here to Michel Gouin). Indeed, each time your opponent gave you a punch and thus exerted a force on your face, your face exerted the same force on the fist of your opponent. Poor little fist! You can see this pain in this video.
https://www.youtube.com/watch?v=crdbnEMCt58

The above examples showed that there are indeed two forces, but it was rather difficult to say whether the two forces have the same magnitude. It is easier to see that these two forces have an equal magnitude in the following video. Many collisions between two carts of different masses can be seen. Again, when cart A exerts a force on cart B during the collision, then cart B also exerts a force on cart A in the opposite direction. A force then acts on each cart. What makes this video interesting is the presence of a ring in front of each cart, which deforms when a force acts on the cart. The greater the force on the cart, the more the ring is deformed. Note that during each collision, the deformation of the rings of the two carts is always identical, indicating that the force on each cart is the same as predicted by Newton's third law.
https://www.youtube.com/watch?v=KB3Y-pSGHos

The first astronauts had great difficulty working in space because of Newton's third law. For example, whenever an astronaut had to exert a force to turn a screw, there was a force exerted on the astronaut that made him turn in the opposite direction, which really complicated the task. This documentary will give you more details on this subject.
https://www.youtube.com/watch?v=PrJnWTcW55s

Even if they are told that the forces are the same when two objects collide, most people would still say that, in a collision between a truck and a small car, a greater force acts on the car than on the truck. In the video, it looks like a greater force act on the smaller car than on the larger car.
https://www.youtube.com/watch?v=NCelD0qr8Do
The force and the effect of this force must not be confused. The force is the same on both cars, but the acceleration is not the same. According to $F = ma$, the least massive car undergoes the greatest acceleration. Occupants of this lighter car will, therefore, be more affected than those in the heavier car. In a collision between a truck and a small car, the same force acts on both vehicles, but the acceleration of the car is colossal while the acceleration of the truck is small. The velocity of the car then changes enormously whereas the velocity of the truck changes a little.

3rd Law Mistakes



www.lhup.edu/~dsimanek/scenario/miscon.htm

In cartoons, the characters sometimes try to move faster in a sailboat by blowing into the sails. There is also this demonstration on youtube.
https://www.youtube.com/watch?v=PY-4K4e5s6w

This idea cannot work because of Newton's third law. Suppose there is a fan blowing air into the sail. It is true the air exerts a force on the sail towards the front of the boat that would make the boat move forward. However, there's a problem. If the fan exerts a force on the air towards the front of the vessel, then the air must exert a force on the fan towards the back of the boat, which pushes the boat backwards. This force directed backwards cancels the force directed forwards made by the air on the sail (assuming the sail is 100% effective).

Here's a demonstration of this on the next video.
https://www.youtube.com/watch?v=8HoQH0nAiHs
Initially, a small piece of cardboard acts as a sail, and the small vehicle is not moving since the force on the cardboard and the force on the fan cancel each other. When the cardboard is removed, the only remaining force is the force on the fan and the vehicle begins to accelerate backwards. The moral of the story: instead of sending air into the sail, the sailor would be better to use the fan to propel the air towards the back of the boat so that there is a force directed forwards acting on the boat. This is how the type of boat featured in this video works.
https://www.youtube.com/watch?v=abLcIiyXBPs

The "force" in Star Wars movies does not seem to respect Newton's third law. When Count Dooku uses the force to push Obiwan in a fight, there should also be a force made by Obiwan on Dooku according to Newton's third law. This force does not seem to be present in this clip.
https://www.youtube.com/watch?v=kmIkpRkgaZk
But then again, it's only a movie.

SUMMARY OF EQUATIONS

Newton's First Law or Law of Inertia

If there is no net external force exerted on an object, then
the velocity of the object is constant (magnitude and direction).

Net Force or Resulting Force

$$\vec{F}_{net} = \sum \vec{F}$$

Newton's Second Law

$$\sum \vec{F} = m\vec{a}$$

In components:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

Newton's Third Law

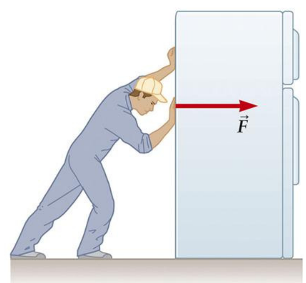
The forces always come from the interaction between 2 objects. Every object that interacts is subjected to a force and the forces have the same magnitude and are always in opposite directions.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

EXERCISES

3.4 Newton's Second Law

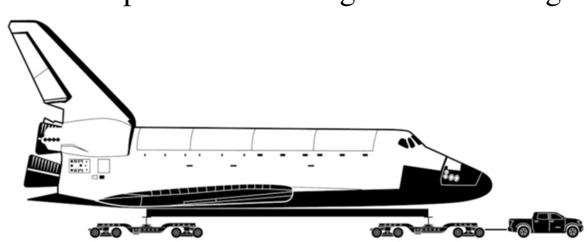
1. Rajiv pushes on an 80 kg refrigerator with a 120 N force. Assuming there is no friction force between the fridge and the floor, what is the acceleration of the refrigerator?



www.chegg.com/homework-help/questions-and-answers/person-shown-pushing-refrigerator-sits-level-floor-assume-floor-slippery-horizontal-force--q1442657

2. The stopping distance of a 1200 kg car is 80 m when it goes at 100 km/h. What is the magnitude of the force exerted on the car while it brakes?

