Chapter 14 solutions

1. a) The eccentricity is

$$\begin{aligned} e &= \frac{v_p^2 r_p}{GM_c} - 1 \\ &= \frac{\left(70{,}000\,\tfrac{m}{s}\right)^2 \left(5\times 10^{10}\,m\right)}{6.674\times 10^{-11}\,\tfrac{Nm^2}{kg^2}\cdot 2\times 10^{30}\,kg} - 1 \\ &= 0.8355 \end{aligned}$$

b) Since the eccentricity is between 0 and 1, the orbit is elliptical.

c) The distance is

$$\begin{aligned} r &= r_p \frac{1+e}{1+e\cos\theta} \\ &= 5\times 10^{10}\,m\,\frac{1+0.8355}{1+0.8355\cdot\cos 90^\circ} \\ &= 9.177\times 10^{10}\,m \end{aligned}$$

This is 91.77 million km.

2. The time interval between the two configurations a and b equals one quarter of the period of Planet X. So it is equal to

$$t = \frac{1}{4}2\pi\sqrt{\frac{r_X^3}{GM}}$$

We need to find out what percentage of the period of the planet Y this time corresponds. Then, the ratio of this time and the period of the planet Y must be calculated. We'll call this ratio f.

$$\begin{aligned} f &= \frac{t}{T_Y} \\ &= \frac{\frac{1}{4}2\pi\sqrt{\frac{r_X^3}{GM}}}{2\pi\sqrt{\frac{r_Y^3}{GM}}} \\ &= \frac{\sqrt{r_X^3}}{4\sqrt{r_Y^3}} \\ &= \frac{1}{4}\sqrt{\frac{r_X^3}{r_Y^3}} \end{aligned}$$

Since $r_X = 3r_Y$, the ratio is

$$\begin{aligned} f &= \frac{1}{4}\sqrt{\frac{(3r_Y)^3}{r_Y^3}} \\ &= \frac{1}{4}\sqrt{\frac{27r_Y^3}{r_Y^3}} \\ &= \frac{1}{4}\sqrt{27} \\ &= 1.299 \end{aligned}$$

Therefore, the angular displacement of the planet Y is

$$\theta = 1.299 \cdot 360° = 467.7°$$

3. The distance is

$$\begin{aligned} r_p &= a(1-e) \\ &= 1.496\times10^{11}m(1-0.01671) \\ &= 1.471\times10^{11}m \end{aligned}$$

4. The distance is

$$\begin{aligned} r_a &= a(1+e) \\ &= 1.496\times10^{11}m\cdot(1+0.01671) \\ &= 1.521\times10^{11}m \end{aligned}$$

5. The speed is

$$\begin{aligned} v_p &= \sqrt{\frac{GM}{a}\frac{1+e}{1-e}} \\ &= \sqrt{\frac{6.674\times 10^{-11}\,\frac{Nm^2}{kg^2}\cdot 1.9885\times 10^{30}\,kg}{1.496\times 10^{11}\,m}\frac{1+0.01671}{1-0.01671}} \\ &= 30,286\,\tfrac{m}{s} \end{aligned}$$

6. The speed is

$$\begin{aligned} v_A &= \sqrt{\frac{GM}{a}\frac{1+e}{1-e}} \\ &= \sqrt{\frac{6.674\times 10^{-11}\,\frac{Nm^2}{kg^2}\cdot 1.9885\times 10^{30}\,kg}{1.496\times 10^{11}\,m}\frac{1-0.01671}{1+0.01671}} \\ &= 29,291\,\tfrac{m}{s} \end{aligned}$$

7. The period is

$$\begin{aligned} T &= 2\pi\sqrt{\frac{a^3}{GM}} \\ &= 2\pi\sqrt{\frac{\left(1.496\times 10^{11}\,m\right)^3}{6.674\times 10^{-11}\,\frac{Nm^2}{kg^2}\cdot 1.9885\times 10^{30}\,kg}} \\ &= 3.156\times 10^7\,s \\ &= 365.26\,j \end{aligned}$$

8. The energy is

$$\begin{aligned} E_{mec} &= -\frac{GMm}{2a} \\ &= -\frac{6.674\times 10^{-11}\,\frac{Nm^2}{kg^2}\cdot 1.9885\times 10^{30}\,kg\cdot 5.98\times 10^{24}\,kg}{2\cdot 1.496\times 10^{11}\,m} \\ &= -2.652\times 10^{33}\,J \end{aligned}$$

9. Using the values at perihelion, the angular momentum is

$$\begin{aligned} L &= mv_p r_p \sin 90° \\ &= 5.98\times10^{24}kg \cdot 30,286\tfrac{m}{s}\cdot\left(1.471\times10^{11}m\right)\sin 90° \\ &= 2.664\times10^{40}\tfrac{kgm^2}{s} \end{aligned}$$

