10 MOMENTUM

At liftoff, a rocket initially at rest starts to eject gas with a speed of v_{exp} = 2000 m/s. The gas is ejected at a rate of 1000 kg/s. The initial mass of the rocket is 100 tons, including 60 tons of gas to be ejected. What is the speed of the rocket 30 seconds after liftoff?

rootfun.net/snapsshot/apollo-15-mission-gallery-images/attachment/missile-of-apollo-15-clearing-the-tower/

Discover the answer to this question in this chapter.

10.1 IMPULSE

Impulse Given by a Constant Force

First, the definition of impulse will be given. The usefulness of this quantity will be shown later.

If a constant force is exerted on an object during time Δt, then the impulse given to the object is

Impulse Given to an Object

$$\vec{I} = \vec{F}\Delta t$$

In components:

$$I_x = F_x\Delta t \qquad I_y = F_y\Delta t \qquad I_z = F_z\Delta t$$

The unit for impulse is Ns or kg m/s. No other name was given to this group of units.

If several forces are acting on an object, the sum of impulses given by each of the forces is the net impulse.

Net Impulse

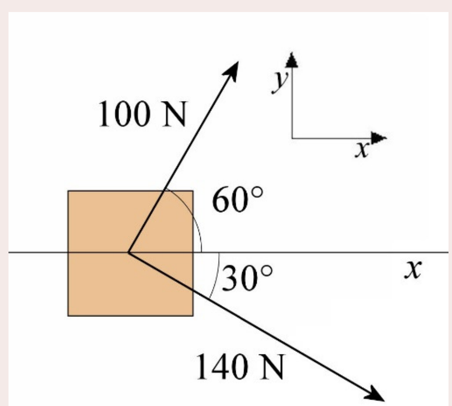
$$\vec{I}_{nette} = \sum \vec{I}$$

In components:

$$I_{x\,net} = \sum I_x \qquad I_{y\,net} = \sum I_y \qquad I_{z\,net} = \sum I_z$$

Example 10.1.1

What are the x and y-components of the net impulse given to this box in 3 seconds?



The components of the impulse given by each force will be calculated.

The components of the impulse given by the 100 N force are

$$I_{1x} = F_{1x}\Delta t = (100N \cdot \cos 60°) \cdot 3s = 150 \tfrac{kgm}{s}$$
$$I_{1y} = F_{1y}\Delta t = (100N \cdot \sin 60°) \cdot 3s = 259.8 \tfrac{kgm}{s}$$

The components of the impulse given by the 140 N force are

$$I_{2x} = F_{2x}\Delta t = (140N \cdot \cos(-30°)) \cdot 3s = 363.7 \tfrac{kgm}{s}$$
$$I_{2y} = F_{2y}\Delta t = (140N \cdot \sin(-30°)) \cdot 3s = -210 \tfrac{kgm}{s}$$

Therefore, the net impulse is

$$I_{x\,net} = 150 \tfrac{kgm}{s} + 363.7 \tfrac{kgm}{s} = 513.7 \tfrac{kgm}{s}$$
$$I_{y\,net} = 259.8 \tfrac{kgm}{s} + -210 \tfrac{kgm}{s} = 49.8 \tfrac{kgm}{s}$$

Impulse Given by a Variable Force

If the magnitude of the force changes, the calculation of the impulse must be divided into parts where the force is constant. The impulses given in each part are then added together.

Impulse Given by a Variable Force Acting on an Object

$$\vec{I} = \sum_{F \text{ constant}} \vec{F}\Delta t$$

In components:

$$I_x = \sum_{F \text{ constant}} F_x\Delta t \qquad I_y = \sum_{\text{F constant}} F_y\Delta t \qquad I_z = \sum_{F \text{ constant}} F_z\Delta t$$

Example 10.1.2

A force acts on an object. The force is 5 N towards the right for 5 seconds and then 3 N towards the left for 1 second. What is the impulse given to the object?

As the force changes, the calculation must be divided into part. The impulse given during the first part is

$$\begin{aligned} I_{1x} &= F_{1x}\Delta t \\ &= 5N \cdot 5s \\ &= 25 \tfrac{kgm}{s} \end{aligned}$$

The impulse given during the second part is

$$\begin{aligned} I_{2x} &= F_{2x}\Delta t \\ &= -3N \cdot 1s \\ &= -3\tfrac{kgm}{s} \end{aligned}$$

Therefore, the total impulse given to the object is 25 kgm/s + –3 kgm/s = 22 kgm/s.

If the force is constantly changing, can the calculation still be divided into parts where the force is constant? Actually, it can. The time interval must be divided into very short time intervals, so short that they become infinitesimally small. Then, the components of the impulse given during these short times also become infinitesimally small and are

$$dI_x = F_x dt \qquad dI_y = F_y dt \qquad dI_z = F_z dt$$

If all these infinitesimally small impulses are then summed (with an integral), the total impulse given is obtained.

Impulse Given by a Variable Force on an Object (Most General Formula)

$$\vec{I} = \int_t^{t'} \vec{F} dt$$

In components:

$$I_x = \int_t^{t'} F_x dt \qquad I_y = \int_t^{t'} F_y dt \qquad I_z = \int_t^{t'} F_z dt$$

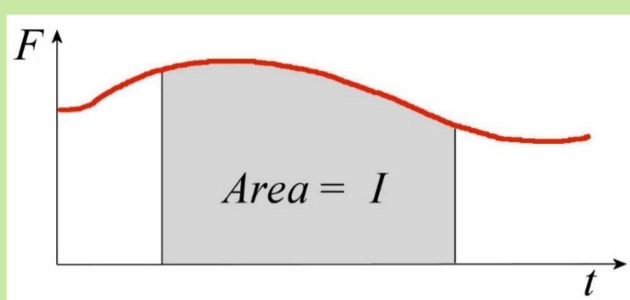
Graphical Representation of Impulse

Since the impulse is the integral

$$I = \int_t^{t'} F dt$$

and this integral is the area under the curve of F on a force-versus-time graph. We have

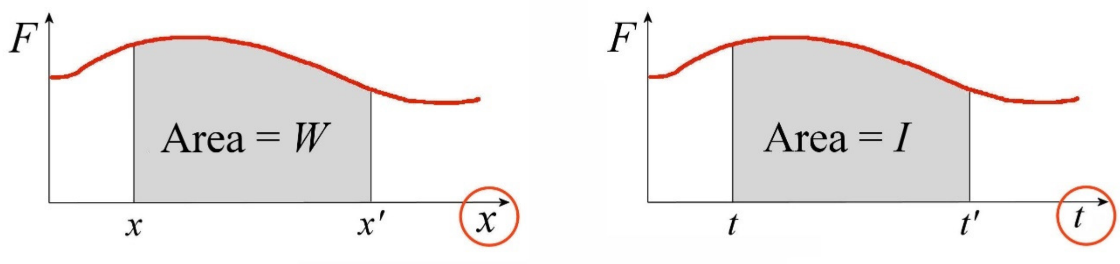
The impulse given to an object is the area under the curve of the force acting on the object as a function of time



Remember that the area is negative if it is below the time axis.

The work was also equal to the area under the curve of the force, but there is a crucial distinction between work and impulse.

- The work is the area under the curve of F on a force-versus-**position** graph.
- The impulse is the area under the curve of F on a force-versus-**time** graph.



10.2 IMPULSE-MOMENTUM THEOREM

Proof of the Theorem

Let's find out now why it can be useful to calculate the impulse given to an object. This all starts with the definition of impulse.

$$I_{x\,net} = \int_t^{t'} F_{x\,net}\,dt \qquad I_{y\,net} = \int_t^{t'} F_{y\,net}\,dt \qquad I_{z\,net} = \int_t^{t'} F_{z\,net}\,dt$$

We will work only on the x-component. The results are similar for the other components.

Since $\vec{F}_{net} = m\vec{a}$ and since the acceleration is the derivative of velocity, the impulse is

$$\begin{aligned} I_{x\,net} &= \int_t^{t'} F_{x\,net}\,dt \\ &= \int_t^{t'} ma_x\,dt \\ &= \int_t^{t'} m\frac{dv_x}{dt}\,dt \\ &= \int_{v_x}^{v_x'} m\,dv_x \end{aligned}$$

$$= \left[mv_x \right]_{v_x}^{v'_x}$$
$$= mv'_x - mv_x$$

The name *momentum* was given to this new quantity. It is denoted *p*.

$$p_x = mv_x$$

(No one seems to know why the letter *p* is used. Perhaps this is because Leibniz gave the name *progress* to this quantity in 1689. The *p* may also come from the fact that several Latin terms associated with collisions start with *p* and those has given us words like *percussion*.)

Using the same definition for the other components, we have

Momentum

$$\vec{p} = m\vec{v}$$

In components:

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z$$

The unit of momentum is also the kg m/s.

(The magnitude of this vector *mv* is used in physics since a long time ago. The impetus of medieval theories was often defined as weight multiplied by speed. Obviously, its role changed a lot with Newtonian physics.)

The momentum components are calculated with the velocity components. In two dimensions, this means that

$$p_x = mv_x = mv\cos\theta$$
$$p_y = mv_y = mv\sin\theta$$

With this definition of momentum, the momentum equation becomes

$$I_{x\,net} = mv'_x - mv_x$$
$$= p'_x - p_x$$

With the other components giving similar results, the following theorem is thus obtained.

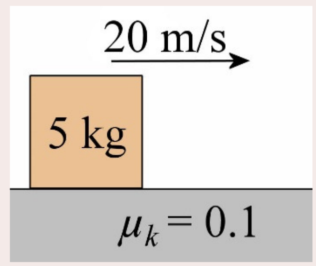
Impulse-Momentum Theorem

$$\vec{I}_{net} = \Delta\vec{p}$$

In components:

$$I_{x\,net} = \Delta p_x \qquad I_{y\,net} = \Delta p_y \qquad I_{z\,net} = \Delta p_z$$

Example 10.2.1



A 5 kg box slides on a horizontal surface with an initial velocity of 20 m/s. The coefficient of friction between the surface and the box is 0.1. What is the speed of the box 5 seconds later?

I_{net} Calculation

In order to find the impulse given to the box, the forces acting on the box must be found. The forces are:

1) The weight (49 N) directed downwards.
2) The normal force (49 N) directed upwards.
3) The friction force (0.1 x 49 N = 4.9 N) directed towards the left.

To calculate the *x*-component of the velocity, those vertical forces do not matter.

The only force with an *x*-component is the friction force. The *x*-component of the net impulse is, therefore,

$$\begin{aligned} I_{fx} &= F_{fx}\Delta t \\ &= (-4.9N)\cdot 5s \\ &= -24.5\tfrac{kgm}{s} \end{aligned}$$

Δp Calculation

The *x*-component of the momentum change is

$$\begin{aligned} \Delta p_x &= mv_x' - mv_x \\ &= 5kg\cdot v_x' - 5kg\cdot 20\tfrac{m}{s} \\ &= 5kg\cdot v_x' - 100\tfrac{kgm}{s} \end{aligned}$$

Impulse-Momentum Theorem Application

$$I_{x\,net} = \Delta p_x$$

$$-24.5\tfrac{kgm}{s} = 5kg\cdot v' - 100\tfrac{kgm}{s}$$
$$v' = 15.1\tfrac{m}{s}$$

Example 10.2.2

A 10 kg box with an initial velocity of 1 m/s towards the left is subjected to a variable force of $F_x = 6\frac{N}{s}\cdot t + 2N$. What will be the velocity of the box at t = 10 s?

I_{net} Calculation

In order to find the impulse given to the box, the forces acting on the box must be found. The forces are:

1) The weight (98 N) directed downwards.
2) The normal force (98 N) directed upwards.
3) The force $F_x = 6\frac{N}{s}\cdot t + 2N$

To calculate the x-component of the velocity, those vertical forces do not matter.

The only force with an x-component is the variable force. The x-component of the net impulse given between t = 0 s and t = 10 s is

$$\begin{aligned}
I_x &= \int_{0s}^{10s}\left(6\tfrac{N}{s}\cdot t + 2N\right)dt \\
&= \left(3\tfrac{N}{s}\cdot t^2 + 2N\cdot t\right)\Big|_{0s}^{10s} \\
&= \left(3\tfrac{N}{s}\cdot(10s)^2 + 2N\cdot 10s\right) - \left(3\tfrac{N}{s}\cdot(0s)^2 + 2N\cdot 0s\right) \\
&= 320\tfrac{kgm}{s}
\end{aligned}$$

Δp Calculation

The x-component of the momentum change is

$$\begin{aligned}
\Delta p_x &= mv'_x - mv_x \\
&= 10kg\cdot v'_x - 10kg\cdot\left(-1\tfrac{m}{s}\right) \\
&= 10kg\cdot v'_x + 10\tfrac{kgm}{s}
\end{aligned}$$

Impulse-Momentum Theorem Application

$$I_{x\,net} = \Delta p_x$$
$$320\tfrac{kgm}{s} = 10kg \cdot v' + 10\tfrac{kgm}{s}$$
$$v' = 31\tfrac{m}{s}$$

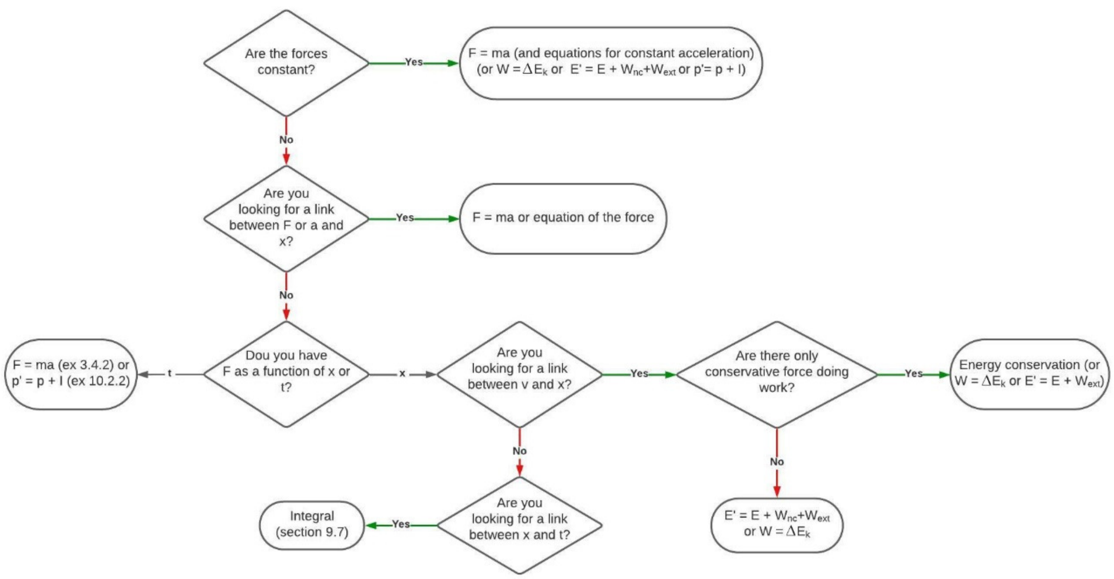
This is the same speed as found in example 3.4.2 (it was the same question).

This method is quite correct, but it is still not widely used. Usually, we would have solved this problem by finding the acceleration with Newton's second law and then we would have found the velocity from the acceleration (with an integral).

Which Method Should Be Used to Solve a Problem?

So far, we have seen 4 methods to solve problems (Newton's 2nd law, $W_{net}=\Delta E_k$, energy conservation and $I_{net}=\Delta p$) and maybe you're wondering how you can know which one to take.

Here is a small diagram that can help you choose the easiest method.



If there is a variable force acting and this force is given as a function of **position**, the problem can be more easily solved with $W_{net}=\Delta E_k$ or energy conservation.

If there is a variable force acting and this force is given as a function of **time**, the problem can be more easily solved with $F = ma$ or $I_{net}=\Delta p$.

If the force is constant, all methods can be used, but it will often be easier to arrive at a solution using $W_{net}=\Delta E_k$ or energy conservation if you they ask you to calculate what will happen after a certain **distance** and it will often be easier to arrive at a solution using $F = ma$ or $I_{net}=\Delta p$ if you they ask you to calculate what will happen after a certain **time.**

10.3 A NEW VERSION OF NEWTON'S SECOND LAW

A New Version with Momentum

In a previous chapter, power was defined as being the work done divided by the time required to do this work. Let's see what happens if the impulse is divided by the time required to give this impulse to an object. For a constant force, this is

$$\frac{\vec{I}}{\Delta t}=\frac{\vec{F}\Delta t}{\Delta t}=\vec{F}$$

The resulting quantity is not new; it is simply the force acting on the object.

Since $\vec{I}_{net}=\Delta\vec{p}$, this also means that

$$\vec{F}_{net}=\frac{\vec{I}_{net}}{\Delta t}$$

is

Relationship between Force and Momentum (Constant Force)

$$\vec{F}_{net}=\frac{\Delta\vec{p}}{\Delta t}$$

If the force is not constant, the impulse can be calculated for an infinitesimally small time *dt*. During such a short time, the variation of momentum is also infinitesimally small and is *dp.* The end result is another version of Newton's second law.

Newton's Second Law

$$\vec{F}_{net}=\frac{d\vec{p}}{dt}$$

Leonard Euler proposed this other version of Newton's second law in 1752. This new version is the same as $F_{net}= ma$ since

$$\vec{F}_{net}=\frac{d\vec{p}}{dt}$$

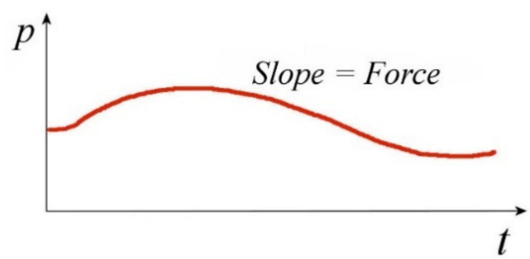
$$\vec{F}_{net}=\frac{d(m\vec{v})}{dt}$$

$$\vec{F}_{net} = m\frac{d\vec{v}}{dt}$$
$$\vec{F}_{net} = m\vec{a}$$

(Since the mass was taken out of the derivative, it is possible to think that $F = dp/dt$ is more general than $F = ma$ since we arrive at $F = ma$ only when the mass is constant. However, this is not the case since the mass is always constant for a closed system. Both versions are equally general. This will be discussed again in Chapter 11.)