3. The engine of an 1800 kg Toyota Tundra makes enough force to go from 0 to 28.8 km/h in one second when it is not towing a trailer. How long does it take for the truck to reach a speed of 10 km/h if it now pulls a 133 000 kg trailer if the engine still generates the same maximum force? (Of course, the friction between the road and the trailer and the air friction on the trailer are neglected.)



news.pickuptrucks.com/2012/09/new-tundra-to-max-tow-space-shuttle-endeavour-to-final-resting-place.html

You don't think that a Toyota Tundra can haul the space shuttle! Have a look at this.
https://www.youtube.com/watch?v=26CtnvXKY3w

4. An F-18 has a mass of 23 500 kg at takeoff. There are two engines that provide 48.9 kN each at full throttle. What is the minimum runway length needed for this aircraft if it has to reach a speed of 80 m/s to take off?

5. A 1000 kg car is subjected to a variable force given by

$$F_x = 120\tfrac{N}{s}\cdot t + 500N$$

and there are no other forces in the direction of the x-axis. At t = 0 s, the velocity of the car is 5 m/s in the direction of the positive x's, and its initial position is x = 0 m.

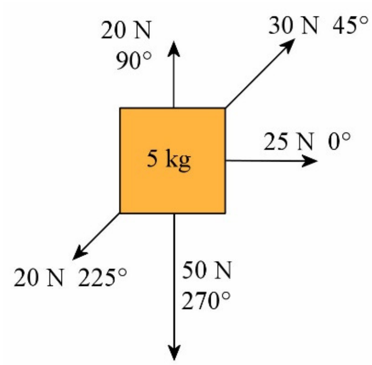
a) What is the velocity of the car at t = 30 s?
b) What is the displacement of the car between t = 0 s and t = 30 s?
c) What is the velocity of the car at x = 400 m?
(For this question, you should come across a rather difficult equation to solve to find t when the object is at x = 400 m. To easily solve this equation, I suggest you go to the Wolfram alpha website
https://www.wolframalpha.com/
and use the *solve* command. For example, if you want to solve the equation $2x^2 + 0.5x + 5 = 20$, write

solve 2*x^2+0.5*x+5=20

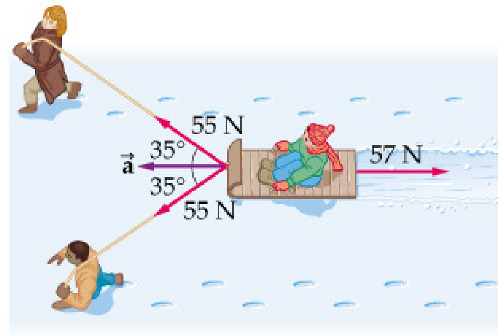
Use periods, not commas, for numbers. Then, you will get

Click *approximate forms* if you want Wolfram to calculate the value of the solutions.)

6. The forces shown in the diagram act on a 5 kg object. What is its acceleration of the object (magnitude and direction)?

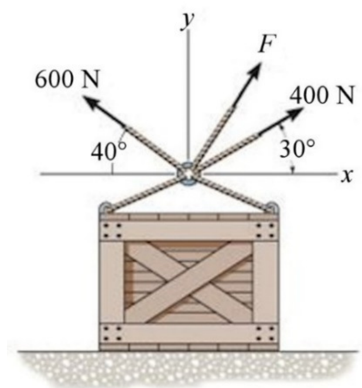


7. Little Aaron on his 2 kg sleigh is pulled by his parents, Alfred, and Gertrude. Initially, Aaron's sled is at rest. In the first 2 seconds of the motion, the sleigh travelled a distance of 6 m with a constant acceleration. What is Aaron's mass?



www.chegg.com/homework-help/questions-and-answers/22-kg-child-ride-teenagers-pull-32-kg-sled-ropes-indicated-figure-figure -1--teenagers-pull-q2939212

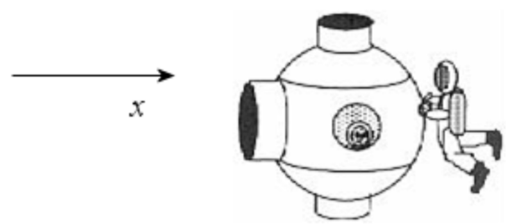
8. The net force exerted by the ropes tied to the box is an 850 N force directed straight up. What is the magnitude and the direction of the unknown force F?



www.chegg.com/homework-help/questions-and-answers/result-force-vertically-upward-850n-i-f1-319n-i-having-trouble-finding-theta-best-way-angl-q1954937

3.5 Newton's Third Law

9. A 100 kg astronaut pushes his 2500 kg spacecraft with a force of 200 N for 0.5 seconds. Initially, the spacecraft and astronaut are at rest.



www.ahsd.org/science/stroyan/hphys/CH4/ch4wq/p2ch4wq3.htm

a) What is the velocity of the spaceship after the push?
b) What is the velocity of the astronaut after the push?

Challenges

(Questions more difficult than the exam questions.)

10. A 2 kg object at $x = 0$ has a velocity of 10 m/s towards the positive x-axis. However, a force equal to

$$F = -8 \tfrac{N}{m} x$$

acts on this object. What will the distance travelled by this object be when its speed reaches zero?

11. The speed of a 2 kg object is given by

$$v = 2\frac{1}{sm}x^2$$

What is the formula of the force acting on the object as a function of its position?

ANSWERS

3.4 Newton's Second Law

1. 1.5 m/s²
2. 5787 N
3. 26 s
4. 768.9 m
5. a) 74 m/s b) 915 m c) 41.93 m/s
6. 7.885 m/s² at -35.56°
7. 9.04 kg
8. 287.5 N at 66.8°

3.5 Newton's Third Law

9. a) -0.04 m/s b) 1 m/s

Challenges

10. 5 m
11. $F = 16\frac{kg}{s^2m^2}x^3$