10. a) The speed is

$$\begin{aligned} v &= \sqrt{GM_c\left(\frac{2}{r}-\frac{1}{a}\right)} \\ &= \sqrt{6.674\times10^{-11}\tfrac{Nm^2}{kg^2}\cdot 1.9885\times10^{30}kg\cdot\left(\frac{2}{1.5\times10^{11}m}-\frac{1}{1.496\times10^{11}m}\right)} \\ &= 29.705\tfrac{km}{s} \end{aligned}$$

b) The angle is found with the following formula

$$r = \frac{a\left(1-e^2\right)}{1+e\cos\theta}$$

Thus,

$$\begin{aligned} r &= \frac{a\left(1-e^2\right)}{1+e\cos\theta} \\ 1.5\times10^{11}m &= \frac{1.496\times10^{11}m\cdot\left(1-0.01671^2\right)}{1+0.01671\cdot\cos\theta} \end{aligned}$$

If this equation is solved for θ, we obtain

$$\begin{aligned} \cos\theta &= -0,1762 \\ \theta &= 100,2° \end{aligned}$$

c) Since the angular momentum is conserved, we must have

$$L = mvr\sin\theta$$

Therefore,

$$L = mvr\sin\theta$$
$$2.664\times10^{40}\,\tfrac{kgm^2}{s} = 5.98\times10^{24}kg\cdot 29{,}705\,\tfrac{m}{s}\cdot 1.5\times10^{11}m\cdot\sin\theta$$
$$\sin\theta = 0.99984$$
$$\theta = 89.0° \quad\text{or}\quad 91.0°$$

11. 1) In the first possibility, an orbit with the following characteristics is followed.

$$r_p = 1.5\times10^{11}m$$
$$r_a = 7.8\times10^{11}m$$

The semi-major axis of this transfer orbit is

$$a = \frac{r_a + r_p}{2} = \frac{7.8\times10^{11}m + 1.5\times10^{11}m}{2} = 4.65\times10^{11}m$$

Therefore, the time is

$$\begin{aligned} T &= \frac{1}{2}2\pi\sqrt{\frac{a^3}{GM_c}} \\ &= \pi\sqrt{\frac{\left(4.65\times10^{11}m\right)^3}{6.674\times10^{-11}\,\tfrac{Nm^2}{kg^2}\cdot 2\times10^{30}kg}} \\ &= 8.622\times10^7 s \\ &= 997.9\ days \end{aligned}$$

2) In the second option, a transfer orbit from the Earth to Venus with the following characteristics is first followed.

$$r_p = 1\times10^{11}m$$
$$r_a = 1.5\times10^{11}m$$

The semi-major axis of this transfer orbit is

$$a = \frac{r_a + r_p}{2} = \frac{1.5\times10^{11}m + 1\times10^{11}m}{2} = 1.25\times10^{11}m$$

Thus, the time for this first part is

$$\begin{aligned}T &= \frac{1}{2}2\pi\sqrt{\frac{a^3}{GM_c}}\\ &= \pi\sqrt{\frac{\left(1.25\times10^{11}m\right)^3}{6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot2\times10^{30}kg}}\\ &= 1.202\times10^7 s\\ &= 139.1\,days\end{aligned}$$

Then, a transfer orbit from Venus to Jupiter with the following characteristics is followed.

$$\begin{aligned}r_p &= 1\times10^{11}m\\ r_a &= 7.8\times10^{11}m\end{aligned}$$

The semi-major axis of this transfer orbit is

$$a = \frac{r_a + r_p}{2} = \frac{7,8\times10^{11}m + 1\times10^{11}m}{2} = 4,4\times10^{11}m$$

The time taken to travel this second part is

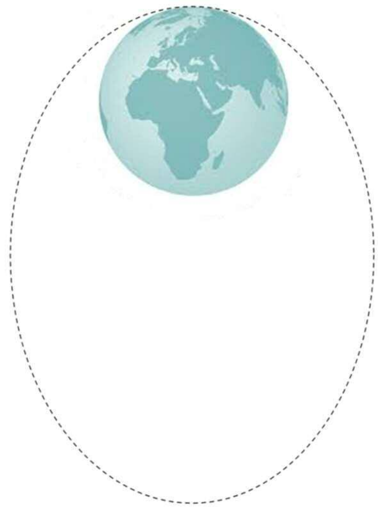
$$\begin{aligned}T &= \frac{1}{2}2\pi\sqrt{\frac{a^3}{GM_c}}\\ &= \pi\sqrt{\frac{\left(4.4\times10^{11}m\right)^3}{6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot2\times10^{30}kg}}\\ &= 7.936\times10^7 s\\ &= 918.6\,days\end{aligned}$$

Therefore, the total time of this second trip is

139.1 days + 918.6 days = 1057.7 days

Thus, the first possibility is the best.

12. By launching tangentially, we have the following orbit.



With this orbit, r_p is the radius of the Earth and r_a is the greatest distance between the centre of the Earth and the object.