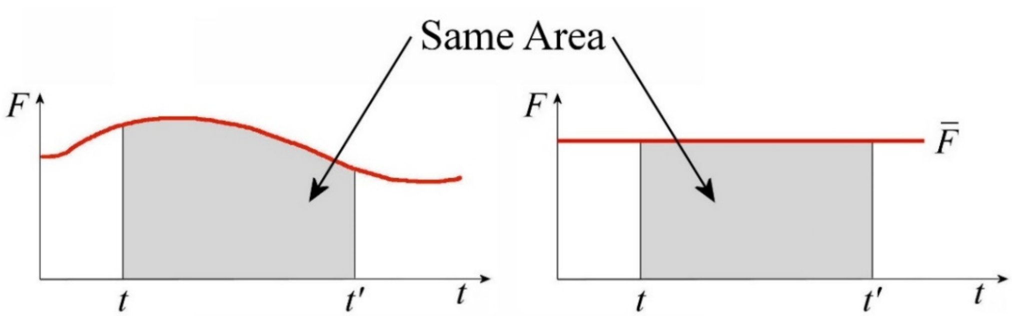
Graphical Interpretation

If force is the derivative of momentum, then the force exerted on the object is the slope on a graph of momentum-versus-time.



Average Force

The average force is defined as being a constant force that gives the same impulse to an object during the same time the force has acted. This means that the area under the curve must be the same for these two graphs.



(By the way, this is how the average value of any function on an interval is defined in mathematics.) As the area is equal to the impulse, this means that

$$\vec{I} = \bar{\vec{F}}\Delta t$$
$$\Delta\vec{p} = \bar{\vec{F}}\Delta t$$

The same results are obtained for the other components. Therefore,

Average Force Acting on an Object

$$\vec{\overline{F}} = \frac{\Delta \vec{p}}{\Delta t}$$

In components:

$$\overline{F}_x = \frac{\Delta p_x}{\Delta t} \qquad \overline{F}_y = \frac{\Delta p_y}{\Delta t} \qquad \overline{F}_z = \frac{\Delta p_z}{\Delta t}$$

Example 10.3.1

A 150 g baseball moving towards the left at 45 m/s is hit by a bat. After the collision, the ball goes at 60 m/s towards the right. What is the average force exerted on the ball if the impact between the ball and the bat lasts 0.01 s?

The average force is calculated with the following formula.

$$\overline{F}_x = \frac{\Delta p_x}{\Delta t}$$

With a positive x-axis pointing towards the right, the impulse given to the ball is

$$\begin{aligned} \Delta p_x &= p_x' - p_x \\ &= mv_x' - mv_x \\ &= 0.15kg \cdot 60\tfrac{m}{s} - 0.15kg \cdot \left(-45\tfrac{m}{s}\right) \\ &= 15.75\tfrac{kgm}{s} \end{aligned}$$

Therefore, the average force is

$$\begin{aligned} \overline{F}_x &= \frac{\Delta p_x}{\Delta t} \\ &= \frac{15.75\frac{kgm}{s}}{0.01s} \\ &= 1575N \end{aligned}$$

The average force is then 1575 N towards the right (since the result is positive).

Common Mistake: Wrong Sign for *p* or *v*

Momentum is a vector. Therefore, its direction is important. Be sure to define clearly positive directions with axes. If the component of the momentum (or the velocity) is in the direction of your axis, it is positive, and if it is in the opposite direction to your axis, it is negative.

This is why many would have used 45 m/s rather than -45 m/s as the initial velocity of the ball in the previous example. That would have been a mistake.

Example 10.3.2

A car travelling at 72 km/h hits a wall. During the accident, the passenger stops over a distance of 1 m (because the front of the car crumples 1 m). What is the average force exerted on a 60 kg passenger during the collision?

The average force is calculated with the following formula.

$$\overline{F}_x = \frac{\Delta p_x}{\Delta t}$$

First, the change in momentum is calculated.

$$\begin{aligned}\Delta p_x &= p'_x - p_x \\ &= mv'_x - mv_x \\ &= 0 - 60kg \cdot 20\tfrac{m}{s} \\ &= -1200\tfrac{kgm}{s}\end{aligned}$$

Then, the duration of the collision is needed. This time is not given, but it can be calculated using kinematics formulas.

$$x = x_0 + \frac{1}{2}\left(v_0 + v\right)t$$

$$1m = 0m + \frac{1}{2}\left(20\tfrac{m}{s} + 0\right)t$$

$$t = 0.1s$$

The average force can then be calculated.

$$\begin{aligned}\overline{F}_x &= \frac{\Delta p_x}{\Delta t} \\ &= \frac{-1200\tfrac{kgm}{s}}{0.1s} \\ &= -12{,}000N\end{aligned}$$

The force is negative because an axis in the direction of the velocity was used. A negative force means that the force is in the opposite direction to the velocity, so towards the rear of the car.

Note that this force corresponds to nearly 20 times the weight of the person. This person thus experiences around 20 g's during the collision. It is possible to survive this because a human being can survive a crash if he experiences less than about 100 g's.

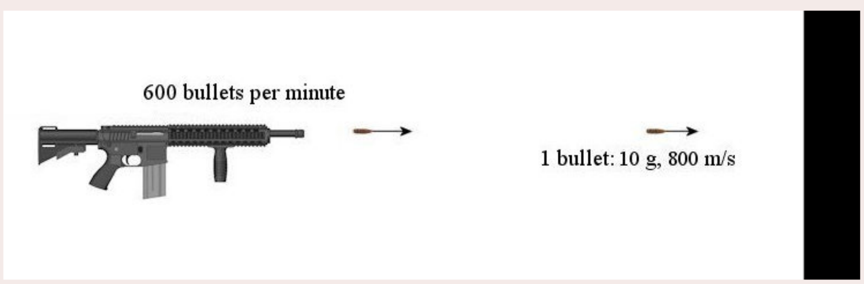
If the passenger is not wearing a seatbelt, the result changes a lot, not because the change of momentum increases but because Δt will be much smaller. Without a seatbelt, the car starts to slow down whereas the person continues to move according to Newton's first law. He stops only when he comes into contact with the steering wheel or the windshield. The stopping distance is then about one centimetre, and the stopping time is about 0.001s. This means that the average force is then about 1.2 million N, or about 2000 times the weight of the person. No one can survive such large force acting on his body.

The airbag does the same thing as the seatbelt; it prevents you from continuing your motion and crash on the dashboard. It forces you to slow down at the same rate as the car in order to reduce the average force.

Also, it would be ridiculous to have a car so stiff that it does not crumple on impact. If a car were so rigid so that it would crumple only a centimetre when it hits a wall, it would amount to the situation where the stopping time was only 0.001 s, and the force would be 1,200,000 N, even if the passengers are wearing their seatbelt.

Example 10.3.3

Bullets are shot at a wall with a machine gun. 600 bullets are shot per minute. Each ball has a mass of 10 g and a speed of 800 m/s. What is the average force that the bullets exert on the wall?



www.deviantart.com/morelikethis/135198329

The average force on the wall would be calculated with

$$\overline{F} = \frac{\Delta p_x}{\Delta t}$$

However, this seems difficult since the wall will remain in place (since it is attached to the Earth) and its momentum will not change. This problem can be circumvented by calculating the average force made on the bullets to stop them. (This force is made by the wall.) Then the force on the wall can be found since the force made by the wall on the bullets is the same magnitude as the force made by the bullets on the wall according to Newton's 3rd law.

Let's calculate the average force on the 600 bullets fired during one minute (this choice of duration is quite arbitrary, we could have taken 10 balls in 10 seconds). With positive axis pointing to the right, we have

$$\begin{aligned}\overline{F}_{x\ on\ bullets} &= \frac{\Delta p_x}{\Delta t} \\ &= \frac{p_x' - p_x}{\Delta t} \\ &= \frac{0 - 600 \cdot 0.01kg \cdot 800\frac{m}{s}}{60s} \\ &= -80N\end{aligned}$$

The final momentum is zero since the balls are now at rest in the wall. A force of 80 N directed towards the left is thus obtained. Remember that this is an average force. The force is zero when no bullets are hitting the wall and rises suddenly when a bullet hits. The average of this changing force is 80 N.

According to Newton's third law, the bullets exert an average force of 80 N towards the right on the wall if the wall exerts an average force of 80 N towards the left on bullets. The answer is, therefore: 80 N towards the right.

10.4 MOMENTUM CONSERVATION

Proof of the Conservation Principle

The impulse-momentum theorem is

$$\Delta\vec{p} = \vec{I}_{net}$$

Since the net impulse is

$$\vec{I}_{net} = \vec{F}_{net}\Delta t$$

the theorem becomes

$$\Delta\vec{p} = \vec{F}_{net}\Delta t$$

Thus, if the net force is zero, the following results are obtained.

$$\Delta\vec{p} = 0 \qquad \rightarrow \qquad \vec{p} = \text{constant}$$

This result is not really surprising since it was already known by Newton's first law that the velocity is constant if the sum of the forces acting on an object is zero. If the velocity is constant, then the momentum is constant.

However, an interesting result is obtained if the same steps are made by summing the impulse exerted on all the objects of a system. This sum is

$$\sum_{system} \vec{I}_{net} = \sum_{system} \Delta\vec{p}$$
$$\sum_{system} \vec{I}_{net} = \sum_{system} \left(\vec{p}' - \vec{p}\right)$$
$$\sum_{system} \vec{F}_{net}\Delta t = \sum_{system} \vec{p}' - \sum_{system} \vec{p}$$

To the right, these are the sums of the momentum of all the objects in the system. This is the total momentum of the system that will be denoted p_{tot}. The equation is then

$$\sum_{system} \vec{F}_{net}\Delta t = \vec{p}'_{tot} - \vec{p}_{tot}$$

As Δt is the same for all the forces, it can be taken outside the sum.

$$\left(\sum_{system} \vec{F}_{net}\right)\Delta t = \vec{p}'_{tot} - \vec{p}_{tot}$$

The sum of the net forces acting on the system is simply the sum of all the force acting on the system. Therefore, the equation can be written as

$$\left(\sum_{system} \vec{F}\right)\Delta t = \vec{p}'_{tot} - \vec{p}_{tot}$$

The forces acting on an object of the system can be internal (made by another object in the system) or external (made by an object from outside the system).

$$\left(\sum_{system} \vec{F}_{ext}\right)\Delta t + \left(\sum_{system} \vec{F}_{int}\right)\Delta t = \vec{p}'_{tot} - \vec{p}_{tot}$$

However, if object A exerts a force on object B, then object B exerts a force of the same magnitude and opposite direction on object A according to Newton's third law. This means that when the sum of the internal forces acting on all objects in the system is done, these two forces always cancel each other because the two objects are part of the system. It also means that the sum of external forces is not necessarily zero since only one of the objects is part of the system and that the two forces associated by Newton's third law cannot cancel each other. The equation thus becomes

$$\left(\sum_{system} \vec{F}_{ext}\right)\Delta t = \vec{p}'_{tot} - \vec{p}_{tot}$$

If the sum of the external forces is zero, then

$$0 = \vec{p}'_{tot} - \vec{p}_{tot}$$

This means that the total momentum of the system remains constant then. Therefore,

Momentum Conservation Law

$$\vec{p}_{tot} = \vec{p}'_{tot} \qquad \text{if} \qquad \sum \vec{F}_{ext} = 0$$

In components:

$$p_{x\,tot} = p'_{x\,tot} \qquad \text{if} \qquad \sum_{system} F_{x\,ext} = 0$$

$$p_{y\,tot} = p'_{y\,tot} \qquad \text{if} \qquad \sum_{system} F_{y\,ext} = 0$$

$$p_{z\,tot} = p'_{z\,tot} \qquad \text{if} \qquad \sum_{system} F_{z\,ext} = 0$$

Remember, however, that this law of conservation is not always true: **It is only true if the sum of all external forces is zero**. There may be external forces, but the sum of these forces must be zero for the law of momentum conservation to hold.

Note that these three equations for the components are independent of each other. This means that it is possible for the *x*-component of momentum to be conserved while the *y*-component is not conserved.

Application of the Law

This law of conservation can now be used for problem-solving. Certain types of problems can be simply solved with this law, including collision problems.

Resolution Method

1) The components of the total momentum of the system are calculated at some instant. These components are noted p_x and p_y. In one dimension, there is only p_x. In two dimensions, both p_x and p_y are used. There might be a third dimension, but we will not go so far in these notes.

2) The components of the total momentum of the system are calculated at another instant. These components are noted p'_x and p'_y.

3) The conservation law is then applied

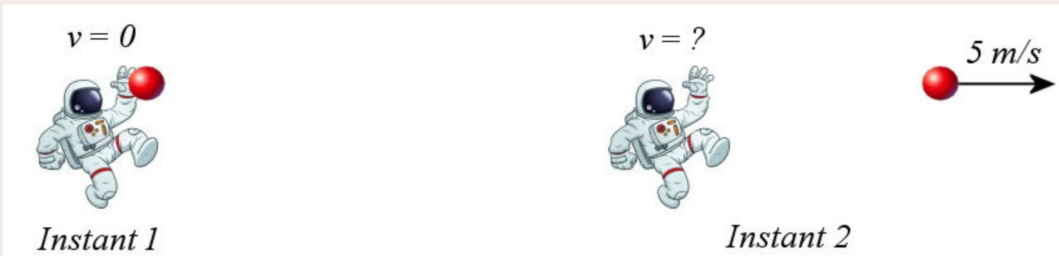
$$p_x = p'_x$$
$$p_y = p'_y$$

4) The equations are solved.

Remember that in order to apply correctly the law of conservation of momentum in this form, the net external force must be zero. There may be external forces, but their sum must vanish.

Example 10.4.1

An 80 kg astronaut in space holds of 4 kg bowling ball in his hands. He is initially motionless in space. The astronaut then throws the bowling ball so that the ball now moves with a velocity of 5 m/s. What is the velocity of the astronaut after he had thrown the ball?



www.colourbox.com/vector/cartoon-astronaut-floating-vector-7024296?u

Momentum at Instant 1

Originally (instant 1), the *x*-component in the total momentum of the system (person and ball) is

$$p_x = 0 + 0 = 0$$

since the astronaut and the ball are both at rest.

Momentum at Instant 2

At instant 2, the *x*-component in the total momentum of the system is

$$\begin{aligned} p_x' &= m_A v_A' + m_B v_B' \\ &= 80kg \cdot v_A' + 4kg \cdot 5\tfrac{m}{s} \\ &= 80kg \cdot v_A' + 20\tfrac{kgm}{s} \end{aligned}$$

Momentum Conservation

The momentum conservation law can now be applied as there is no external force. The only forces here are the forces between the ball and the astronauts. Since both these objects are part of the system, these forces are internal forces. The conservation law then gives

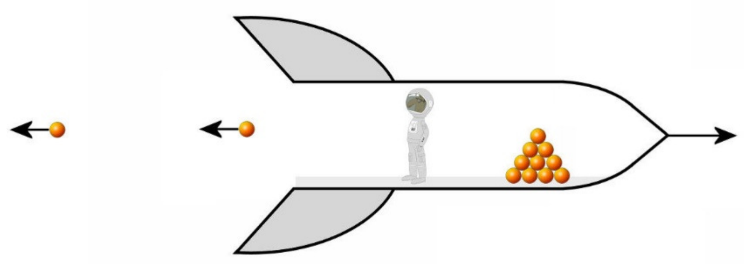
$$\begin{aligned} p_x &= p_x' \\ 0 &= 80kg \cdot v_A' + 20\tfrac{kgm}{s} \end{aligned}$$

$$v_A' = -0.25 \tfrac{m}{s}$$

Thus, the astronaut moves at 0.25 m/s towards the left.

It is not so surprising to have an astronaut moving towards the left according to Newton's third law. When the astronaut pushes the ball towards the right to give it speed, the ball pushes on the astronaut towards the left with a force of the same magnitude. This force then gives a speed the astronaut towards the left.

This is basically how a rocket is propelled. The astronaut's speed increases when he throws a ball. Every time he throws a ball (always in the same direction), its speed increases. Here's a bad spaceship model, propelled by bowling balls. The astronaut in the rocket simply throws bowling balls to give speed to the spacecraft!



Whenever the astronaut launches a bowling ball, the speed of the ship increases a little. The more bowling balls he throws and the faster he throws them, the faster the spaceship will move at the end.

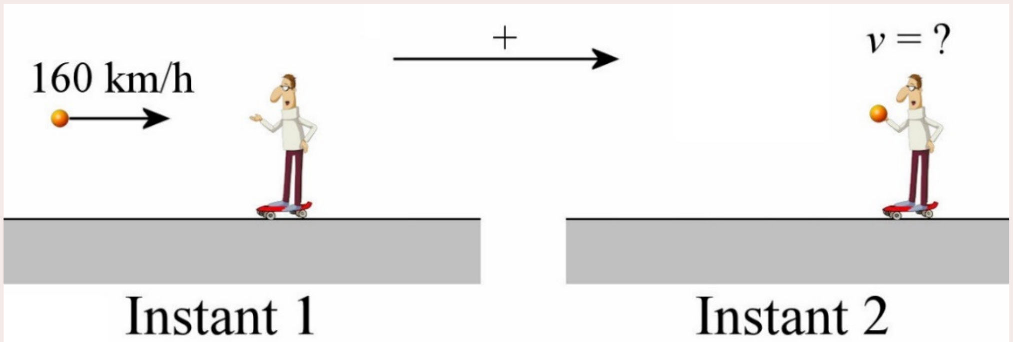
This is not exactly how it is done but it is not that far from the truth. Instead of throwing bowling balls, gas molecules are ejected. Due to its low mass, one thrown molecule does not increase the speed of the ship much. However, if enough of them are ejected, the spaceship could end up with a great speed. This is how Dr. Smith propelled himself in this video.
https://www.youtube.com/watch?v=Z3xyqfCZmSU

This is also what happens when you let go an inflated balloon. When the air is ejected in one direction, the balloon is propelled in the opposite direction. This idea can be used to propel a small vehicle.
http://www.youtube.com/watch?v=OewYUTDcQ2E

A larger speed can be achieved if the gas molecules are ejected with more speed, which can be done by heating the gas. An exothermic chemical reaction between substances can form a very hot gas. Ejected in one direction, this gas propels the vehicle in the opposite direction. This is what can be seen in this video
http://www.youtube.com/watch?v=TYEtQGLzvkI
or this one
https://www.youtube.com/watch?v=uuYoYl5kyVE
We will return later to this subject and calculate the speed of a rocket ejecting gas.

Example 10.4.2

A 60 kg person at rest on a skateboard catches a baseball ($m = 0.135$ kg) going at 160 km/h. What will the speed of the person (with the ball in his hands) be after the catch?



Momentum at Instant 1

At instant 1, the momentum of the system (person and ball) is (using the axis shown in the diagram)

$$\begin{aligned} p_x &= m_{ball}v_{ball} + m_{person}v_{person} \\ &= 0.135kg \cdot 44.44 \tfrac{m}{s} + 0 \\ &= 6 \tfrac{kgm}{s} \end{aligned}$$

Momentum at Instant 2

At instant 2, the momentum of the system is

$$\begin{aligned} p'_x &= m_{ball}v'_{ball} + m_{person}v'_{person} \\ &= 0.135kg \cdot v' + 60kg \cdot v' \\ &= (60.135kg) \cdot v' \end{aligned}$$

Momentum Conservation

The velocity can be found with the law of momentum conservation.

$$\begin{aligned} p_x &= p'_x \\ 6 \tfrac{kgm}{s} &= (60.135kg) \cdot v' \\ v' &= 0.09976 \tfrac{m}{s} \end{aligned}$$

In this example, it is important to note that there are external forces: the weight and normal force. As the system is formed of the ball and the person, the forces made by the Earth and the ground are external forces. However, the conservation law can still be applied, since these two forces cancel each other, and the sum of the external forces is zero.