With the conservation of energy, we have

$$\frac{1}{2}mv^2-\frac{GMm}{R}=-\frac{GMm}{2a}$$

Since the speed is

$$\begin{aligned}v&=0.85v_{esc}\\&=0.85\sqrt{\frac{2GM}{R}}\end{aligned}$$

we obtain

$$\begin{aligned}\frac{1}{2}m\left(0.85\sqrt{\frac{2GM}{R}}\right)^2-\frac{GMm}{R}&=-\frac{GMm}{2a}\\ \frac{1}{2}m(0.85)^2\frac{2GM}{R}-\frac{GMm}{R}&=-\frac{GMm}{2a}\\ (0.85)^2\frac{1}{R}-\frac{1}{R}&=-\frac{1}{2a}\\ \left((0.85)^2-1\right)\frac{1}{R}&=-\frac{1}{2a}\\ a&=\frac{R}{2\left(1-(0.85)^2\right)}\\ a&=\frac{200R}{111}\end{aligned}$$

Since

$$r_p + r_a = 2a$$

We arrive at

$$r_a + R = 2\frac{200R}{111}$$
$$r_a = \left(\frac{400}{111} - 1\right)R$$
$$r_a = \frac{289}{111}R$$

This is the distance between the centre of the Earth and the object. The distance between the object and the surface is

$$d = r_a - R$$
$$d = \frac{289}{111}R - R$$
$$d = \frac{178}{111}R$$
$$d = \frac{178}{111}\cdot 6380km$$
$$d = 10,231km$$

13. To find the time, Kepler's second law is used.

$$\Delta A = \frac{\sqrt{GM_c r_p\left(1+e\right)}}{2}\Delta t$$

If the comet makes one full revolution, the law gives

$$A_{ellipse} = \frac{\sqrt{GM_c r_p\left(1+e\right)}}{2}T$$

If the comet passes from point 1 to point 2, the law gives

$$A_{gray} = \frac{\sqrt{GM_c r_p\left(1+e\right)}}{2}t$$

By dividing these two equations one by the other, we obtain

$$\frac{A_{gray}}{A_{ellipse}} = \frac{t}{T}$$

(Which is not surprising, since Kepler's law says that the areas are the same in equal times. The ratio of areas is thus equal to the ratio of times.)

It remains to find the grey area. This area is

$$\begin{aligned} A &= A_{\text{moitié de l'ellipse}} - A_{\text{triangle}} \\ &= \frac{\pi a^2\sqrt{1-e^2}}{2} - \frac{(2b)\cdot c}{2} \\ &= \frac{\pi a^2\sqrt{1-e^2}}{2} - bc \\ &= \frac{\pi a^2\sqrt{1-e^2}}{2} - a\sqrt{1-e^2}\cdot ae \\ &= \pi a^2\sqrt{1-e^2}\left(\frac{1}{2}-\frac{e}{\pi}\right) \end{aligned}$$

Therefore

$$\frac{A_{gray}}{A_{ellipse}} = \frac{t}{T}$$

$$\frac{\pi a^2\sqrt{1-e^2}\left(\frac{1}{2}-\frac{e}{\pi}\right)}{\pi a^2\sqrt{1-e^2}} = \frac{t}{T}$$

$$\left(\frac{1}{2}-\frac{e}{\pi}\right) = \frac{t}{T}$$

Thus, the time is

$$\begin{aligned} \left(\frac{1}{2}-\frac{e}{\pi}\right) &= \frac{t}{T} \\ \left(\frac{1}{2}-\frac{0.87}{\pi}\right) &= \frac{t}{50a} \\ t &= 11.15a \end{aligned}$$

14. We have 3 equations and three unknowns.

$$v_P = \sqrt{\frac{GM_c}{a}\frac{1+e}{1-e}} \quad \to \quad 2200\tfrac{m}{s} = \sqrt{\frac{GM_c}{a}\frac{1+e}{1-e}}$$

$$v_A = \sqrt{\frac{GM_c}{a}\frac{1-e}{1+e}} \quad \to \quad 1700\tfrac{m}{s} = \sqrt{\frac{GM_c}{a}\frac{1-e}{1+e}}$$

$$T = 2\pi\sqrt{\frac{a^3}{GM_c}} \quad \to \quad 3{,}888{,}000s = 2\pi\sqrt{\frac{a^3}{GM_c}}$$

To solve this system, let's first multiply the first equation by the second equation.

$$2200\tfrac{m}{s}\cdot 1700\tfrac{m}{s} = \sqrt{\frac{GM_c}{a}\frac{1+e}{1-e}}\cdot\sqrt{\frac{GM_c}{a}\frac{1-e}{1+e}}$$

$$3.74\times 10^6\tfrac{m^2}{s^2} = \frac{GM_c}{a}$$

If the square root of this result is now multiplied by the third equation, we obtain

$$\sqrt{3.74\times 10^6\tfrac{m^2}{s^2}}\cdot 3{,}888{,}000s = \sqrt{\frac{GM_c}{a}}\cdot 2\pi\sqrt{\frac{a^3}{GM_c}}$$

$$7.519034\times 10^9 m = 2\pi\sqrt{\frac{GM_c}{a}\frac{a^3}{GM_c}}$$

$$7.519034\times 10^9 m = 2\pi a$$

$$a = 1.196691\times 10^9 m$$

$$a = 1{,}196{,}691km$$

From this result, the mass of the planet can be found.

$$3{,}888{,}000s = 2\pi\sqrt{\frac{a^3}{GM_c}}$$

$$3{,}888{,}000s = 2\pi\sqrt{\frac{\left(1.196691\times 10^9 m\right)^3}{6.674\times 10^{-11}\tfrac{Nm^2}{kg^2}M_c}}$$

$$M_c = 6.706\times 10^{25}kg$$

The eccentricity can then be found with one of the formulas for the speed. It can also be found by dividing the two speed equations.

$$\frac{2200\,\frac{m}{s}}{1700\,\frac{m}{s}}=\frac{\sqrt{\frac{GM_c}{a}\frac{1+e}{1-e}}}{\sqrt{\frac{GM_c}{a}\frac{1-e}{1+e}}}$$

$$\frac{2200\,\frac{m}{s}}{1700\,\frac{m}{s}}=\sqrt{\frac{GM_c}{a}\frac{1+e}{1-e}}\cdot\sqrt{\frac{a}{GM_c}\frac{1+e}{1-e}}$$

$$\frac{22}{17}=\frac{1+e}{1-e}$$

$$22(1-e)=17(1+e)$$

$$22-22e=17+17e$$

$$5=39e$$

$$e=\frac{5}{39}=0.1282$$

15. a) The speed is

$$v=\sqrt{\frac{2GM_{Sun}}{r_p}}$$

$$=\sqrt{\frac{2\cdot 6.674\times 10^{-11}\,\frac{Nm^2}{kg^2}\cdot 2\times 10^{30}\,kg}{5\times 10^{10}\,m}}$$

$$=73.07\,\frac{km}{s}$$

b) As that mechanical energy is zero on a parabolic trajectory, the speed can be found with the conservation of energy

$$0=\frac{1}{2}mv^2+\frac{-GM_cm}{r}$$

$$0=\frac{1}{2}v^2+\frac{-GM_c}{r}$$

$$0=\frac{1}{2}v^2+\frac{-6.674\times 10^{-11}\,\frac{Nm^2}{kg^2}\cdot 2\times 10^{30}\,kg}{2\times 10^{11}\,m}$$

$$v=36.53\,\frac{km}{s}$$

16. The speed can be found with the eccentricity formula.

$$e=\frac{v_p^2 r_p}{GM_c}-1$$
$$1.2=\frac{v_p^2\left(5\times10^{10}m\right)}{6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot2\times10^{30}kg}-1$$
$$v_p=76.64\frac{km}{s}$$

b) The speed can be found with the conservation of energy. On this trajectory, energy is

$$E_{mec}=-\frac{GM_c m\left(1-e\right)}{2r_p}$$

Therefore,

$$-\frac{GM_c m\left(1-e\right)}{2r_p}=\frac{1}{2}mv^2+\frac{-GM_c m}{r}$$
$$-\frac{GM_c\left(1-e\right)}{2r_p}=\frac{1}{2}v^2+\frac{-GM_c}{r}$$
$$\frac{-6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot2\times10^{30}kg\cdot\left(1-1.2\right)}{2\cdot5\times10^{10}m}=\frac{1}{2}v^2+\frac{-6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot2\times10^{30}kg}{2\times10^{11}m}$$
$$2.6696\times10^6\frac{J}{kg}=\frac{1}{2}v^2+-6.674\times10^6\frac{J}{kg}$$
$$v=36.61\frac{km}{s}$$

17. a) To know the shape of the orbit, the sign of the mechanical energy of the comet will be calculated. This energy is

$$E_{mec}=\frac{1}{2}mv^2-\frac{GM_T m}{r}$$

We don't have the mass of the comet, but anyway, we only want to know that the sign of the energy. So we have

$$\begin{aligned} E_{mec} &= m\left(\frac{1}{2}v^2 - \frac{GM_T}{r}\right) \\ &= m\left(\frac{1}{2}\left(100\tfrac{m}{s}\right)^2 - \frac{6.674\times10^{-11}\tfrac{Nm^2}{kg^2}\cdot 5.98\times10^{24}kg}{10^{11}m}\right) \\ &= m\left(1008.948\tfrac{m^2}{s^2}\right) \end{aligned}$$

As the value is positive, the orbit is hyperbolic.

b) We know that

$$E_{mec} = m\left(1008.948\tfrac{m^2}{s^2}\right)$$

At the point closest to the Earth, we have

$$\begin{aligned} m\left(1008.948\tfrac{m^2}{s^2}\right) &= \frac{1}{2}mv_p^2 - \frac{GM_T m}{r_p} \\ 1008.948\tfrac{m^2}{s^2} &= \frac{1}{2}v_p^2 - \frac{GM_T}{r_p} \end{aligned}$$

A second equation is required to solve. This second equation comes from the conservation of angular momentum.

$$\begin{aligned} vr\sin\psi &= v_p r_p \\ 100\tfrac{m}{s}\cdot 10^{11}m\cdot\sin 0.5° &= v_p r_p \\ 8.7265\times10^{10}\tfrac{m^2}{s} &= v_p r_p \\ v_p &= \frac{8.7265\times10^{10}\tfrac{m^2}{s}}{r_p} \end{aligned}$$

Substituting this value into the energy equation, we obtain

$$1008.948\,\tfrac{m^2}{s^2} = \frac{1}{2}v_p^2 - \frac{GM_T}{r_p}$$

$$1008.948\,\tfrac{m^2}{s^2} = \frac{1}{2}\left(\frac{8.7265\times10^{10}\,\tfrac{m^2}{s}}{r_p}\right)^2 - \frac{GM_T}{r_p}$$

$$1008.948\,\tfrac{m^2}{s^2} = \frac{3.8076\times10^{21}\,\tfrac{m^4}{s^2}}{r_p^2} - \frac{3.991052\times10^{14}\,\tfrac{m^3}{s^2}}{r_p}$$

$$1008.948\,\tfrac{m^2}{s^2}\,r_p^2 = 3.8076\times10^{21}\,\tfrac{m^4}{s^2} - 3.991052\times10^{14}\,\tfrac{m^3}{s^2}\,r_p$$

$$1008.948\cdot r_p^2 + 3.991052\times10^{14}\,m\cdot r_p - 3.8076\times10^{21}\,m^2 = 0$$