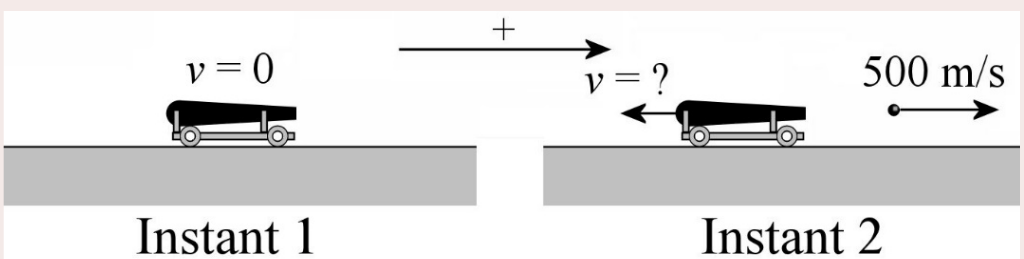
It can also be seen that by receiving an object, a person gains speed. This shows that a person can change his speed not only by throwing something but also by catching something. By combining these two methods, the person in this video can move.
http://www.youtube.com/watch?v=KeNye0nTqmM
She is on a kind of air cushion in order to eliminate the friction force which is an external force.

Gun Recoil

If there is an explosion, the force made by the gas released by the explosion is an internal force between objects in the system if the gas is included in the system. Take the example of a cannon shooting a shell. Initially, the gun, the shell, and the explosive charge are at rest, and the total momentum is zero. When the load explodes, and the shell is propelled, the total momentum of this system must remain zero. To simplify, the masses of the explosive charge and of the released gas are neglected so that only the momentum of the gun and the shell are considered. If the axis is in the direction of the motion of the shell, then the momentum of the shell is positive. The momentum of the cannon must then be negative (for the sum to be zero), which means that the gun moves in a direction opposite to the shell velocity. This motion is the recoil of the gun.

Example 10.4.3

A 500 kg cannon, initially at rest, fires a 5 kg shell with a speed of 500 m/s. What is the velocity of the canon after the firing of the shell?



Momentum at Instant 1

At instant 1, the momentum is (using a positive axis pointing towards the right)

$$\begin{aligned} p_x &= m_{cannon} v_{cannon} + m_{shell} v_{shell} \\ &= 0 + 0 \\ &= 0 \end{aligned}$$

Momentum at Instant 2

At instant 2, the total momentum is

$$\begin{aligned} p'_x &= m_{cannon}v'_{cannon} + m_{shell}v'_{shell} \\ &= 500kg \cdot v'_{cannon} + 5kg \cdot 500\tfrac{m}{s} \\ &= 500kg \cdot v'_{cannon} + 2500\tfrac{kgm}{s} \end{aligned}$$

Momentum Conservation

Using the law of momentum conservation, the velocity is

$$\begin{aligned} p_x &= p'_x \\ 0 &= 500kg \cdot v'_{cannon} + 2500\tfrac{kgm}{s} \\ v'_{cannon} &= -5\tfrac{m}{s} \end{aligned}$$

The gun then recoils at 5 m/s.

An example of gun recoil can be seen in this video.
http://www.youtube.com/watch?v=b9qvt_LS5sg
Do not remain behind a gun when it is fired. Otherwise, here's what could happen to you.
https://www.youtube.com/watch?v=EefIsRsu-qU

Guns and rifles also recoil when fired. The larger the momentum of the bullet, the larger the recoil is because both momentums must cancel each other since the initial momentum was zero. With a gun that gives a large momentum to the bullets, the recoil can be difficult to control.
https://www.youtube.com/watch?v=VLAIBclQIFQ

Movie Mistakes

When people get shot in a movie, they are often thrown back over a significant distance after impact. This is the case in this excerpt from the film "Martyrs", a French horror film. No need to watch the whole clip, the first 20 seconds is sufficient.
http://www.youtube.com/watch?v=Jm6O8CNURcU
It is possible for a human being to be thrown back as far as is the father of this family, but the bullet must then have a gigantic momentum. As the 70 kg father (approximately) is being thrown back at 5 m/s (approximately), he receives a momentum roughly equal to 70 kg x 5 m/s = 350 kgm/s. This is really a colossal momentum since it is equivalent to a 100 g projectile going ten times faster than sound! Even so, let's assume this might be possible.

Before the shot is fired, the momentum is zero (shooter, rifle, and bullet at rest). If the projectile is fired with a momentum of 350 kgm/s, the rifle and the shooter must recoil with a momentum of 350 kgm/s in the opposite direction so that the total momentum of the system remains zero.

If the shooter and rifle received 350 kgm/s of momentum, then they would be thrown back with great speed. You surely notice that it is the same momentum as the one received by the victim. Thus, the shooter should be thrown backwards as violently as the one who received the shot. This is clearly not what happened in the clip.

This is what really happens when someone fires a gun giving a considerable momentum to a bullet.
http://www.youtube.com/watch?v=0MOpQeQgEO8
This gun gives a momentum of 37 kgm/s to the bullet. Imagine if it had given 350 kgm/s!

Other Examples of the Application of the Law of Momentum Conservation

Example 10.4.4

A 10 kg dog is on a 30 kg raft. Initially, the raft and dog are motionless. Then the dog starts walking towards the left with a speed of 6 m/s. What is the velocity of the raft?

aaugh.com/wordpress/2010/12/momentous-peanuts/

Momentum at Instant 1

Initially, the total momentum of the system (dog and raft) is (using a positive axis towards the right)

$$\begin{aligned} p_x &= m_{raft}v_{raft} + m_{dog}v_{dog} \\ &= 0+0 \\ &= 0 \end{aligned}$$

Momentum at Instant 2

When the dog is walking towards the left, the total momentum of the system is

$$\begin{aligned} p'_x &= m_{raft}v'_{raft} + m_{dog}v'_{dog} \\ &= 30kg \cdot v'_{raft} + 10kg \cdot \left(-6\tfrac{m}{s}\right) \\ &= 30kg \cdot v'_{raft} - 60\tfrac{kgm}{s} \end{aligned}$$

Momentum Conservation

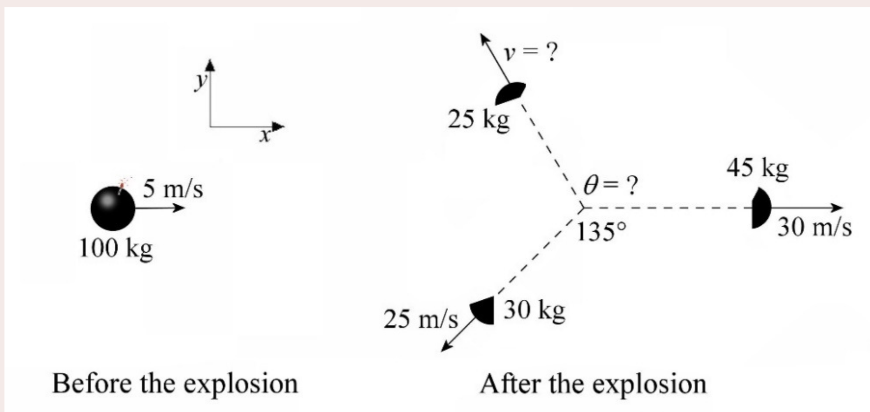
The law of momentum conservation then gives

$$p_x = p'_x$$
$$0 = 30kg \cdot v'_{raft} + 10kg \cdot \left(-6 \tfrac{m}{s}\right)$$
$$v'_{raft} = 2 \tfrac{m}{s}$$

The raft is thus moving at 2 m/s towards the right.

Example 10.4.5

A 100 kg bomb moving at 5 m/s explodes into three fragments. If the speed and the direction of the velocities of the 30 kg and 45 kg fragments are those indicated in the diagram, what is the velocity (magnitude and direction) of the 25 kg fragment?



Let's start with the *x*-component of the momentum.

x-component of the Momentum at Instant 1

Initially, the *x*-component of the total momentum is

$$\begin{aligned} p_x &= mv_x \\ &= 100kg \cdot 5 \tfrac{m}{s} \\ &= 500 \tfrac{kgm}{s} \end{aligned}$$

x-component of the Momentum at Instant 2

After the explosion, the *x*-component of the total momentum is

$$\begin{aligned} p'_x &= m_1 v'_{1x} + m_2 v'_{2x} + m_3 v'_{3x} \\ &= 45kg \cdot 30 \tfrac{m}{s} + 30kg \cdot 25 \tfrac{m}{s} \cdot \cos\left(-135°\right) + 25kg \cdot v'_{3x} \end{aligned}$$

$$= 819.67\tfrac{kgm}{s} + 25kg \cdot v'_{3x}$$

Momentum Conservation

The law of momentum conservation then gives

$$\begin{aligned} p_x &= p'_x \\ 500\tfrac{kgm}{s} &= 819.67\tfrac{kgm}{s} + 25kg \cdot v'_{3x} \\ v'_{3x} &= -12.79\tfrac{m}{s} \end{aligned}$$

Now, let's look at the y-component of the total momentum.

y-component of the Momentum at Instant 1

Initially, the y-component of the total momentum is

$$\begin{aligned} p_y &= mv_y \\ &= 100kg \cdot 0\tfrac{m}{s} \\ &= 0 \end{aligned}$$

y-component of the Momentum at Instant 2

After the explosion, the y-component of the total momentum is

$$\begin{aligned} p'_y &= m_1 v'_{1y} + m_2 v'_{2y} + m_3 v'_{3y} \\ &= 45kg \cdot 0\tfrac{m}{s} + 30kg \cdot 25\tfrac{m}{s} \sin(-135°) + 25kg \cdot v'_{3y} \\ &= -530.33\tfrac{kgm}{s} + 25kg \cdot v'_{3y} \end{aligned}$$

Momentum Conservation

The law of momentum conservation then gives

$$\begin{aligned} p_y &= p'_y \\ 0 &= -530.33\tfrac{kgm}{s} + 25kg \cdot v'_{3y} \\ v'_{3y} &= 21.21\tfrac{m}{s} \end{aligned}$$

Speed and Direction of Velocity

From the components, the speed and the direction of the velocity can be found.

$$v_3' = \sqrt{v_{3x}'^2 + v_{3y}'^2}$$
$$= \sqrt{\left(-12.79\tfrac{m}{s}\right)^2 + \left(21.21\tfrac{m}{s}\right)^2}$$
$$= 24.77\tfrac{m}{s}$$

$$\theta = \arctan\frac{v_{3y}'}{v_{3x}'}$$
$$= \arctan\frac{21.21\frac{m}{s}}{-12.79\frac{m}{s}}$$
$$= 121.1°$$

(180° has been added to the answer given by the calculator because v'_{3x} is negative.)

10.5 COLLISIONS

Momentum Conservation

During a collision, there are forces between the colliding objects. If the system consists of the colliding objects, these forces are internal forces and the total momentum must be conserved in the collision. Therefore,

Momentum and Collisions

In a collision, the total momentum of the system is conserved.

$$\vec{p}_{tot} = \vec{p}'_{tot}$$

In components:

$$p_{tot\ x} = p'_{tot\ x} \qquad p_{tot\ y} = p'_{tot\ y} \qquad p_{tot\ z} = p'_{tot\ z}$$

Descartes had already postulated the conservation of momentum in collisions in 1644. Although he measured the quantity of motion with *mv*, the concept was more related to kinetic energy since he was not taking into account the direction of speed. In addition, the idea was only a speculation justified by the perfection of God and, therefore, it is hard to argue that Descartes really discovered momentum conservation.

Momentum conservation in collision was discovered in 1666 when the Royal Society of London launched the contest to find the laws of collisions. Three scientists (John Wallis, Christopher Wren and Christiaan Huygens) presented the results of their work in 1668. All of them came to the conclusion that the momentum *mv* is conserved in collisions provided that the direction of speed is taken into account (in other words, to take into account the fact that momentum is a vector).

This law is sometimes applied to a collision even if there is an external force. This can be done because the latter is often negligible compared to the forces between objects during the collision. Thus, when a baseball player strikes a ball, the law of conservation of momentum is applied despite the presence of the force of gravity on the bat and on the ball (which are external forces). The forces exerted on the ball and the bat are so much larger than the forces of gravity that the effect of the gravitational forces during the collision can be neglected.

However, the law of momentum conservation is not sufficient to fully resolve collision problems, some other information must be provided. It is necessary to know how the two objects behave when they come into contact. Both objects can stick together or can bounce off each other upon contact.

Elastic, Inelastic and Completely Inelastic Collisions

Elastic collision

In an elastic collision, the two colliding objects bounce off each other without any loss of kinetic energy. To illustrate, imagine that a ball is dropped and then bounce off the floor. If the collision is elastic, then the kinetic energy of the ball just after the collision is the same as it was just before the collision. As the kinetic energy before the collision comes from gravitational energy and the kinetic energy after the collision returns to gravitational energy, the gravitational energies at the highest point are the same before and after the collision, which means that, after the collision, the ball will ascend to the same height it was released.

Elastic Collision

In an elastic collision, the kinetic energy of the system is conserved.

$$E_{k\,tot} = E'_{k\,tot}$$

Huygens was the first to show in 1669 that kinetic energy is conserved in an elastic collision. It was the first time that kinetic energy appeared in physics. For a long time, it was called *vis viva* (*living force)* and its formula was mv^2 (the ½ was not there). When the kinetic energy theorem was formulated in the 1820's, Coriolis suggested adding ½ in the vis viva formula to give priority to the concept of work. It was only in 1850 that Lord Kelvin gave the name of *kinetic energy* to $\frac{1}{2}mv^2$.

(Since both quantities mv and $\frac{1}{2}mv^2$ are conserved in an elastic collision, we can understand why scientists of the decades before Newton, who were trying to define force by studying collisions, came up with definitions in which force is proportional to v or v^2.)

Inelastic Collision

Most of the time, collisions are not elastic. As some kinetic energy is lost in a collision (in the form of a permanent deformation or sound or heat, for example), the total kinetic energy is lower after the collision. The conservation of kinetic energy thus cannot be used in this kind of problem.

To solve an inelastic collision problem, only the equation of momentum conservation can be used. However, since there are two velocities to find (the velocity of each object after the collision), and there is only one equation, some additional information must be given. For example, this additional information can be the velocity of one of the objects after the collision or the fraction of kinetic energy lost in the collision.

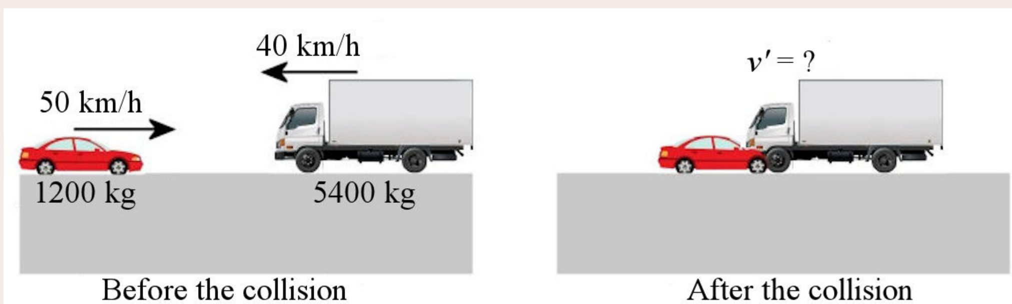
Completely Inelastic Collision

In a completely inelastic collision, the two objects stick together after the collision and, therefore, have the same velocity. In such a case, the law of momentum conservation is sufficient to solve the problem.

Examples for One-Dimensional Collisions

Example 10.5.1

The two vehicles shown in the diagram are involved in a collision. What will the velocity of the vehicles be after the collision if they stick together?



fr.depositphotos.com/2577683/stock-illustration-Car.html
www.canstockphoto.com/delivery-cargo-truck-13682465.html

Since this is a completely inelastic collision, only the momentum is conserved.

Momentum Before the Collision

Before the collision, the *x*-component of the total momentum is

$$\begin{aligned} p_x &= m_1 v_1 + m_2 v_2 \\ &= 1200kg \cdot 13.89 \tfrac{m}{s} + 5400kg \cdot \left(-11.11 \tfrac{m}{s}\right) \\ &= -43.333 \tfrac{kgm}{s} \end{aligned}$$

Momentum After the Collision

After the collision, we have a single object of 6600 kg, the *x*-component of its momentum is

$$\begin{aligned} p'_x &= mv' \\ &= 6600kg \cdot v' \end{aligned}$$

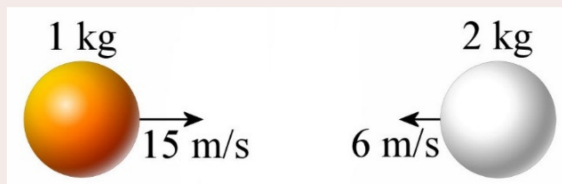
<u>Momentum Conservation</u>

The law of momentum conservation then gives

$$\begin{aligned} p_x &= p'_x \\ -43.333 \tfrac{kgm}{s} &= 6600kg \cdot v' \\ v' &= -6.566 \tfrac{m}{s} \end{aligned}$$

Example 10.5.2

A 1 kg ball moving at 15 m/s towards the right collides head-on with a 2 kg ball moving towards the left at 6 m/s. What are the velocities of the balls after the collision if the collision is elastic?



Since this is an elastic collision, the momentum and the kinetic energy are both conserved.

The law of momentum conservation gives

$$\begin{aligned} p_x &= p'_x \\ m_1 v_1 + m_2 v_2 &= m_1 v'_1 + m_2 v'_2 \\ 1kg \cdot 15 \tfrac{m}{s} + 2kg \cdot \left(-6 \tfrac{m}{2}\right) &= 1kg \cdot v'_1 + 2kg \cdot v'_2 \\ 3 \tfrac{m}{s} &= v'_1 + 2v'_2 \end{aligned}$$

The conservation of kinetic energy gives

$$\begin{aligned} E_k &= E'_k \\ \cancel{\tfrac{1}{2}} m_1 v_1^2 + \cancel{\tfrac{1}{2}} m_2 v_2^2 &= \cancel{\tfrac{1}{2}} m_1 v_1'^2 + \cancel{\tfrac{1}{2}} m_2 v_2'^2 \\ 1kg \cdot \left(15 \tfrac{m}{s}\right)^2 + 2kg \cdot \left(6 \tfrac{m}{s}\right)^2 &= 1kg \cdot v_1'^2 + 2kg \cdot v_2'^2 \\ 297 \tfrac{m^2}{s^2} &= v_1'^2 + 2v_2'^2 \end{aligned}$$

These two equations must be solved in order to find the velocities. We'll start by solving for v'_1 in the momentum equation

$$v'_1 = 3 \tfrac{m}{s} - 2v'_2$$

and then substitute this value into the kinetic energy equation.

$$297\tfrac{m^2}{s^2} = v_1'^2 + 2v_2'^2$$
$$297\tfrac{m^2}{s^2} = \left(3\tfrac{m}{s} - 2v_2'\right)^2 + 2v_2'^2$$
$$297\tfrac{m^2}{s^2} = 9\tfrac{m^2}{s^2} - 12\tfrac{m}{s}\cdot v_2' + 4v_2'^2 + 2v_2'^2$$
$$6v_2'^2 - 12\tfrac{m}{s}\cdot v_2' - 288\tfrac{m^2}{s^2} = 0$$

This quadratic equation can then be solved. The solutions are

$$v_2' = 8\tfrac{m}{s} \qquad \text{and} \qquad v_2' = -6\tfrac{m}{s}$$

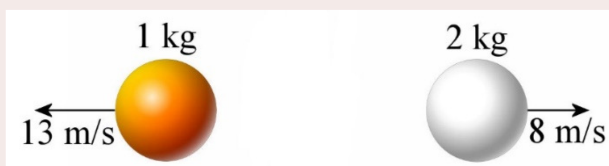
There are always two solutions, and one of them is always identical to the velocity before the collision. The solutions of the conservation law correspond to all the velocities for which the momentum and the kinetic energy are the same as they were before the collision. Obviously, the velocities before the collision give the same momentum and kinetic energy as the ones before the collision! The velocity after the collision is the solution which is different from the initial velocity. Therefore,

$$v_2' = 8\tfrac{m}{s}$$

Then the velocity of the other ball can be found. The momentum equation is used, because we'll have to guess the sign of the velocity if the kinetic energy equation is used because the velocity is squared. Therefore, the velocity of the other ball is

$$3\tfrac{m}{s} = v_1' + 2v_2'$$
$$3\tfrac{m}{s} = v_1' + 2\cdot 8\tfrac{m}{s}$$
$$v_1' = -13\tfrac{m}{s}$$

The velocities after the elastic collision are shown in this diagram.



Example 10.5.3

In an inelastic collision, a 1 kg ball moving at 5 m/s towards the right collides head-on with a 2 kg ball moving towards the left at 2 m/s. After the collision, the velocity of the 1 kg ball is 4 m/s towards the left.

a) What is the velocity of the 2 kg ball after the collision?

Since this is an inelastic collision, only the momentum is conserved. The law of momentum conservation gives

$$p_x = p'_x$$
$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$$
$$1kg \cdot 5\tfrac{m}{s} + 2kg \cdot \left(-2\tfrac{m}{s}\right) = 1kg \cdot \left(-4\tfrac{m}{s}\right) + 2kg \cdot v'_2$$
$$1\tfrac{kgm}{s} = -4\tfrac{kgm}{s} + 2kg \cdot v'_2$$
$$v'_2 = 2.5\tfrac{m}{s}$$

b) What fraction of the kinetic energy is lost in the collision?

The initial total kinetic energy of the balls is

$$\begin{aligned} E_k &= \tfrac{1}{2} \cdot 1kg \cdot \left(5\tfrac{m}{s}\right)^2 + \tfrac{1}{2} \cdot 2kg \cdot \left(2\tfrac{m}{s}\right)^2 \\ &= 16.5J \end{aligned}$$

The total kinetic energy of the balls after the collision is

$$\begin{aligned} E'_k &= \tfrac{1}{2} \cdot 1kg \cdot \left(4\tfrac{m}{s}\right)^2 + \tfrac{1}{2} \cdot 2kg \cdot \left(2.5\tfrac{m}{s}\right)^2 \\ &= 14.25J \end{aligned}$$

(Of course, the final kinetic energy after the collision can never be larger than the kinetic energy before the collision.) Therefore, the energy loss is

$$\begin{aligned} \Delta E_k &= E'_k - E_k \\ &= 14.25J - 16.5J \\ &= -2.25J \end{aligned}$$

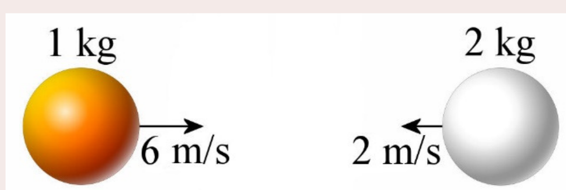
The fraction lost is then

$$\begin{aligned} \frac{\Delta E_k}{E_{ki}} &= \frac{-2.25J}{16.5J} \\ &= -0.136 \end{aligned}$$

(It is negative because it is a loss.) Therefore, 13.6% of the mechanical energy is lost in this collision.

Example 10.5.4

In an inelastic collision, a 1 kg ball moving at 6 m/s towards the right collides head-on with a 2 kg ball moving towards the left at 2 m/s. In the collision, 50% of the kinetic energy is lost. What are the speeds of the balls after the collision?



Since this is an inelastic collision, only the momentum is conserved.

The law of momentum conservation gives

$$\begin{aligned} p_x &= p'_x \\ m_1 v_1 + m_2 v_2 &= m_1 v'_1 + m_2 v'_2 \\ 1kg \cdot 6\tfrac{m}{s} + 2kg \cdot \left(-2\tfrac{m}{2}\right) &= 1kg \cdot v'_1 + 2kg \cdot v'_2 \\ 2\tfrac{m}{s} &= v'_1 + 2v'_2 \end{aligned}$$

We have only one equation to find 2 speeds. However, the information we have on energy will allow us to make a second equation. We know that the final kinetic energy is equal to 50% of the initial energy. We can therefore calculate the initial kinetic energy and then equal the final kinetic energy to half of the initial kinetic energy. This equation of final kinetic energy will be our 2nd equation.

The initial kinetic energy is

$$\begin{aligned} E_k &= \tfrac{1}{2} \cdot 1kg \cdot \left(6\tfrac{m}{s}\right)^2 + \tfrac{1}{2} \cdot 2kg \cdot \left(2\tfrac{m}{s}\right)^2 \\ &= 22J \end{aligned}$$

Since half of the energy is lost in the collisions, the kinetic energy after the collision is 11 J. Therefore,

$$\begin{aligned} E'_k &= \tfrac{1}{2} m_1 v_1'^2 + \tfrac{1}{2} m_2 v_2'^2 \\ 11J &= \tfrac{1}{2} \cdot 1kg \cdot v_1'^2 + \tfrac{1}{2} \cdot 2kg \cdot v_2'^2 \\ 22\tfrac{m^2}{s^2} &= v_1'^2 + 2v_2'^2 \end{aligned}$$

This is the 2nd equation.

These two equations must be solved in order to find the velocities.

$$\begin{aligned} 2\tfrac{m}{s} &= v'_1 + 2v'_2 \\ 22\tfrac{m^2}{s^2} &= v_1'^2 + 2v_2'^2 \end{aligned}$$

We'll start by solving for v'_1 in the momentum equation

$$v'_1 = 2\tfrac{m}{s} - 2v'_2$$

and then substitute this value into the kinetic energy equation.

$$\begin{aligned} 22\tfrac{m^2}{s^2} &= v_1'^2 + 2v_2'^2 \\ 22\tfrac{m^2}{s^2} &= \left(2\tfrac{m}{s} - 2v'_2\right)^2 + 2v_2'^2 \\ 22\tfrac{m^2}{s^2} &= 4\tfrac{m^2}{s^2} - 8\tfrac{m}{s} \cdot v'_2 + 4v_2'^2 + 2v_2'^2 \end{aligned}$$

$$6v_2'^2 - 8\tfrac{m}{s}\cdot v_2' - 18\tfrac{m^2}{s^2} = 0$$

This equation can be solved to obtain

$$v_2' = 2.52258\tfrac{m}{s} \qquad \text{and} \qquad v_2' = -1.18925\tfrac{m}{s}$$

From those values, the speed of the other ball can be found with $2\tfrac{m}{s} = v_1' + 2v_2'$. The following solutions are then obtained.

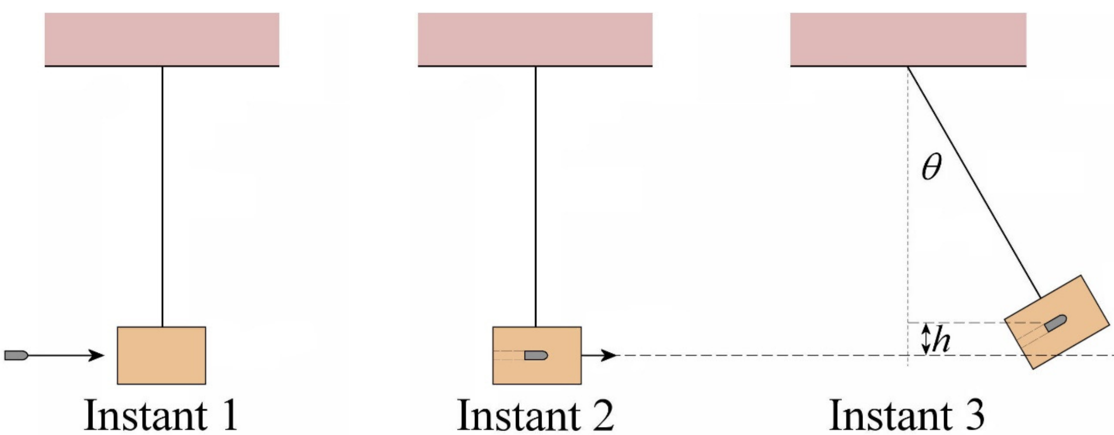
$$\begin{matrix} v_1' = -3.04517\tfrac{m}{s} \\ v_2' = 2.52258\tfrac{m}{s} \end{matrix} \qquad \text{and} \qquad \begin{matrix} v_1' = 4.37850\tfrac{m}{s} \\ v_2' = -1.18925\tfrac{m}{s} \end{matrix}$$

In these two solutions, there is one (the second) where the signs of the velocities are identical to the signs before the collision. As it is impossible to have velocities after the collision that are both in the same direction as the initial velocities, this solution must be rejected. (This second solution is actually the initial situation with the same momentum, but half of its kinetic energy.)

Thus, those are the velocities after the collision.

Ballistic Pendulum

A ballistic pendulum is used to measure the speed of certain objects, like rifle bullets, for example. A rifle bullet is fired into a pendulum initially at rest and the maximum angle reached by the pendulum is measured.



http://www.youtube.com/watch?NR=1&v=cVjxn4KjOb4
With the maximum angle, the speed of the rifle bullet before colliding with the pendulum can be obtained.

There are actually two parts to this problem.

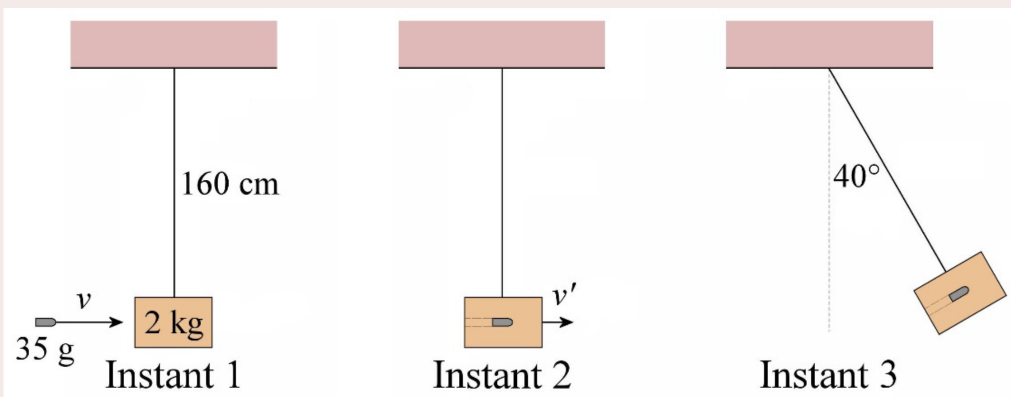
1) First, there's a collision between the ball and the block. During this phase (between instant 1 and instant 2), there is momentum conservation (as in any collision) while mechanical energy is not conserved (as in any inelastic collisions since part of the ball's kinetic energy turns into heat, sound and block deformation).

2) Then there is a pendulum motion. During this phase (between instant 2 and instant 3), the mechanical energy is conserved (as for any pendulums if friction is neglected), but the momentum is not conserved (since there are external forces such as gravity and rope tension).

It is therefore necessary to consider each of these parts using different conservation laws.

Example 10.5.5

A bullet (m_1= 35 g) is fired into a wooden block (m_2 = 2 kg) which is hanging at the end of a 160 cm long rope. The pendulum then swings up to an angle of 40°. What was the speed of the bullet?



As the only information available concerns the final configuration, we will start from the final position (instant 3) to go back to the initial configuration (instant 1).

We will therefore start by studying the pendulum motion (between instants 2 and 3) using the conservation of mechanical energy.

(There will be no prime for the quantities used at instant 2, one prime for the quantities used at instant 2, and two primes for the quantities used instant 3.)

Pendulum Motion

Energy at Instant 2

Immediately after the collision (instant 2), the mechanical energy is (placing the $y = 0$ at the lowest point of the pendulum)

$$\begin{aligned} E' &= \tfrac{1}{2} m_{block} v'^2 + m_{block} g y' \\ &= \tfrac{1}{2} m_{block} v'^2 \end{aligned}$$

Energy at Instant 3

When the angle is maximum (instant 3), the mechanical energy is

$$\begin{aligned} E'' &= \tfrac{1}{2} m_{block} v''^2 + m_{block} g y'' \\ &= m_{block} g y_{\max} \end{aligned}$$

Mechanical Energy Conservation

The law of mechanical energy conservation then gives

$$\begin{aligned} E' &= E'' \\ \tfrac{1}{2} \cancel{m_{block}} v'^2 &= \cancel{m_{block}} g y_{\max} \\ v' &= \sqrt{2 g y_{\max}} \end{aligned}$$

The maximum height reached by the pendulum can be calculated from the maximum angle with

$$\begin{aligned} y_{\max} &= L\left(1 - \cos\theta_{\max}\right) \\ &= 1.6m \cdot \left(1 - \cos 40°\right) \\ &= 0.3743m \end{aligned}$$

The speed of the block right after the collision is then

$$\begin{aligned} v' &= \sqrt{2 g y_{\max}} \\ &= \sqrt{2 \cdot 9.8 \tfrac{m}{s^2} \cdot 0.3743m} \\ &= 2.709 \tfrac{m}{s} \end{aligned}$$

Collision

The collision between the bullet and the wooden block can now be examined (between instants 1 and 2) using momentum conservation.

Momentum at Instant 1

Before the collision (instant 1), the *x*-component of the total momentum of the bullet-block system is

$$\begin{aligned} p_x &= m_{bullet}v_{bullet} + m_{block}v_{block} \\ &= 0.035kg \cdot v_{bullet} + 0 \end{aligned}$$

Momentum at Instant 2

After the collision (instant 2), we have a single object of 2.035 kg (the block with the embedded bullet). The *x*-component of the total momentum is

$$\begin{aligned} p_x' &= mv' \\ &= 2.035kg \cdot 2.709\tfrac{m}{s} \\ &= 5.512\tfrac{kgm}{s} \end{aligned}$$

Momentum Conservation

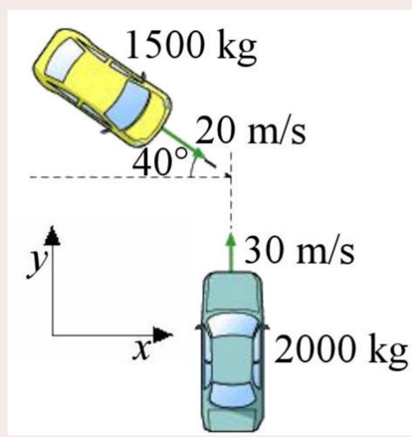
The law of momentum conservation then gives us the speed of the ball.

$$\begin{aligned} p_x &= p_x' \\ 0.035kg \cdot v_{bullet} &= 5.512\tfrac{kgm}{s} \\ v_{bullet} &= 157.5\tfrac{m}{s} \end{aligned}$$

Examples for Two-Dimensional Collisions

Example 10.5.6

A 2000 kg car moving north at 30 m/s collides with a 1500 kg car moving at 20 m/s in the direction shown in the diagram. If both cars stick together after the collision, what is the velocity (magnitude and direction) of the cars after the collision?



www.physicsforums.com/showthread.php?t=193963

As this is a completely inelastic collision (since the two objects stick together after the collision) only the momentum is conserved.

Conservation of the x-component of the Momentum

Let's start with the conservation of the *x*-component of the momentum.

$$p_x = p_x'$$

$$m_1 v_{1x} + m_2 v_{2x} = m_{tot} v'_x$$
$$2000kg \cdot 0 \tfrac{m}{s} + 1500kg \cdot 20 \tfrac{m}{s} \cdot \cos(-40°) = 3500kg \cdot v'_x$$
$$22,981.3 \tfrac{kgm}{s} = 3500kg \cdot v'_x$$
$$v'_x = 6.566 \tfrac{m}{s}$$

<u>Conservation of the y-component of the Momentum</u>

Then, the conservation of the y-component of the momentum gives

$$p_y = p'_y$$
$$m_1 v_{1y} + m_2 v_{2y} = m_{tot} v'_y$$
$$2000kg \cdot 30 \tfrac{m}{s} + 1500kg \cdot 20 \tfrac{m}{s} \cdot \sin(-40°) = 3500kg \cdot v'_y$$
$$40,716.4 \tfrac{kgm}{s} = 3500kg \cdot v'_y$$
$$v'_y = 11.633 \tfrac{m}{s}$$

<u>Final Velocity</u>

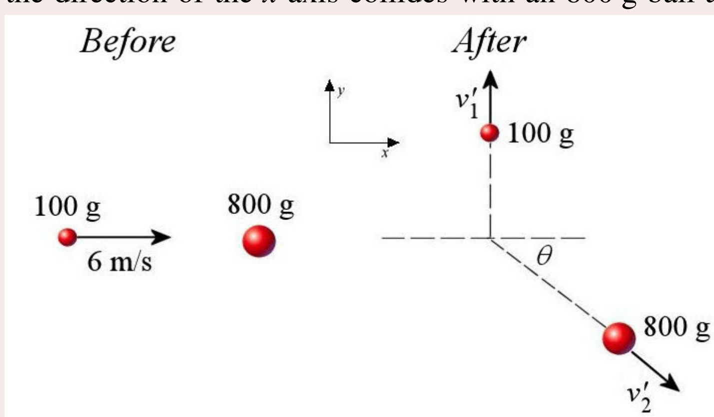
Thus, the speed and the direction of the velocity are

$$\begin{aligned} v' &= \sqrt{v_x'^2 + v_y'^2} \\ &= \sqrt{\left(6.566 \tfrac{m}{s}\right)^2 + \left(11.633 \tfrac{m}{s}\right)^2} \\ &= 13.36 \tfrac{m}{s} \end{aligned}$$

$$\begin{aligned} \theta &= \arctan \frac{v'_y}{v'_x} \\ &= \arctan \frac{11.633 \tfrac{m}{s}}{6.566 \tfrac{m}{s}} \\ &= 60.56° \end{aligned}$$

Example 10.5.7

A 100 g ball moving at 6 m/s in the direction of the x-axis collides with an 800 g ball at rest. After the collision, the 100 g ball is heading in the direction of the y-axis. Determine the speed of the 100 g ball and the velocity (magnitude and direction) of the 800 g ball if the collision is elastic.