The solution of the quadratic equation is

$$\begin{aligned} r_p &= \frac{-3.991052\times10^{14}\,m \pm \sqrt{\left(3.991052\times10^{14}\,m\right)^2 - 4\cdot1008.948\cdot3.8076\times10^{21}\,m^2}}{2\cdot1008.948} \\ &= \frac{-3.991052\times10^{14}\,m \pm 3.991245\times10^{14}\,m}{2\cdot1008.948} \\ &= \frac{1.9251\times10^{10}\,m}{2\cdot1008.948} \\ &= 9\,540\,165\,m \\ &= 9540.2\,km \end{aligned}$$

As this value is larger than the radius of the Earth, the comet does not strike the Earth. It passes, however, very close (at 3160 km of the surface).

c) The eccentricity can be found with the energy formula

$$E_{mec} = -\frac{GM_c m(1-e)}{2r_p}$$

$$1008.948\,\tfrac{m^2}{s^2}\cdot m = -\frac{6.674\times10^{-11}\,\tfrac{Nm^2}{kg^2}\cdot5.98\times10^{24}\,kg\cdot m(1-e)}{2\left(9.5402\times10^{6}\,m\right)}$$

$$(1-e) = -4.82\times10^{-5}$$

$$e = 1.0000482$$

18. The field is

$$\begin{aligned} g &= \frac{GM}{r^2} \\ &= \frac{6.674\times10^{-11} \frac{Nm^2}{kg^2}\cdot 5.98\times10^{24}kg}{\left(6.48\times10^{6}m\right)^2} \\ &= 9.505\,\tfrac{N}{kg} \end{aligned}$$

19. The field is

$$\begin{aligned} g &= \frac{GMr}{R^3} \\ &= \frac{6.674\times10^{-11} \frac{Nm^2}{kg^2}\cdot 5.98\times10^{24}kg\cdot 5.38\times10^{6}m}{\left(6.38\times10^{6}m\right)^3} \\ &= 8.27\,\tfrac{N}{kg} \end{aligned}$$

20. a) The field is

$$\begin{aligned} g &= \frac{GM}{r^2} \\ &= \frac{6.674\times10^{-11} \frac{Nm^2}{kg^2}\cdot 7.35\times10^{22}kg}{\left(1.738\times10^{6}m\right)^2} \\ &= 1.624\,\tfrac{N}{kg} \end{aligned}$$

b) The weight os the person is

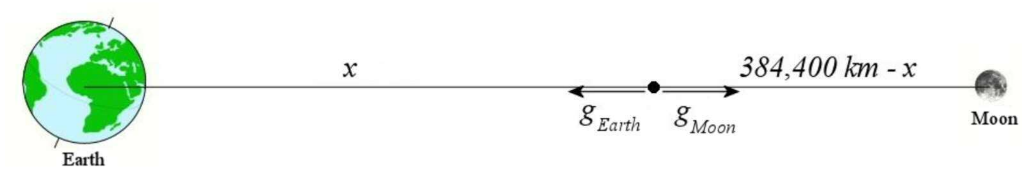
$$\begin{aligned} w &= mg \\ &= 70kg\cdot 1.624\,\tfrac{N}{kg} \\ &= 113.68N \end{aligned}$$

c) As the weight of this person on Earth is 70 kg x 9.8 N/kg = 686 N, the ratio between the weight on the Moon and the weight on the Earth is

$$\frac{113.68N}{686N} = 0.166$$

The weight on the Moon is therefore only 16.6% of the weight of the person on Earth.

21. We have the following configuration.



The sum of the fields is

$$g = -\frac{GM_{Earth}}{x^2} + \frac{GM_{Moon}}{\left(3.844\times 10^8 m - x\right)^2}$$

Since we want a vanishing field, we must have

$$0 = -\frac{GM_{Earth}}{x^2} + \frac{GM_{Moon}}{\left(3.844\times 10^8 m - x\right)^2}$$

$$\frac{GM_{Earth}}{x^2} = \frac{GM_{Moon}}{\left(3.844\times 10^8 m - x\right)^2}$$

$$\frac{M_{Earth}}{x^2} = \frac{M_{Moon}}{\left(3.844\times 10^8 m - x\right)^2}$$

$$\left(3.844\times 10^8 m - x\right)^2 = \frac{M_{Moon}}{M_{Earth}} x^2$$

$$\left(3.844\times 10^8 m - x\right)^2 = \frac{7.35\times 10^{22} kg}{5.98\times 10^{24} kg} x^2$$

$$\left(3,844\times 10^8 m - x\right)^2 = 0.01229 x^2$$

$$1.4776\times 10^{17} m^2 - 7.688\times 10^8 m\cdot x + x^2 = 0.01229 x^2$$

$$1.4776\times 10^{17} m^2 - 7.688\times 10^8 m + 0.9877 x^2 = 0$$

The solution of this quadratic equation is

$$x = 3.46 \times 10^8 \text{ m}$$

The field is, therefore, zero 346,000 km from the Earth.