Since this is an elastic collision, the momentum and the kinetic energy are both conserved. Thus, there will be 3 equations: the conservation law for the *x* and *y* components of the momentum and the law of conservation of kinetic energy (since the collision is elastic). Three equations are required since there are three quantities to find: two speeds and the direction of the velocity of the 800 g ball.

Conservation of the x-component of the Momentum

The equation for the conservation of the *x*-component of the momentum is

$$p_x = p'_x$$
$$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_1 v'_{2x}$$
$$0.1kg \cdot 6\tfrac{m}{s} + 0 = 0 + 0.8kg \cdot v'_{2x}$$
$$v'_{2x} = 0.75\tfrac{m}{s}$$

Conservation of the y-component of the Momentum

The equation for the conservation of the *y*-component of the momentum is

$$p_y = p'_y$$
$$m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_1 v'_{2y}$$
$$0 + 0 = 0.1kg \cdot v'_{1y} + 0.8kg \cdot v'_{2y}$$
$$v'_{1y} = -8 v'_{2y}$$

Energy Conservation

The equation for the conservation of kinetic energy is

$$E_k = E'_k$$
$$\cancel{\tfrac{1}{2}} m_1 v_1^2 + \cancel{\tfrac{1}{2}} m_2 v_2^2 = \cancel{\tfrac{1}{2}} m_1 v_1'^2 + \cancel{\tfrac{1}{2}} m_2 v_2'^2$$
$$0.1kg \cdot \left(6\tfrac{m}{s}\right)^2 + 0 = 0.1kg \cdot v_1'^2 + 0.8kg \cdot v_2'^2$$
$$36\tfrac{m^2}{s^2} = v_1'^2 + 8v_2'^2$$

Solving the Equations

To solve these equations, it must be remembered that $v^2 = v_x^2 + v_y^2$**.** The energy equation then becomes

$$36\tfrac{m^2}{s^2} = v_1'^2 + 8v_2'^2$$
$$36\tfrac{m^2}{s^2} = \left(\cancel{v_{1x}'^2} + v_{1y}'^2\right) + 8\left(v_{2x}'^2 + v_{2y}'^2\right)$$

The two results previously obtained from the momentum conservation equations are then substituted in this equation.

$$36\tfrac{m^2}{s^2} = \left(v_{1y}'^2\right) + 8\left(v_{2x}'^2 + v_{2y}'^2\right)$$

$$36\tfrac{m^2}{s^2} = \left(v_{1y}'^2\right) + 8\left(\left(0.75\tfrac{m}{s}\right)^2 + \left(-\frac{v_{1y}'}{8}\right)^2\right)$$

$$31.5\tfrac{m^2}{s^2} = v_{1y}'^2 + \frac{v_{1y}'^2}{8}$$

$$31.5\tfrac{m^2}{s^2} = \frac{9v_{1y}'^2}{8}$$

$$v_{1y}' = 5.292\tfrac{m}{s}$$

The last velocity component needed is then found with the y-component of the momentum equation.

$$\begin{aligned} v_{2y}' &= -\frac{v_{1y}'}{8} \\ &= -0.6614\tfrac{m}{s} \end{aligned}$$

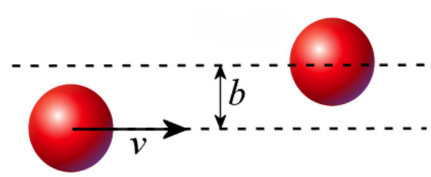
Therefore, the magnitude and direction of the 800 g ball velocity are

$$\begin{aligned} v_2' &= \sqrt{v_{2x}'^2 + v_{2y}'^2} \\ &= \sqrt{\left(0.75\tfrac{m}{s}\right)^2 + \left(-0.6614\tfrac{m}{s}\right)^2} \\ &= 1\tfrac{m}{s} \end{aligned}$$

$$\begin{aligned} \theta_2 &= \arctan\frac{v_{2y}'}{v_{2x}'} \\ &= \arctan\frac{-0.6614}{0.75} \\ &= -41.41° \end{aligned}$$

The 100 g ball thus moves at 5.292 m/s, and the 800 g ball moves at 1 m/s at - 41.41°.

For elastic collisions, one information concerning the balls after the collision must be given. In the last example, it was known that the 100 g ball was moving in the direction of the positive y-axis. If no additional information were given, four unknowns would have to be found with three equations (two momentum equations and one kinetic energy equation). Since it is impossible to find more unknown than the number of equations, it would be impossible to solve the problem.



Does this mean that it is impossible to know the result of the collisions with only the laws of physics? Of course not. The additional data given actually informs us about the way the balls hit each other. Depending on the value of b in the diagram (called the impact parameter) the result of the collision is different. This value of b may be the additional data given, and the direction and speed of the two objects after the collision could be calculated using this information. As this is quite tricky, the speed or the direction of the velocity of one of the objects after the collision is given in these notes. This is equivalent to giving the value of *b*.

10.6 IMPULSE-MOMENTUM THEOREM FOR A SYSTEM

It is possible to use momentum to solve a problem even if momentum is not conserved because there are external forces. With a non-vanishing external net force, the momentum change is (equation obtained in section 10.4)

$$\left(\sum_{system} \vec{F}_{ext}\right)\Delta t = \vec{p}'_{tot} - \vec{p}_{tot}$$

With this equation, it was determined that momentum is conserved if the sum of the external forces is zero. The equation also tells us what to do if this sum is not zero. We can write this equation in the following form.

$$\vec{p}_{tot} + \left(\sum_{system} \vec{F}_{ext}\right)\Delta t = \vec{p}'_{tot}$$

As the last term is the net impulse given to the system by external forces, the result is

Impulse-Momentum Theorem for a System

$$\vec{p}_{tot} + \vec{I}_{ext\ net} = \vec{p}'_{tot}$$

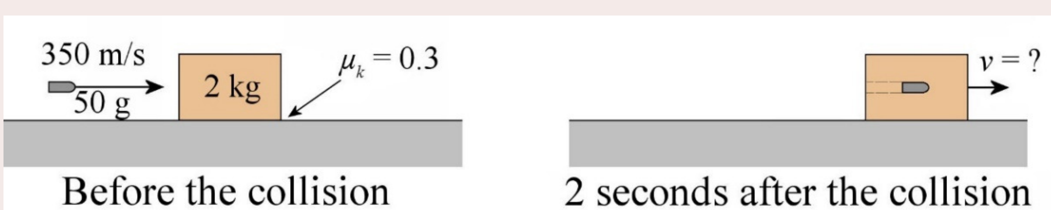
In components:

$$p_{tot\ x} + I_{net\ ext\ x} = p'_{tot\ x} \qquad p_{tot\ y} + I_{net\ ext\ y} = p'_{tot\ y} \qquad p_{tot\ z} + I_{net\ ext\ z} = p'_{tot\ z}$$

where $I_{net\ ext}$ are net impulses made by external forces.

Example 10.6.1

A bullet is fired into a wooden block and gets lodged in it (which corresponds to a completely inelastic collision). After the collision, the block slides on the floor. The kinetic friction coefficient between the block and the floor is 0.3. What is the speed of block 2 seconds after being hit by the bullet?



If the system consists of the block and the bullet, the friction force is an external force. The problem can then be solved with the formula

$$p_{tot\ x} + I_{net\ ext\ x} = p'_{tot\ x}$$

The momentum at instant 1 (before the collision) is

$$\begin{aligned} p_{tot\ x} &= m_{block} v_{block\ x} + m_{bullet} v_{bullet\ x} \\ &= 2kg \cdot 0 \tfrac{m}{s} + 0.05kg \cdot 350 \tfrac{m}{s} \\ &= 17.5 \tfrac{kgm}{s} \end{aligned}$$

The momentum at instant 2 (2 seconds after the collision) is

$$\begin{aligned} p'_{tot\ x} &= m_{tot} v'_x \\ &= 2.05kg \cdot v' \end{aligned}$$

The impulse given by the external force (friction force here) in 2 seconds is

$$\begin{aligned} I_{net\ ext\ x} &= \sum F_{ext\ x} \Delta t \\ &= -F_f \Delta t \end{aligned}$$

There's a minus sign because the force is towards the left. Using the friction force formula, the impulse is then

$$\begin{aligned} I_{net\ ext\ x} &= -\mu F_N \Delta t \\ &= -\mu mg \Delta t \\ &= -0.3 \cdot 2.05kg \cdot 9.8 \tfrac{N}{kg} \cdot 2s \\ &= -12.054 \tfrac{kgm}{s} \end{aligned}$$

Thus, the theorem gives

$$\begin{gathered} p_{tot\ x} + I_{net\ ext\ x} = p'_{tot\ x} \\ 17.5 \tfrac{kgm}{s} - 12.054 \tfrac{kgm}{s} = 2.05kg \cdot v' \\ 5.446 \tfrac{kgm}{s} = 2.05kg \cdot v' \\ v' = 2.657 \tfrac{m}{s} \end{gathered}$$

10.7 MOMENTUM AND KINETIC ENERGY

Relationship Between Momentum and Kinetic Energy

The kinetic energy can be found directly from the momentum without finding the speed since there is a formula to pass directly from one to the other. This formula is obtained in the following way.

$$E_k = \frac{mv^2}{2}$$

$$E_k = \frac{m^2v^2}{2m}$$

$$E_k = \frac{(mv)^2}{2m}$$

This leads to

Relationship Between Momentum and Kinetic Energy

$$E_k = \frac{p^2}{2m}$$

Three Differences Between Kinetic Energy and Momentum

1) Collisions

During a collision

- Momentum is always conserved.
- Kinetic energy is conserved only if the collision is elastic.

2) The Direction of the Velocity

The direction of the velocity is important, but only for the momentum since only this quantity is a vector. Only the speed matters for the kinetic energy, the direction of the velocity has no significance.

- Momentum is a vector
- Kinetic energy is a scalar

So, it is possible to change the momentum without changing the kinetic energy if the direction of the velocity changes while the speed stays the same. However, it is impossible to change the kinetic energy without changing the momentum.

3) Rate of Change of Momentum and Kinetic Energy

The equation

$$W_{net} = \Delta E_k$$

indicates that the work corresponds to the change in kinetic energy. The rate at which kinetic energy is added is the power.

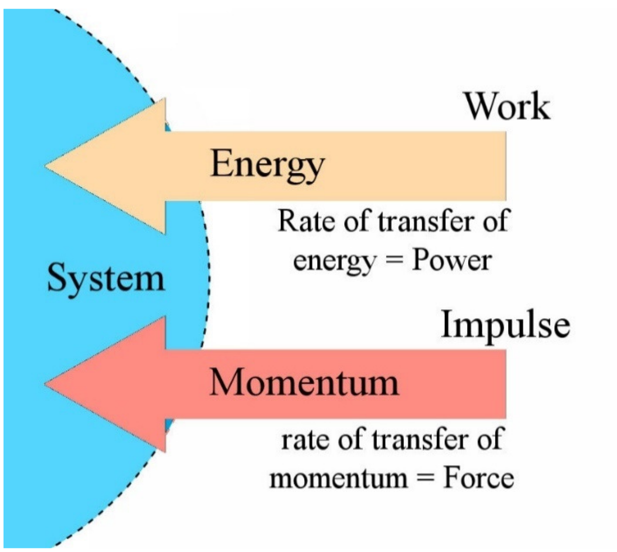
$$P = \frac{dW}{dt}$$

The equation

$$\vec{I}_{net} = \Delta\vec{p}$$

indicates that the impulse corresponds to the change in momentum. The rate at which momentum is added is the force.

$$F = \frac{dp}{dt}$$



This diagram summarizes these four formulas.

Vis Viva Controversy

It is perfectly legitimate to ask which of these two quantities, mv or $\frac{1}{2}mv^2$, is more important in physics. This question was even at the origin of the vis viva controversy (*vis viva* was the name given to mv^2 at that time).

The debate began in 1686 when Leibniz launched an attack on Descartes's idea that the quantity *mv* was conserved in the universe. Leibniz then asserts that Descartes was wrong and that mv^2 should be used instead.

Among the supporters of *mv*, we note Isaac Newton, Denis Papin, Jean-Jacques Dortous de Mairan, Samuel Clarke, Edme Mariotte, Pierre Varignon and Voltaire. Among the supporters of mv^2, we note Leibniz, Johann and Daniel Bernoulli, Christian Wolff, Willelm Jacob 's Gravesande, Pieter van Musschenbroek and the Marquise du Châtelet (who did so much to introduce Newton's ideas in France).

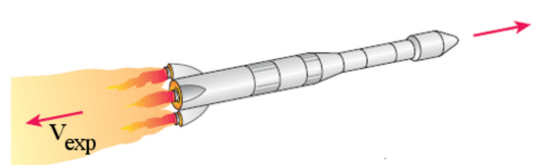
Each side then tries to convince the other that their quantity was the best. Many arguments used in favour of one or the other went beyond simple collisions arguments. For example, it was argued that the momentum was more fundamental because the flight time of an object is thrown upwards doubles if the momentum doubles. On the other side, it was argued that the kinetic energy was more fundamental since the maximum height of an object thrown upwards doubles is the kinetic energy doubles.

Actually, no quantity is more fundamental than the other; it depends on what is sought. Each of these quantities has its place in physics. If time is an important aspect of the problem, momentum seems to be more important. For example, if car A has twice the momentum than car B, then car A take twice as much time to stop then car B. On the other side, if the distance is an important aspect of the problem, the kinetic energy seems to be more important. For example, if car A has twice as much kinetic energy as car B, then the stopping distance of the car A is twice the stopping distance of car B.

10.8 ROCKET AND AIRCRAFT PROPULSION

Rockets

A mass changing object, i.e. a rocket expelling gas with an expulsion speed of v_{exp}, will now be considered. Note that v_{exp} is the speed relative to the rocket and not the speed relative to the ground. The gas is expelled at a rate R, which corresponds to the number of kilograms of gas expelled each second.

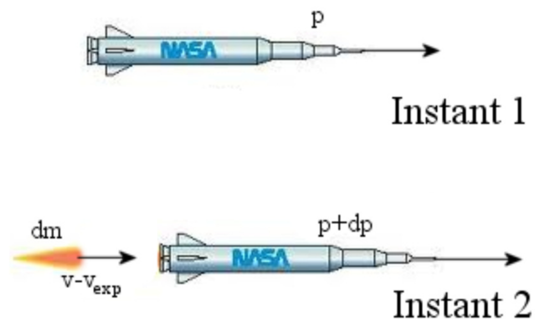


revisionworld.co.uk/a2-level-level-revision/physics/force-motion/momentum-second-law/momentum-second-law-0

Thrust Force of Rocket Engines

In the diagram to the right, a rocket expels a small amount of gas. The total momentum of the system (rocket and gas) must be the same before and after the ejection. This means that

$$p = p'$$
$$p_{rocket} + p_{gas} = p'_{rocket} + p'_{gas}$$
$$p_{rocket} + vdm = \left(p_{rocket} + dp_{rocket}\right) + dm\left(v - v_{\exp}\right)$$
$$0 = dp_{rocket} - v_{\exp}dm$$
$$dp_{rocket} = v_{\exp}dm$$



nothingnerdy.wikispaces.com/FORCES+CHANGE+MOMENTUM

As the force on the rocket is dp_{rocket}/dt, the equation becomes

$$dp_{rocket} = v_{\exp}dm$$
$$\frac{dp_{rocket}}{dt} = v_{\exp}\frac{dm}{dt}$$
$$F_{thrust} = v_{\exp}\frac{dm}{dt}$$

This *dm/dt* is the rate at which the mass of the rocket changes. This is actually the rate of mass ejection of the gas (*R*).

Thrust Force Rocket Engines

$$F_{thrust} = v_{\exp}R$$

Speed of the Rocket

The force is constant if the expelled gas velocity and rate of mass ejection are constant. However, it is tricky to find the final velocity of the rocket because the mass of the rocket decreases constantly. This means that the acceleration of the rocket is not constant. The acceleration of the rocket is

$$F_{thrust} = Ma$$
$$a = \frac{F_{thrust}}{M}$$

where *M* is the mass of the rocket (which is constantly changing). Since the thrust force is

$$F_{thrust} = v_{\exp}R$$

The acceleration is

$$a = \frac{v_{\exp}R}{M}$$

As the mass M decreases at rate R, the mass as a function of time is

$$M = m - Rt$$

where m is the initial mass of the rocket. The acceleration equation then gives

$$a = \frac{v_{\exp} R}{m - Rt}$$
$$\frac{dv}{dt} = v_{\exp} \frac{R}{m - Rt}$$
$$dv = v_{\exp} \frac{R}{m - Rt} dt$$

Integrating from the initiation of thrust ($t = 0$) to a time T gives the speed at time T.

$$\int_v^{v'} dv = v_{\exp} \int_0^T \frac{R}{m - Rt} dt$$
$$\left[v\right]_v^{v'} = -v_{\exp} \left[\ln\left(m - Rt\right)\right]_0^T$$
$$v' - v = -v_{\exp} \left(\ln\left(m - RT\right) - \ln m\right)$$
$$v' = v + v_{\exp} \ln \frac{m}{m - RT}$$

The speed is then

Speed of a Rocket Expelling Gas during Time *T*

$$v' = v + v_{\exp} \ln \frac{m}{m - RT}$$
$$v' = v + v_{\exp} \ln \frac{m}{m'}$$

In the second equation m' is the mass of the rocket at time T, which is $m - RT$.

Example 10.8.1

A rocket, initially at rest in space, begins to eject gas at v_{exp} = 2000 m/s. The gas is ejected at the rate of 1000 kg/s. The initial mass of the rocket is 100 tons, which includes 60 tons of gas to be ejected.

a) What is the thrust force?

The force is

$$\begin{aligned} F_{thrust} &= Rv_{\exp} \\ &= 1000\tfrac{kg}{s}\cdot 2000\tfrac{m}{s} \\ &= 2,000,000N \end{aligned}$$

b) What is the speed of the rocket once all the gas is ejected?

The final speed of the rocket is

$$\begin{aligned} v' &= v + v_{\exp}\ln\frac{m}{m'} \\ &= 0 + 2000\tfrac{m}{s}\cdot\ln\frac{100\,tons}{40\,tons} \\ &= 1833\tfrac{m}{s} \end{aligned}$$

c) How long did it take for the rocket to reach this speed?

As 60,000 kg of gas are ejected at a rate of 1000 kg/s, the ejection lasted

$$\begin{aligned} t &= \frac{60,000kg}{1000\tfrac{kg}{s}} \\ &= 60s \end{aligned}$$

Example 10.8.2

At liftoff, a rocket initially at rest starts to eject gas with a speed of v_{exp} = 2000 m/s. The gas is ejected at a rate of 1000 kg/s. The initial mass of the rocket is 100 tons, including 60 tons of gas to be ejected.

a) What is the initial acceleration of the rocket?

There are two forces exerted on the rocket: the force of gravity and the thrust force of the engines. Therefore, the force equation gives

$$\begin{aligned} \sum F_y &= ma_y \\ -F_g + F_{thrust} &= ma_y \\ -mg + Rv_{\exp} &= ma_y \\ -100,000kg\cdot 9.8\tfrac{N}{kg} + 1000\tfrac{kg}{s}\cdot 2000\tfrac{m}{s} &= 100,000kg\cdot a_y \\ a_y &= 10.2\tfrac{m}{s^2} \end{aligned}$$

As this acceleration is positive, it is directed upwards, and the rocket managed to lift off. The acceleration could not be negative (downwards). If the rocket failed to lift off, a normal force is added so that $a = 0$.

Note that this 10.2 m/s² acceleration is not constant. It increases as the mass of the spacecraft decreases.

b) What is the speed of the rocket 30 seconds after liftoff?

Here, the formula of the speed of a rocket cannot be used directly because the gravitational force also acts on the rocket. However, it may be used if a method unknown up to now is employed.

Up to now, the sum of the forces was done in order to find the acceleration and then the velocity of an object. Actually, it is possible to do this sum later in the solution. The acceleration made be each force can be found, and these accelerations may then be added to find the net acceleration and the velocity. It is also possible to find the acceleration and then the velocity resulting from the action of each force and then add these velocities to obtain the net velocity of the object. These methods all give the same result. It does not really matter if you sum the forces, the acceleration or the velocities, provided that the sum is done at some point.

This trick will be used here because we know the velocity given by each of the forces acting on the rocket. The engine thrust force gives an upwards speed of

$$v_1' = v + v_{\text{exp}} \ln \frac{m}{m'}$$

and the gravitational force gives a downwards speed of

$$v_2' = -9.8 \tfrac{m}{s^2} \cdot t$$

(Since all objects fall with an acceleration of 9.8 m/s².) The resulting speed is

$$v' = v + v_{\text{exp}} \ln \frac{m}{m'} - 9.8 \tfrac{m}{s^2} \cdot t$$

After 30 seconds, the mass of the rocket is 70 tons (since 1 ton is ejected each second). The speed is then

$$\begin{aligned} v' &= v + v_{\text{exp}} \ln \frac{m}{m'} - 9{,}8 \tfrac{m}{s^2} \cdot t \\ &= 0 + 2000 \tfrac{m}{s} \cdot \ln \frac{100\,tons}{70\,tons} - 9.8 \tfrac{m}{s^2} \cdot 30s \\ &= 419.4 \tfrac{m}{s} \end{aligned}$$

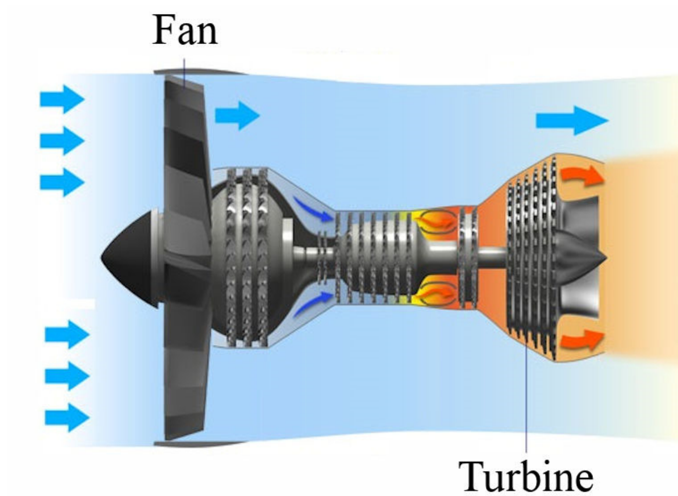
Aircraft

A propeller engine simply pushes air from the atmosphere towards the rear of the aircraft. The ejected gas does not come from fuel in the aircraft, but from the atmosphere.

A turbofan combines both types of thrust. The burning fuel is ejected backwards as for a jet engine, but, near the exit, the gas ejected at high speed turns a turbine which turns a huge fan at the entrance of the turbofan. This fan acts as a propeller that pushes air towards the back of the plane. So, part of the thrust comes from the gas ejected by the combustion of fuel (as for a jet engine), and part of the thrust comes from the air of the atmosphere pushed backwards by the fan (as for a propeller engine).

The diagram shows the flow of air pushed by the fan (in blue) and the flow of air from the combustion of the fuel (in red).

However, much of the thrust of this engine comes from the air pushed by the fan. So, we're going to simplify by assuming that all the thrust comes from this air pushed by the fan that essentially works like a propeller.



unehistoiredavions.weebly.com/le-reacuteacteur.html

Thrust Force of Aircraft Engines

Thus, for the propeller engine and the turbojet engine, the air pushed by the engine, whether with a propeller or a fan, comes from the air of the atmosphere which has zero speed. The conservation of the momentum then gives

$$p = p'$$

$$p_{aircraft} + p_{gas} = p'_{aircraft} + p'_{gas}$$

$$p_{aircraft} + 0 = \left(p_{aircraft} + dp_{aircraft}\right) + dm\left(v - v_{\exp}\right)$$

$$0 = dp_{aircraft} + \left(v - v_{\exp}\right)dm$$

$$dp_{aircraft} = \left(v_{\exp} - v\right)dm$$

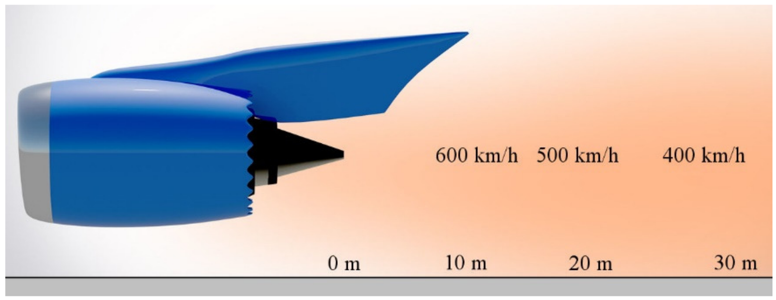
$$\frac{dp_{aircraft}}{dt} = \left(v_{\exp} - v\right)\frac{dm}{dt}$$

As the force is $dp_{aircraft}/dt$ and the rate of expulsion of gases R is dm/dt. We have

Thrust Force of Aircraft Engines

$$F_t = \left(v_{\exp} - v\right) R$$

The diagram to the right shows that the gas ejection speed from a commercial aircraft engine is of the order of 600 km/h just at the exit of the engine when the aircraft is taking off. Even at 30 m distance, the speed is still 400 km/h.



www.boldmethod.com/learn-to-fly/maneuvers/taxiing-behind-jet-blast/?fb_comment_id=2974279795962393_2988416497882056

(This is the expulsion speed for an engine at the beginning of the runway. In flight, the ejection speed is even greater.)

The effects of this very fast air can be seen in these videos.
https://www.youtube.com/watch?v=DFP4xl0V0mk
https://www.youtube.com/watch?v=GqVjD3nBSQg

Example 10.8.3

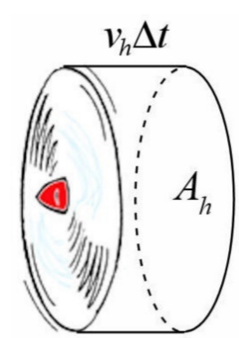
1350 kg of air per second pass in an Airbus A350-1000 engine. Air is expelled from the engine at a speed of 320 m/s. What is the magnitude of the thrust of the engine when the aircraft is stopped on the runway?

The force is

$$\begin{aligned} F_t &= \left(v_{\exp} - v\right) R \\ &= \left(320\tfrac{m}{s} - 0\tfrac{m}{s}\right) \cdot 1350\tfrac{kg}{s} \\ &= 432{,}000 N \end{aligned}$$

Alternative Version of the Thrust Formula

Another version of this equation can also be made for propeller engines and turbojet engines. To find this new formula, let's look at the air ejected by the propeller or fan during a time Δt. Simplifying a little, this air forms a cylinder (diagram).



The quantity of air that has passed through the propeller is equal to the air density multiplied by the volume of this cylinder.

$$m = \rho A_h v_h \Delta t$$

v_h is the speed of the air when it passes through the propeller and A_h is the area of the circle described by the propeller or fan when they rotate. ρ is the air density. This density is 1.29 kg/m³ at 0 °C and 101.3 kPa, and 1.20 kg/m³ at 20 °C and 101.3 kPa (it changes with temperature, pressure, and altitude).

Dividing this equation by time, the rate of passage of the mass R is obtained

$$R = \frac{m}{\Delta t} = \rho A_h v_h$$

It only remains to find the speed of the air when it passes through the propeller. It could be shown that the air that passes through a propeller or fan gains half of its speed before the propeller and the other half of its speed after its passage through the propeller. The speed of the air in the propeller is therefore equal to the average speed.

$$v_h = \frac{v_{\text{exp}} + v}{2}$$

Therefore, the result is

Quantity of Air That Passes Every Second in an Aircraft Engine

$$R = \tfrac{1}{2} \rho A_h \left(v_{\text{exp}} + v \right)$$

The thrust formula

$$F_t = \left(v_{\text{exp}} - v \right) R$$

thus becomes

$$\begin{aligned} F_t &= \left(v_{\text{exp}} - v \right) \rho A_h v_h \\ &= \left(v_{\text{exp}} - v \right) \rho A_h \frac{v_{\text{exp}} + v}{2} \\ &= \frac{1}{2} \left(v_{\text{exp}} - v \right) \left(v_{\text{exp}} + v \right) \rho A_h \end{aligned}$$

The final result is thus

Thrust Force of Aircraft Engines

$$F_t = \frac{1}{2} \left(v_{\text{exp}}^2 - v^2 \right) \rho A_h$$

Example 10.8.4

At take-off, each of Pratt and Whitney's two PW150A engines installed on the Bombardier Q-400 exerts a thrust force of 20,000 N. The propellers have a diameter of 4.1 m. (The density of the air is 1.2 kg/m³.)

www.youtube.com/watch?v=BCqjMhSzhFM

a) What is the expulsion speed of the gases?

The expulsion speed is found with

$$F_t = \frac{1}{2}\left(v_{\exp}^2 - v^2\right)\rho A_h$$

As the thrust depends on A_h, it is necessary to find the circle area described by the propellers as they rotate. Since the radius is 2.05 m, the area is

$$\begin{aligned} A_h &= \pi r^2 \\ &= \pi \cdot (2.05m)^2 \\ &= 13.20m^2 \end{aligned}$$

Therefore,

$$\begin{aligned} F_t &= \frac{1}{2}\left(v_{\exp}^2 - v^2\right)\rho A_h \\ 20,000N &= \frac{1}{2}\cdot\left(v_{\exp}^2 - \left(0\tfrac{m}{s}\right)^2\right)\cdot 1.20\tfrac{kg}{m^3}\cdot 13.2m^2 \\ v_{\exp} &= 50.3\tfrac{m}{s} \end{aligned}$$

b) How many kilograms of air are pushed by an engine every second?

The thrust force is also given by

$$F_t = \left(v_{\exp} - v\right)R$$

Therefore

$$\begin{aligned} 20,000N &= \left(50.3\tfrac{m}{s} - 0\tfrac{m}{s}\right)\cdot R \\ R &= 398\tfrac{kg}{s} \end{aligned}$$

The formula seems to indicate, however, that the Q-400 will not be able to take off since it needs a speed of 60 m/s to take off and the expulsion speed is only 50 m/s. However, the orientation of the propeller blades can be changed to change the expulsion speed. The faster the aircraft goes, the more the expulsion speed made by the propeller blades is increased by increasing the angle of attack of the blades.

SUMMARY OF EQUATIONS

Impulse Given to an Object

$$\vec{I} = \vec{F}\Delta t$$

In components:

$$I_x = F_x\Delta t \qquad I_y = F_y\Delta t \qquad I_z = F_z\Delta t$$

Net Impulse

$$\vec{I}_{nette} = \sum \vec{I}$$

In components:

$$I_{x\,nette} = \sum I_x \qquad I_{y\,nette} = \sum I_y \qquad I_{z\,nette} = \sum I_z$$

Impulse Given by a Variable Force Acting on an Object

$$\vec{I} = \sum_{F\ \text{constant}} \vec{F}\Delta t$$

In components:

$$I_x = \sum_{\text{F constant}} F_x\Delta t \qquad I_y = \sum_{\text{F constant}} F_y\Delta t \qquad I_z = \sum_{\text{F constant}} F_z\Delta t$$

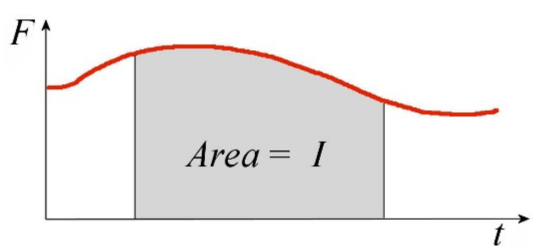
Impulse Given by a Variable Force on an Object (Most General Formula)

$$\vec{I} = \int_t^{t'} \vec{F}dt$$

In components:

$$I_x = \int_t^{t'} F_x dt \qquad I_y = \int_t^{t'} F_y dt \qquad I_z = \int_t^{t'} F_z dt$$

The impulse given to an object is the area under the curve of the force acting on the object as a function of time



Momentum

$$\vec{p} = m\vec{v}$$

In components:

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z$$

Impulse-Momentum Theorem

$$\vec{I}_{net} = \Delta\vec{p}$$

In components:

$$I_{x\,net} = \Delta p_x \qquad I_{y\,net} = \Delta p_y \qquad I_{z\,net} = \Delta p_z$$

Relationship between Force and Momentum (Constant Force)

$$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$$

Newton's Second Law

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

Average Force Acting on an Object

$$\vec{\bar{F}} = \frac{\Delta\vec{p}}{\Delta t}$$

In components:

$$\bar{F}_x = \frac{\Delta p_x}{\Delta t} \qquad \bar{F}_y = \frac{\Delta p_y}{\Delta t} \qquad \bar{F}_z = \frac{\Delta p_z}{\Delta t}$$

Momentum Conservation Law

$$\vec{p}_{tot} = \vec{p}\,'_{tot} \qquad \text{if} \qquad \sum \vec{F}_{ext} = 0$$

In components:

$$p_{x\,tot} = p'_{x\,tot} \qquad \text{if} \qquad \sum_{system} F_{x\,ext} = 0$$

$$p_{y\,tot} = p'_{y\,tot} \qquad \text{if} \qquad \sum_{system} F_{y\,ext} = 0$$

$$p_{z\,tot} = p'_{z\,tot} \qquad \text{if} \qquad \sum_{system} F_{z\,ext} = 0$$

Momentum and Collisions

In a collision, the total momentum of the system is conserved.

$$\vec{p}_{tot} = \vec{p}'_{tot}$$

In components:

$$p_{tot\ x} = p'_{tot\ x} \qquad p_{tot\ y} = p'_{tot\ y} \qquad p_{tot\ z} = p'_{tot\ z}$$

Elastic Collision

In an elastic collision, the kinetic energy of the system is conserved.

$$E_{k\ tot} = E'_{k\ tot}$$

Relationship between Momentum and Kinetic Energy

$$E_k = \frac{p^2}{2m}$$

Impulse-Momentum Theorem for a System

$$\vec{p}_{tot} + \vec{I}_{ext\ net} = \vec{p}'_{tot}$$

In components:

$$p_{tot\ x} + I_{net\ ext\ x} = p'_{tot\ x} \qquad p_{tot\ y} + I_{net\ ext\ y} = p'_{tot\ y} \qquad p_{tot\ z} + I_{net\ ext\ z} = p'_{tot\ z}$$

where $I_{net\ ext}$ are net impulses made by external forces.

Thrust Force of Rocket Engines

$$F_{thrust} = v_{\exp} R$$

Speed of a Rocket Expelling Gas during Time *T*

$$v' = v + v_{\exp} \ln \frac{m}{m - RT}$$

$$v' = v + v_{\exp} \ln \frac{m}{m'}$$

Thrust Force of Aircraft Engines

$$F_t = \left(v_{\exp} - v\right) R$$

$$F_t = \frac{1}{2}\left(v_{\exp}^2 - v^2\right) \rho A_h$$

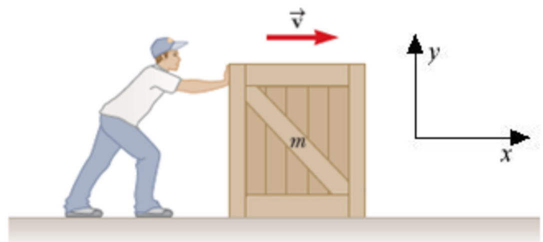
Quantity of Air That Passes Every Second in an Aircraft Engine

$$R = \tfrac{1}{2}\rho A_h \left(v_{\exp} + v\right)$$

EXERCISES

10.1 Impulse

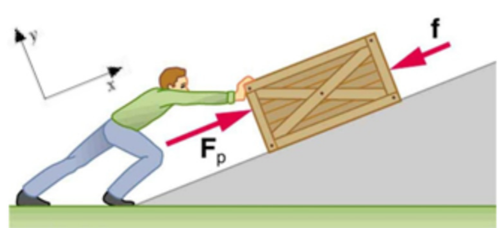
1. Kareem pushes a 120 kg box with an 800 N horizontal force towards the right for 10 seconds. The coefficient of friction between the ground and the box is of 0.3.



www.chegg.com/homework-help/questions-and-answers/man-pushing-crate-mass-m-920-kg-speed-v-0870-m-s-encounters-rough-horizontal-surface-lengt-q3150472

a) What are the components of the impulse given by the gravitational force?
b) What are the components of the impulse given by the normal force?
c) What are the components of the impulse given by the friction force?
d) What are the components of the impulse given by the force exerted by Kareem?