(There is another solution at 432,000 km from the Earth. There, both fields have the same magnitude, but the total field is non-zero because fields are in the same direction.)

22. An x-axis towards the right and a y-axis upwards will be used. The field made by the Moon is towards the left, so towards the negative x. The field made by the Moon is

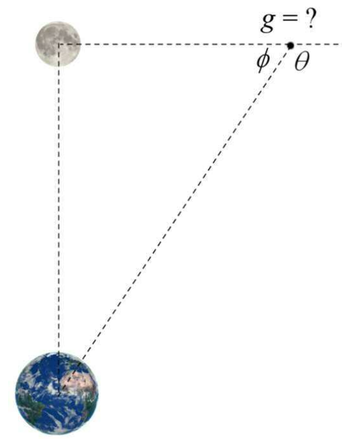
$$\begin{aligned} g_{Mx} &= -\frac{GM}{r^2} \\ &= -\frac{6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot 7.35\times10^{22}kg}{\left(1.5\times10^8 m\right)} \\ &= -2.1801\times10^{-4}\frac{N}{kg} \end{aligned}$$

To find the magnitude of the field made by the Earth, the distance between the Earth and the place where the field is sought must be known. This distance is

$$\begin{aligned} r &= \sqrt{\left(3.844\times10^8 m\right)^2+\left(1.5\times10^8 m\right)^2} \\ &= 4.126\times10^8 m \end{aligned}$$

Therefore, the magnitude of the field made by the Earth is

$$
\begin{aligned}
g_E &= \frac{GM}{r^2} \\
&= \frac{6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot5.98\times10^{24}\,kg}{\left(4.126\times10^{8}\,m\right)} \\
&= 2.344\times10^{-3}\,\tfrac{N}{kg}
\end{aligned}
$$



To resolve this field into components, the angle θ must be known. The angle ϕ is found first

$$
\begin{aligned}
\tan\phi &= \frac{384\,400}{150\,000} \\
\phi &= 68.68°
\end{aligned}
$$

Therefore, the angle θ is

$$
\begin{aligned}
\theta &= 180° - \phi \\
&= 180 - 68.68° \\
&= 111.32°
\end{aligned}
$$

Therefore, the components of the field of the Earth are

$$
\begin{aligned}
g_{Ex} &= 2.344\times10^{-3}\,\tfrac{N}{kg}\cos\left(-111.32°\right) \\
&= -8.521\times10^{-4}\,\tfrac{N}{kg}
\end{aligned}
$$

$$
\begin{aligned}
g_{Ey} &= 2.344\times10^{-3}\,\tfrac{N}{kg}\sin\left(-111.32°\right) \\
&= -2.184\times10^{-3}\,\tfrac{N}{kg}
\end{aligned}
$$

Thus, the components of the total field are

$$
\begin{aligned}
g_x &= g_{Mx} + g_{Mx} \\
&= -2.180\times10^{-4}\,\tfrac{N}{kg} - 8.521\times10^{-4}\,\tfrac{N}{kg} \\
&= -1.070\times10^{-3}\,\tfrac{N}{kg}
\end{aligned}
$$

$$
\begin{aligned}
g_y &= g_{My} + g_{Ey} \\
&= 0 + -2.184\times10^{-3}\,\tfrac{N}{kg} \\
&= -2.184\times10^{-3}\,\tfrac{N}{kg}
\end{aligned}
$$

Therefore, the magnitude of the field is

$$\begin{aligned} g &= \sqrt{g_x^2 + g_y^2} \\ &= \sqrt{\left(1.070\times10^{-3} \tfrac{N}{kg}\right)^2 + \left(2.184\times10^{-3} \tfrac{N}{kg}\right)^2} \\ &= 2.432\times10^{-3} \tfrac{N}{kg} \end{aligned}$$

23. a) The force is

$$\begin{aligned} F &= \frac{GMm}{r^2} \\ &= \frac{6.674\times10^{-11} \frac{Nm^2}{kg^2} \cdot 5.98\times10^{24} kg \cdot 7.35\times10^{22} kg}{\left(3.844\times10^{8} m\right)^2} \\ &= 1.985\times10^{20} N \end{aligned}$$

b) According to Newton's third law, the force is

$$1.985\times10^{20} N$$

24. The tidal force exerted by the Sun is

$$F_{Sun} = \frac{2GM_{Sun} m R_{Earth}}{r_{Sun-Earth}^3}$$

and the tidal force exerted by the Moon is

$$F_{Moon} = \frac{2GM_{Moon} m R_{Earth}}{r_{Moon-Earth}^3}$$

The ratio of these two forces is then

$$\frac{F_{Moon}}{F_{Soleil}} = \frac{\left(\frac{2GM_{Moon}mR_{Earth}}{r^3_{Moon-Earth}}\right)}{\left(\frac{2GM_{Sun}mR_{Earth}}{r^3_{Sun-Earth}}\right)}$$

$$= \frac{\left(\frac{M_{Moon}}{r^3_{Moon-Earth}}\right)}{\left(\frac{M_{Sun}}{r^3_{Sun-Earth}}\right)}$$