2. Gilbert pushes a 30 kg crate uphill on a 20° slope for 20 seconds. The crate has an acceleration of 1 m/s² towards the top of the slope, and there is a frictional force of F_f = 70 N opposed to the motion of the crate.



cnx.org/content/m42150/latest/?collection=col11406/latest

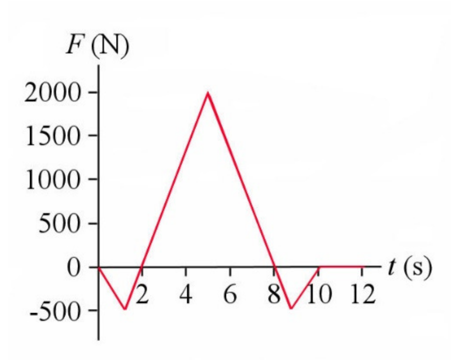
a) What are the components of the impulse given by the gravitational force?
b) What are the components of the impulse given by the friction force?
c) What are the components of the impulse given by the force exerted by Gilbert?
d) What are the components of the impulse given by the normal force?
e) What are the components of the net impulse?

3. The following constant force acts on an object, one after the other.

$$\vec{F}_1 = \left(2\vec{i} + \vec{j} - 4\vec{k}\right)N \text{ is exerted on an object for 3 seconds.}$$
$$\vec{F}_2 = \left(-4\vec{i} + 5\vec{j} + 2\vec{k}\right)N \text{ is exerted on the object for 5 seconds.}$$

What are the 3 components of the impulse given to the object during these 8 seconds?

4. Here is the graph of the x-component of a force acting on an object as a function of time. What is the x-component of the impulse given to the object between t = 0 s and t = 8 s?



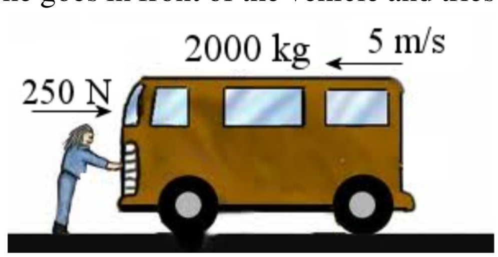
5. The force

$$F_x = 9\tfrac{N}{s^2} \cdot t^2$$

acts on an object. What is the x-component of the impulse given to the object between t = 0 s and t = 5 s?

10.2 Impulse-Momentum Theorem

6. Carole had parked her 2000 kg Westphalia on a slope but the brakes failed. Thus, later, she sees her Westphalia rolling by itself at 5 m/s on a horizontal portion of the road. She then runs to stop her vehicle. She goes in front of the vehicle and tries to stop it by exerting a 250 N force. Using the formula $I_{x\,net}=\Delta p_x$, determine the speed of the Westphalia after Carole has exerted the force for 20 seconds.



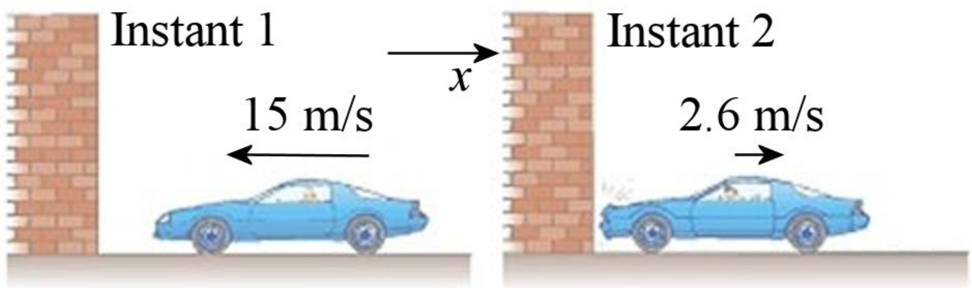
c.codmanacademy.org/branches/physicsofdriving2/index.php?module=pagemaster&PAGE_user_op=view_page&PAGE_id=3&MMN_position=3:3

10.3 A New Version of Newton's Second Law

7. At an archery meeting, Gisele launches a 100 g arrow at 150 m/s. What was the magnitude of the average force on the arrow if the force is exerted for 0.05 seconds on the arrow?

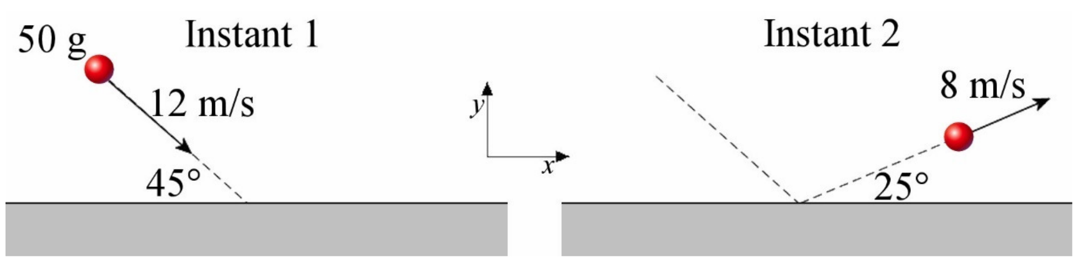
8. A 20 g rifle bullet moving at 900 m/s is stopped by a wooden block. If the bullet stops in 0.004 seconds, what is the magnitude of the average force on the bullet?

9. Justin's car drove into a wall and bounces back a little as shown in the diagram. What was the average force acting on the wall if the mass of the car is 1150 kg and the collision lasted 0.10 s?

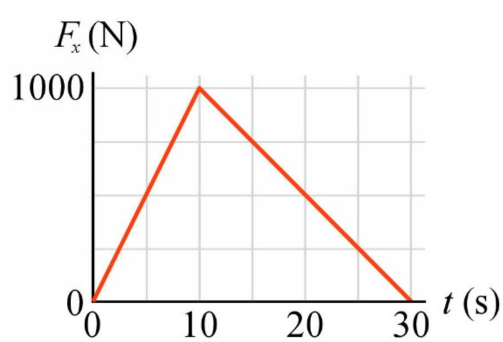


www.kshitij-school.com/Study-Material/Class-11/Physics/Linear-momentum-and-collisions/Impulse-momentum-theorem.aspx

10. A 50 g ball bounces off the ground. The following diagram shows the velocities immediately before and immediately after the collision with the ground. What is the average force (magnitude and direction) exerted on the ball during the collision if it lasts 0.06 s?

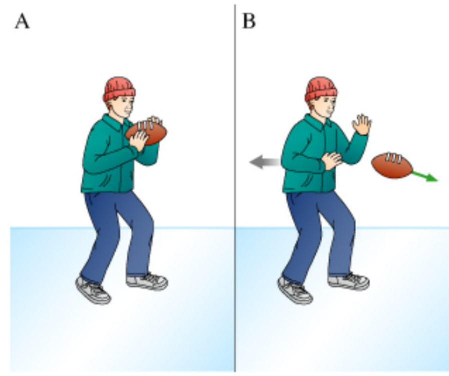


11. Here's a force-versus-time graph of the force acting on an object. What is the average force exerted on the object between $t = 0$ sec and $t = 30$ s?



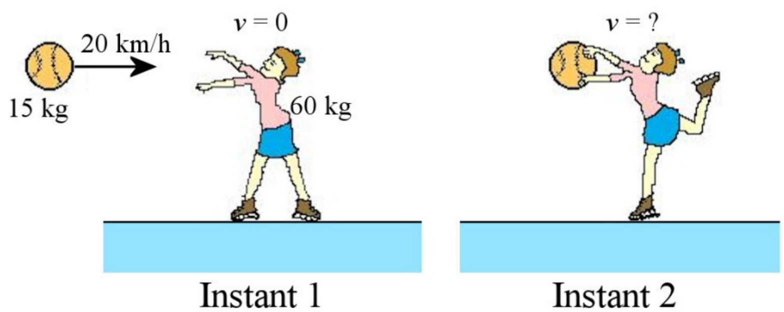
10.4 Momentum Conservation

12. Edward holds a ball while he is at rest on ice (image A). There is no friction between the ice and Edward's boots. Edward then launches the ball with a speed of 20 m/s (image B). What is Edward's velocity after he threw the ball if Edward's mass is 65 kg, and the mass of the ball is 800 g?



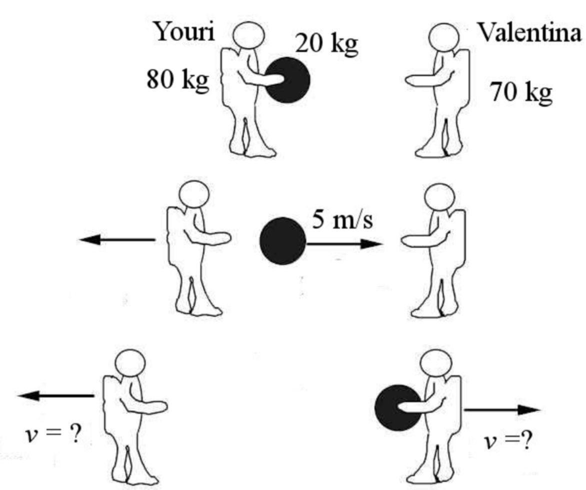
www.masteringphysicssolutions.net/mastering-physics-solutions-catching-a-ball-on-ice/

13. A ball is heading at 20 km/h towards Marie-Sophie, who is at rest (image A). Marie-Sophie is on an icy surface that offers no friction. What is the velocity of Marie-Sophie once she caught the ball (picture B)?



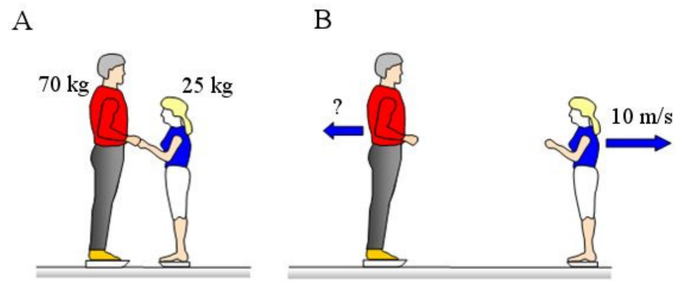
www.physicsclassroom.com/class/momentum/u4l2d.cfm

14. Two astronauts in space far from any planet are initially at rest. Yuri (80 kg) throws a 20 kg canister towards Valentina (70 kg) at a speed of 5 m/s. What are the velocities of the two astronauts once Valentina has grabbed the canister?



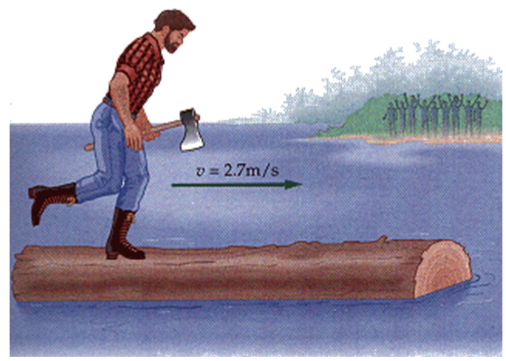
public.wsu.edu/~jtd/Physics205/Chap7/CHAP7_2.htm

15. Helmut and Brünnhilde are both at rest on an icy surface offering no friction (image A). Helmut then pushes Brünnhilde, giving her a speed of 10 m/s (image B). What is the velocity of Helmut?



www.schoolphysics.co.uk/age16-19/Mechanics/Dynamics/text/Conservation_of_momentum%20_and_skaters/index.html

16. McPherson is initially at rest at the end of a floating log also at rest. McPherson then starts walking towards the other end of the log with a speed of 2.7 m/s. How long does it take for McPherson to get to the other end of the log if the mass of McPherson is 94 kg, the mass of the log is 345 kg, and the length of the log is 5 m?

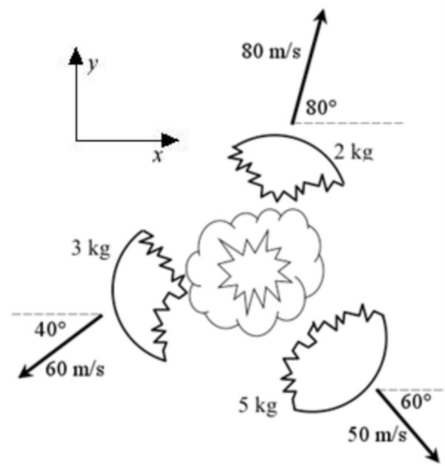


www.chegg.com/homework-help/questions-and-answers/94-kg-lumberjack-stands-endof-345-kg-floating-log-shown-infigure-9-17-log-lumberjack-rest--q388258

17. The HMS Leander was a British warship active between 1780 and 1813 (although she was briefly captured by France for a few months in 1798 and 1799). She was not a huge warship as she was ranked as a fourth-rate vessel (like the ship in the diagram) but there were still 44 guns on board. During a side-by-side fight with another ship, only the guns on one side of the vessel were used. These guns were shooting 22 cannonballs, whose total mass was 180 kg, at the speed of 425 m/s. What was the Leander recoil speed when all 22 guns were fired simultaneously if the mass of the ship was 1,200 tons (which does not include the cannonballs that were fired)?

www.st-george-squadron.com/sgs/wiki/index.php?title=HMS_Golden_Eagle

18. A 10 kg cannonball explodes into three fragments. The diagram shows the speed and direction of the velocity of the three fragments after the explosion. What was the speed and direction of the velocity of the cannonball before the explosion?

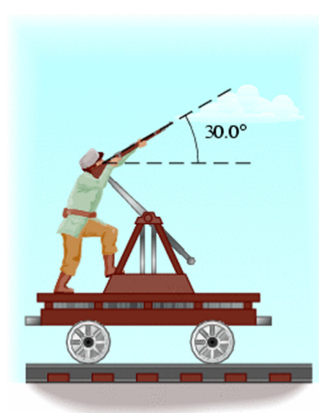


www.zahniser.net/~russell/physics06/index.php?title=Practice%20Momentum%20Test

19. In a nuclear alpha decay, the atomic nucleus ejects an alpha particle (which is composed of two neutrons and two protons) while releasing energy. The released energy is transferred into the kinetic energies of the two particles (the nucleus and the alpha particle) after the disintegration. What are the velocities of the two particles after the disintegration knowing that the energy released is 1.3×10^{-13} J and that, after the disintegration, the masses are 6.64×10^{-27} kg (alpha particle) and 388.6×10^{-27} kg (nucleus)?

cnx.org/content/m42633/latest/?collection=col11406/latest

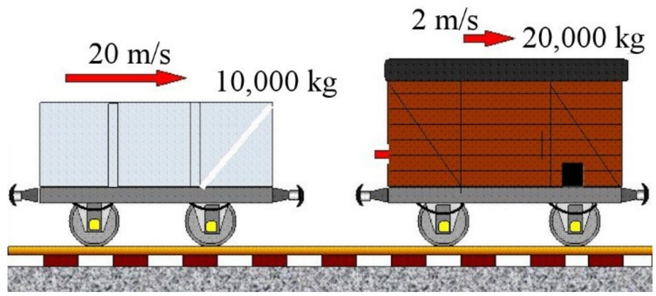
20. Leon fires the rifle shown in the diagram. Initially, the carriage is at rest. The carriage rolls without friction on the track. The bullet has a mass of 30 g and a speed of 900 m/s. What is the recoil velocity of the carriage if the mass of the carriage (including everything on it) is 150 kg?



www.physicsforums.com/showthread.php?t=140750

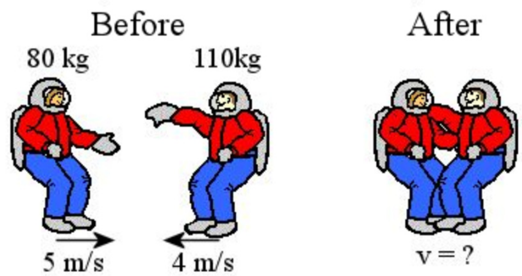
10.5 Collisions

21. The two train cars of the diagram are involved in a completely inelastic collision. What is the velocity of the cars after the collision?



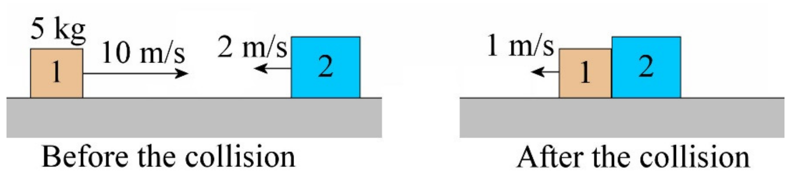
antonine-education.co.uk/Pages/Physics_GCSE/Unit_2/Add_07_Momentum/add_page_07.htm

22. Fabrice and Raphael are two astronauts heading towards each other with the velocities shown in the diagram. When they meet, they cling to each other. What is their velocity after they have clung to each other?

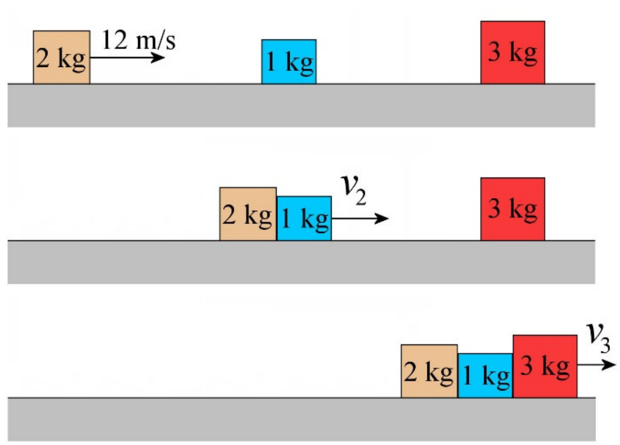


www.physicsclassroom.com/class/momentum/u4l2dd.cfm

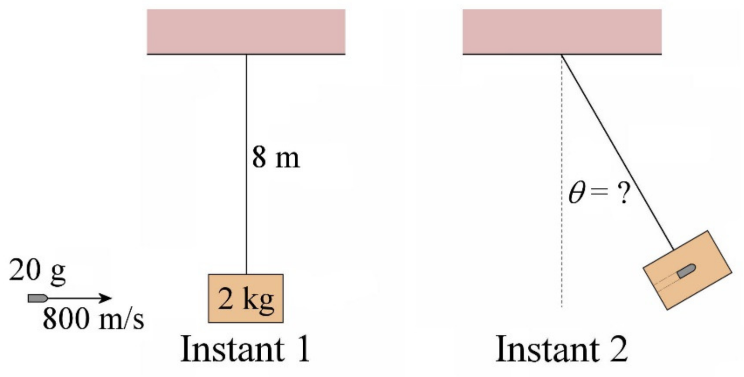
23. A 5 kg block (block 1) moving at 10 m/s collides with another block (block 2) moving at 2 m/s. The speeds and the directions of the velocity of the two blocks are shown in the diagram. After a completely inelastic collision, the blocks have a speed of 1 m/s towards the left. What is the mass of block 2?