$$= \frac{M_{Moon}r^3_{Sun-Earth}}{M_{Sun}r^3_{Moon-Earth}}$$

$$= \frac{7.35\times 10^{22}kg\cdot\left(1.496\times 10^{11}m\right)^3}{1.9885\times 10^{30}kg\cdot\left(3.844\times 10^{8}m\right)^3}$$

$$= 2.18$$

25. We must have

$$F_{tides} = 1\% \text{ of } mg$$

$$\frac{2GM_{Moon}mR_{Earth}}{r^3_{Moon-Earth}} = 0,01mg$$

Therefore,

$$\frac{2GM_{Moon}R_{Earth}}{r^3_{Moon-Earth}} = 0,01g$$

$$r_{Moon-Earth} = \sqrt[3]{\frac{2GM_{Moon}R_{Earth}}{0,01g}}$$

$$r_{Moon-Earth} = \sqrt[3]{\frac{2\cdot 6.674\times 10^{-11}\frac{Nm^2}{kg^2}\cdot 7.35\times 10^{22}kg\cdot 6.38\times 10^{6}m}{0,01\cdot 9.8\frac{N}{kg}}}$$

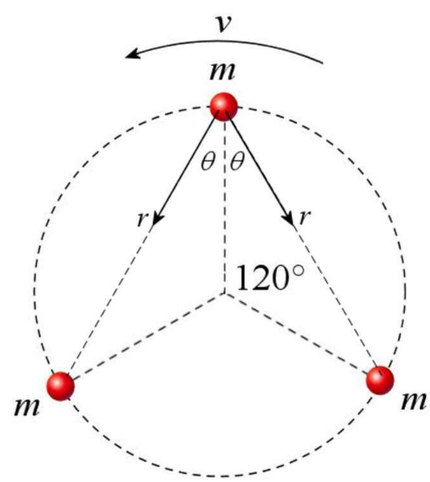
$$r_{Moon-Earth} = 8,611,907m = 8612km$$

26. The distance is

$$\begin{aligned} r &= 2.42285 \sqrt[3]{\frac{\rho_p}{\rho_s}} R_p \\ &= 2.42285 \sqrt[3]{\frac{1408 \frac{kg}{m^3}}{5427 \frac{kg}{m^3}}} \cdot R_{Sun} \\ &= 1.545 R_{Sun} \\ &= 1.545 \cdot 695{,}500 km \\ &= 1{,}074{,}743 \text{ km} \end{aligned}$$

At 46,000,000 km, Mercury is quite safe.

27. The net force acting on each planet has to be found. Each planet is subjected to the gravitational pull exerted by the other two planets. The direction of these forces are



Since the sum of the angles of a triangle must be 180 °, the angle θ must be 30 °.

The distance r is found from the radius of the path (R) with the cosine law

$$\begin{aligned} r^2 &= R^2 + R^2 - 2R^2 \cos(120°) \\ &= R^2 \left(2 - 2\cos(120°)\right) \\ &= 3R^2 \end{aligned}$$

Therefore, the force of gravity made by each planet is

$$F = G\frac{MM}{r^2}$$
$$= G\frac{M^2}{3R^2}$$

The component of this force directed towards the centre of the circle is

$$F_R = F\cos 30°$$
$$= F \cdot \frac{\sqrt{3}}{2}$$
$$= G\frac{M^2}{3R^2}\frac{\sqrt{3}}{2}$$

If the two force components of the force made by the two planets are added, the result is

$$\sum F_R = G\frac{M^2}{3R^2}\frac{\sqrt{3}}{2} + G\frac{M^2}{3R^2}\frac{\sqrt{3}}{2}$$
$$= G\frac{M^2}{3R^2}\sqrt{3}$$
$$= G\frac{M^2}{\sqrt{3}R^2}$$

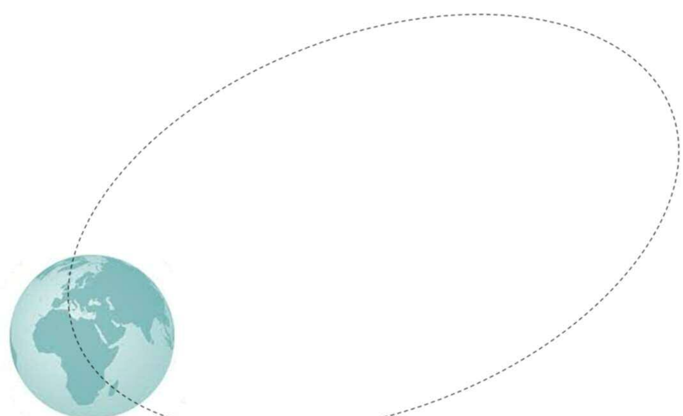
This force must be equal to the centripetal force.

$$M\frac{4\pi^2 R}{T^2} = G\frac{M^2}{3R^2}\sqrt{3}$$

Therefore, the period is

$$M\frac{4\pi^2 R}{T^2} = G\frac{M^2}{\sqrt{3}R^2}$$
$$\frac{1}{T^2} = \frac{GM}{4\pi^2\sqrt{3}R^3}$$
$$T^2 = \frac{4\pi^2\sqrt{3}R^3}{GM}$$
$$T^2 = \frac{4\pi^2\sqrt{3}\left(10^{10}m\right)^3}{6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot10^{24}kg}$$
$$T^2 = 1.02455\times10^{18}s^2$$
$$T = 1.0122\kappa10^9 s$$
$$T = 32.07 ans$$

28. a) When an object is thus thrown from the surface of the Earth, an orbit like this is obtained.



The conservation of energy gives

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{2a}$$

Since the speed is

$$\begin{aligned} v &= 0,85 v_{lib} \\ &= 0,85\sqrt{\frac{2GM}{R}} \end{aligned}$$

we arrive at

$$\frac{1}{2}m\left(0,85\sqrt{\frac{2GM}{R}}\right)^2-\frac{GMm}{R}=-\frac{GMm}{2a}$$
$$\frac{1}{2}m\left(0,85\right)^2\frac{2GM}{R}-\frac{GMm}{R}=-\frac{GMm}{2a}$$
$$\left(0,85\right)^2\frac{1}{R}-\frac{1}{R}=-\frac{1}{2a}$$
$$\left(\left(0,85\right)^2-1\right)\frac{1}{R}=-\frac{1}{2a}$$
$$a=\frac{R}{2\left(1-\left(0,85\right)^2\right)}$$
$$a=\frac{200R}{111}$$

To find the value of r_a, the value of the eccentricity is needed. To find it, Kepler's second law is used.

$$rv\sin\psi=\sqrt{GMr_p\left(1+e\right)}$$

At the position shown in the figure, the angle between the radius and the speed is 135 °. Therefore,

$$Rv\sin 135°=\sqrt{GMr_p\left(1+e\right)}$$
$$Rv\frac{1}{\sqrt{2}}=\sqrt{GMr_p\left(1+e\right)}$$

Since the speed is

$$v=0.85v_{esc}$$
$$=0.85\sqrt{\frac{2GM}{R}}$$

we arrive at

$$R\cdot 0.85\sqrt{\frac{2GM}{R}}\frac{1}{\sqrt{2}}=\sqrt{GMr_p\left(1+e\right)}$$
$$0.85\sqrt{2GMR}\frac{1}{\sqrt{2}}=\sqrt{GMr_p\left(1+e\right)}$$
$$0.85\sqrt{R}=\sqrt{r_p\left(1+e\right)}$$

Since r_p is a (1 – e), this equation becomes

$$0.85\sqrt{R}=\sqrt{a\left(1-e\right)\left(1+e\right)}$$
$$0.85\sqrt{R}=\sqrt{a\left(1-e^2\right)}$$
$$\left(0.85\right)^2 R=a\left(1-e^2\right)$$

Using the value of a previously found

$$a=\frac{200R}{111}$$

the equation becomes

$$\left(0.85\right)^2 R=\frac{200}{111}R\left(1-e^2\right)$$
$$\left(0.85\right)^2\frac{111}{200}=1-e^2$$
$$0.4009875=1-e^2$$
$$e=0.77396$$

Therefore, the maximum distance is

$$\begin{aligned} r_a &= a\left(1+e\right) \\ &= \frac{200R}{111}(1+0.77396) \\ &= 3.1963R \end{aligned}$$

The distance between the object and the surface is then

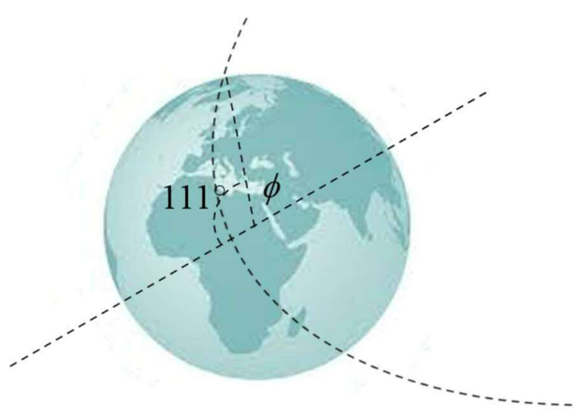
$$\begin{aligned} d &= 3.1963R - R \\ &= 2.1963R \\ &= 2.1963 \cdot 6380km \\ &= 14,013km \end{aligned}$$

b) To know the range, the value of θ on the orbit of the object at launch must be known.

Because initially, the object is at a distance R, the value of θ is found with this formula.

$$r = \frac{a\left(1-e^2\right)}{1+e\cos\theta}$$
$$R = \frac{200}{111}R\frac{1-0.77396^2}{1+0.77396\cos\theta}$$
$$\frac{111}{200} = \frac{1-0.77396^2}{1+0.77396\cos\theta}$$
$$\cos\theta = -0.3585$$
$$\theta = 111.01°$$

We thus have the following situation.



Thus, the angle between the starting point and the maximum distance (ϕ in the figure) is

$$\begin{aligned} \phi &= 180° - 111.01° \\ &= 68.99° \end{aligned}$$

As we have the same angle between the maximum distance and the arrival point, the angle between the point of departure and the point of arrival measured from the centre of the Earth is 137.98 °.

Therefore, the distance between the starting point and the arrival point is

$$\begin{aligned} x &= \frac{137.98°}{360°} \cdot 2\pi \cdot 6380km \\ &= 15,364km \end{aligned}$$