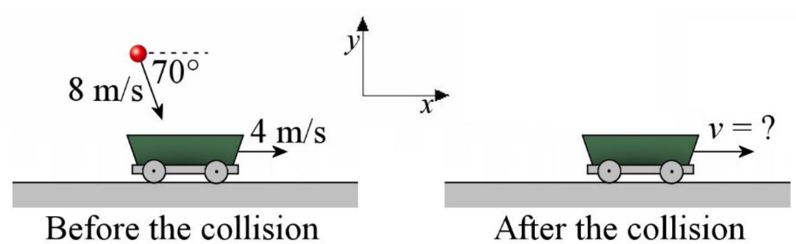
24. In the situation shown in the diagram, find the speed v_2 and v_3 if the blocks are involved in two completely inelastic collisions.



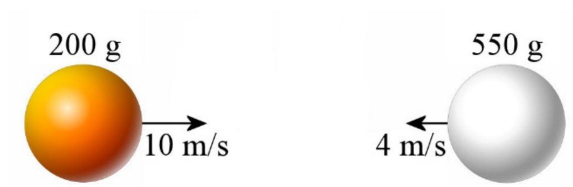
25. In the following situation, what is the maximum angle reached by the pendulum after the collision?



26. With the conservation of the x-component of momentum, determine the velocity of the carriage after the collision with the ball if the ball has a mass of 10 kg and the carriage has a mass of 200 kg. (This is a completely inelastic collision.)

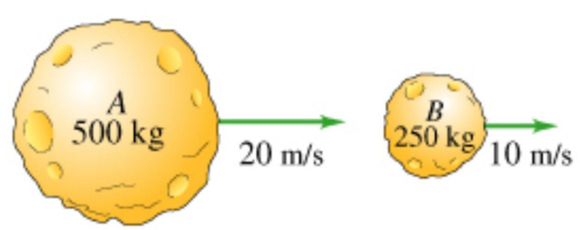


27. What are the velocities of these balls after an elastic collision?



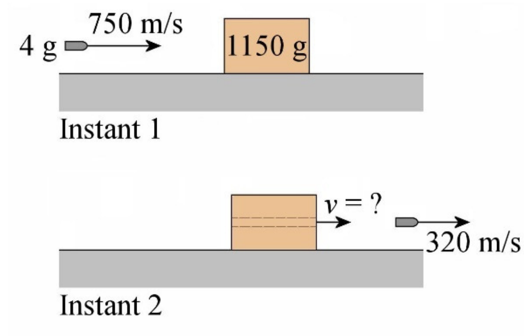
28. The two asteroids shown in the diagram are involved in a collision (it is not known whether the collision is elastic or inelastic). After the collision, the 250 kg asteroid has a speed of 12 m/s.

a) What is the speed of the 500 kg asteroid after the collision?
b) What fraction of the kinetic energy is lost in the collision?
c) What is the change in momentum of the 500 kg asteroid?
d) What is the change in momentum of the 250 kg asteroid?

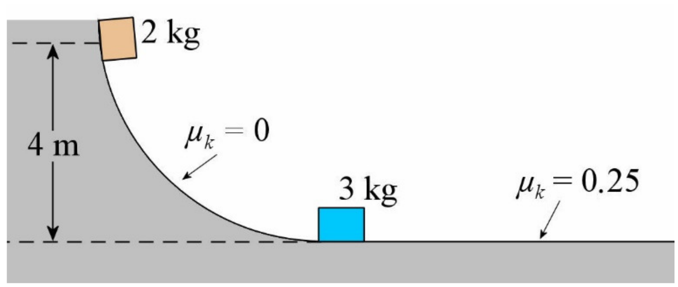


www.chegg.com/homework-help/questions-and-answers/asteroids-drifting-constant-velocity-collide-masses-velocities-asteroids-collision-indicat-q3363246

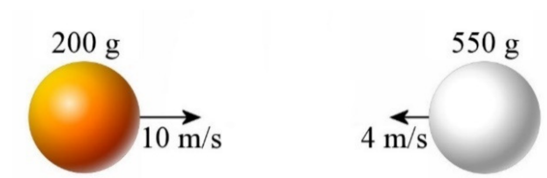
29. A 4 g bullet moves with a speed of 750 m/s. It then hits an 1150 g wooden block. However, the bullet comes out on the other side the block with a speed of 320 m/s. What is the speed of the wooden block after the bullet has passed through it?



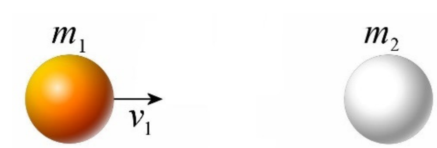
30. In the situation shown in the diagram, the 2 kg block slides on a frictionless surface and then makes a completely inelastic collision with a 3 kg block. The two blocks then slide on a horizontal surface. However, there is some friction between the blocks and the horizontal surface. What will the stopping distance of the blocks be if the friction coefficient is 0.25?



31. What are the velocities of these balls after a collision if 40% of the kinetic energy is lost?



32. In this elastic collision, show that the speeds after the collision are given by



$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 \qquad v_2' = \frac{2m_1}{m_1 + m_2} v_1$$

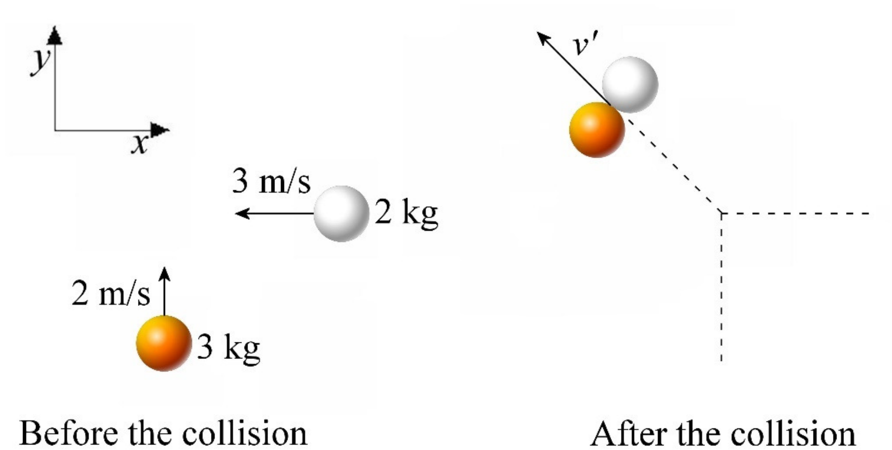
33. In this elastic collision, show that the speeds after the collision are given by

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 \qquad v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

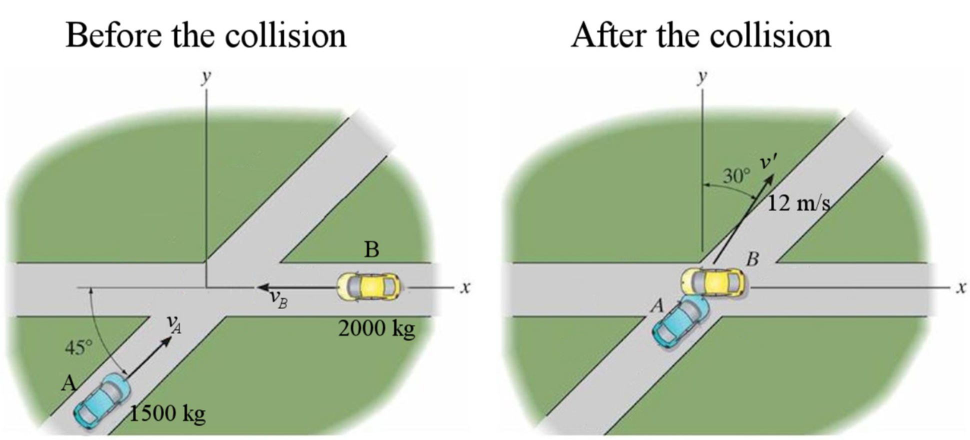
34. In this completely inelastic collision, show that the kinetic energy lost in the collision is given by

$$\Delta E_k = -\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2} v_1^2$$

35. What is the velocity (magnitude and direction) of these balls after they make a completely inelastic collision?

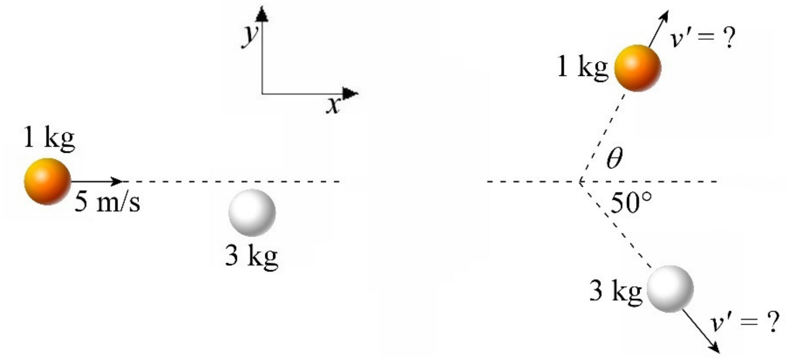


36. Here's a completely inelastic collision between vehicle A, whose mass is 1500 kg, and vehicle B, whose mass is 2000 kg. After the collision, both vehicles stick together and have a speed of 12 m/s in the direction indicated in the diagram (v'). What was the speed of each car before the collision?



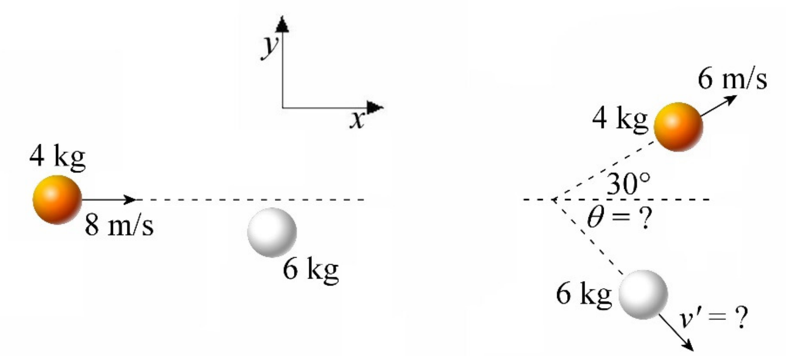
1285500059.reader.chegg.com/homework-help/questions-and-answers/day-driving-m_a-1500-kg-car-going-northeastyou-embarrassingly-smash-instructor-rsquo-s-m_b-q2407067

37. In the following elastic collision, determine the speed of the 3 kg ball and the velocity (magnitude and direction) of the 1 kg ball.



Before the collision After the collision

38. Here's a collision (it is not known whether the collision is elastic or inelastic).



Before the collision After the collision

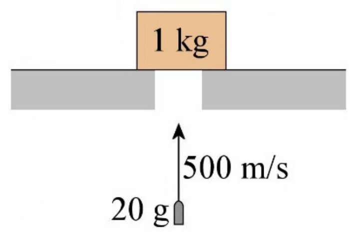
a) What is the velocity of ball 2 (magnitude and direction) after the collision?
b) How much kinetic energy is lost during this collision?
c) Is the collision elastic or inelastic?

10.6 Impulse-Momentum Theorem for a System

39. In the situation shown in the diagram, a 50 N force continuously pushes the 10 kg block which is initially at rest. One second later, the block makes a completely inelastic collision with a 20 kg block. What is the velocity of the blocks 2 seconds after the collision? (There is no friction between the blocks and the ground.)

$t = 0$ s $t = 3$ s

40. A 20 g bullet hits a 1 kg block with a speed of 500 m/s as shown in the diagram. After the collision, the ball is lodged in the block. What is the velocity of the block 0.5 second after the collision?



10.8 Rocket and Aircraft Propulsion

41. The rocket Ariane 5 ejects gas at a speed of 3200 m/s and at the rate of 3400 kg/s. The initial mass of the rocket is 710,000 kg, including fuel.

a) What is the thrust force of the engine?
b) Assume that this rocket is initially at rest in space. What will its speed be if the engine is fired for a minute?
c) Now assume the rocket is fired upwards from the surface of the Earth. What will the speed of the rocket be 30 seconds after the engine was fired?

42. When the engines of a 100-ton rocket are fired for 60 seconds, the speed of the rocket in space changes from 15 km/s to 18 km/s. What is the gas ejection speed if 0.75 tons of fuel is ejected per second?

43. Each of the engines of an Airbus A350 can exert a thrust of 431,000 N on take-off. At the beginning of the take-off (when the Airbus has a very low speed), how many kilograms of air must pass per second through each engine if the gas expulsion speed is 600 km/h?

44. In mid-flight at 828 km/h, each engine of an Airbus A350 exerts a thrust of 200,000 N. How many kilograms of air must pass per second through each engine if the gas expulsion speed is 936 km/h?

45. On take-off, the engine of a Cessna 182 propels the air at a speed of 50 m/s. The propeller has a diameter of 2 m. What is the magnitude of the thrust force at the beginning of the take-off (when the plane is almost stopped on the runway)? (The density of air is 1.2 kg/m³.)

46. In flight at 270 km/h at an altitude of 3000 m (the density of the air is 0.9 kg/m³.), the air expelled by an engine with a propeller having a diameter of 3 m has a speed of 288 km/h. What is the magnitude of the thrust force exerted by this engine?

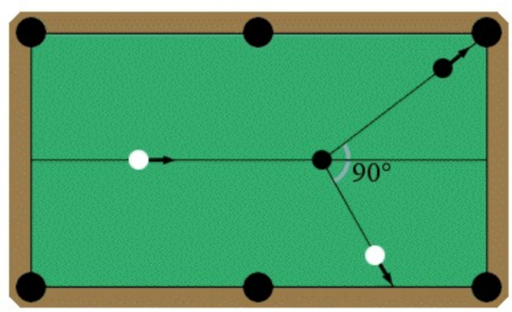
47. In flight at 270 km/h, the engine of a Cessna exerts a thrust of 500 N. The propeller has a diameter of 2 m. How fast is the air expelled when the aircraft is flying at an altitude of 3000 m? (The density of the air is 0.9 kg/m³.)

Challenges

(Questions more difficult than the exam questions.)

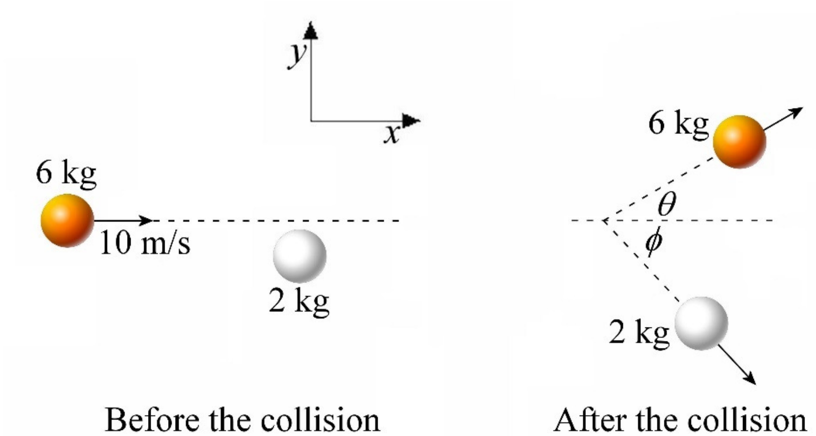
48. In the completely elastic collision shown in the diagram, what will the maximum spring compression be if there is no friction?

49. Show that in a completely elastic collision between two balls the same mass such as the one shown in the diagram, the angle between the trajectories of the 2 balls after the collision is always 90°. (Before the collision, the white ball has some speed, and the black ball is at rest.)



www.sparknotes.com/physics/linearmomentum/collisions/problems_1.html

50. In a completely elastic collision between two balls such as the one shown in the diagram, what is the maximum value that θ can have?



ANSWERS

10.1 Impulse

1. a) I_x = 0 kgm/s I_y = -11,760 kgm/s
   b) I_x = 0 kgm/s I_y = 11,760 kgm/s
   c) I_x = -3528 kgm/s I_y = 0 kgm/s
   d) I_x = 8000 kgm/s I_y = 0 kgm/s
2. a) I_x = -2011.1 kgm/s I_y = -5525.4 kgm/s
   b) I_x = -1400 kgm/s I_y = 0 kgm/s
   c) I_x = 4011.1 kgm/s I_y = 0 kgm/s
   d) I_x = 0 kgm/s I_y = 5525.4 kgm/s
   e) I_x = 600 kgm/s I_y = 0 kgm/s
3. I_x = -14 kgm/s I_y = 28 kgm/s I_z = -2 kgm/s
4. 5500 kgm/s
5. 375 kgm/s

10.2 Impulse-Momentum Theorem

6. 2.5 m/s towards the left

10.3 A New Version of Newton's Second Law

7. 300 N
8. 4500 N
9. -202,400 N (Thus 202,400 N towards the left)
10. 9.942 N at 95.94°
11. 500 N

10.4 Momentum Conservation

12. 0.246 m/s in the direction opposite to the velocity of the ball
13. 4 km/h towards the right
14. Yuri: 1.25 m/s towards the left Valentina: 1.111 m/s towards the right
15. 3.571 m/s towards the left
16. 1.455 s
17. 0.06375 m/s in the direction opposite to the velocity of the cannonballs
18. 17.53 m/s at -85.1°
19. Nucleus: 1.06×10^5 m/s towards the left
    Alpha particle: 6.20×10^6 m/s towards the right
20. 0.1559 m/s towards the left

10.5 Collisions

21. 8 m/s towards the right
22. 0.2105 m/s towards the left
23. 55 kg
24. v_2 = 8 m/s v_3 = 4 m/s
25. 53.1°
26. 3.94 m/s towards the right
27. 550 g ball: 3.467 m/s towards the right
    200 g ball: 10.53 m/s towards the left
28. a) 19 m/s b) 0.0378 c) -500 kgm/s d) 500 kgm/s
29. 1.496 m/s towards the right
30. 2.56 m
31. 550 g ball: 2.623 m/s towards the right
    200 g ball: 8.214 m/s towards the left
32. See the proof in the chapter 10 solutions
33. See the proof in the chapter 10 solutions
34. See the proof in the chapter 10 solutions
35. 1.697 m/s angle with the positive x-axis = 135°
36. Car A: 34.29 m/s Car B: 7.69 m/s
37. v'_1 = 4.154 m/s at 62.8° v'_2 = 1.607 m/s
38. a) 2.738 m/s at -46.9° b) 33,52 J lost c) Inelastic

10.6 Impulse-Momentum Theorem for a System

39. 5 m/s towards the right
40. 4.904 m/s upwards

10.8 Rocket and Aircraft Propulsion

41. a) 1.088×10^7 N b) 1084 m/s c) 202.3 m/s
42. 5018 m/s
43. 2586 kg/s
44. 6667 kg/s
45. 4712 N
46. 2465 N
47. 77.3 m/s

Challenges

48. 25 cm
49. See the proof in the chapter 10 solutions
50. 19.47°